Fundamentals of Acoustics
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Lecture – 12
Important Mathematical Concepts-Bode Plot for Simple Zero

Hello, welcome to Fundamentals of Acoustics. Today is the last day of the second week of this course and what we plan to do today is continue the discussion on Bode plots specifically what we will do today is we will start by developing a Bode plot for magnitude for a simple zero.

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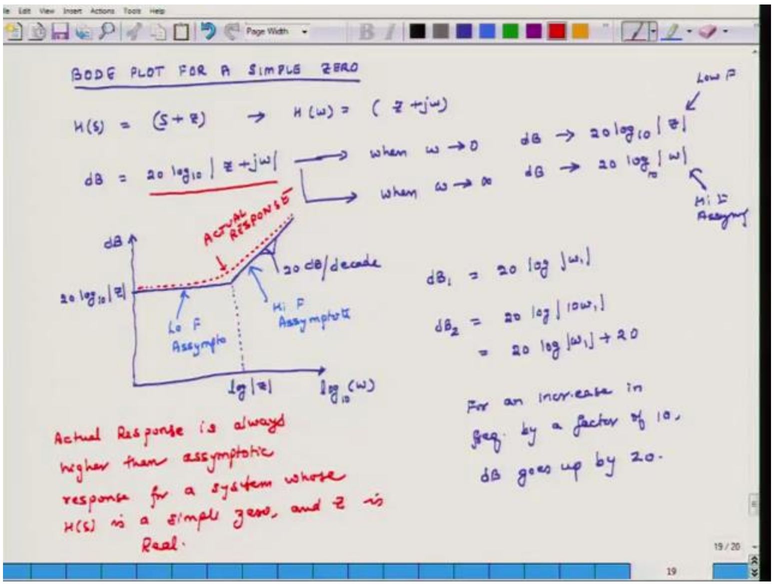

Bode plot for a simple zero. So, in this case, the transfer function H of S is the simple zero that is S plus z. Now we know that when we construct Bode plots, they are in context when the complex frequency equals j omega. So, H of omega equals z plus j omega, in decibels I can express it as d B equals 20 log 10 z plus j omega.

Now, this decibel value of the amplitude or the magnitude of H, what we will do is again as I said we have asymptotic representations of H. So, we will consider 2 scenarios, the first scenario is when omega approaches 0 then d B approaches 20 log of 10 z and when omega approaches infinity then the value of decibel approaches 20 log of omega. So, all both of these are base 10 units. So, this will give me my low frequency asymptote and the second relation will give me my high frequency asymptote. So, now, I will plot the asymptotic response for a simple zero. So, I have on the x axis by plot log of omega to the base 10 and on the y axis I have my low frequency asymptote which is this and the low frequency asymptote is a straight line and its value is. So, this is in decibels its value is 20 log of 10 of z.

And then the high frequency asymptote we see is 20 log of 10 of omega log, log of omega on base 10. So, what we note is that as omega increases the value of decibels increase and it is a straight line on a log axis. So, the curve is something like this, the next thing we have to figure out is what is the point of intersection for the high and the low frequency asymptotes? So, at the point of intersection, the value of high frequency asymptote is same as value of low frequency asymptote and that will happen when omega equals z. So, the corresponding position on the log axis is log of z or z, the next thing we want to know is; what is the slope of this curve? So, we find out the slope let us say d B 1 equals 20 log of omega one d B 2 is 20 and suppose I increase my angular of frequency by a factor of 10 then it is 20 log of 10 omega 1.

This is equal to 20 log of omega 1 plus 20, which means that for an increase in frequency by a factor of 10 d B goes up by 20, I increase the angular of frequency by 10 x d B went up by 20 which means that the slope of this line is 20 decibels per decade actually I am going to write it at different location. So, this slope is 20 d B per decade 20 decibels per decade and finally, we would like to see the actual response curve. So, the purple line is the asymptotic response this is the low frequency asymptote. This is the high frequency asymptote and the actual curve actual response is if we plot this actual equation. So, we see that for any value of omega z plus j omega its absolute magnitude is going to be higher.

When I plot this that is assuming z is real. So, when I plot it, this is my actual response. So, for a simple zero actual response is always higher then asymptotic response for a system whose H of S is a simple 0 and z is real z is real. So, what we have seen is 2 Bode plots in last class, we develop the Bode plot for a simple pole the low frequency response was a straight line, horizontal line and the high frequency was a negative list probe lines slope was negative 20 decibels per decade for a simple zero, the magnitude plot kind of looks like its mirror image the high frequency response is a positive list probe line low frequency response is horizontal line which has zero slope.

These are the magnitude plots for a simple zero and a simple pole, the next thing we were going to do is we will develop their phase plots we will develop their phase plots.

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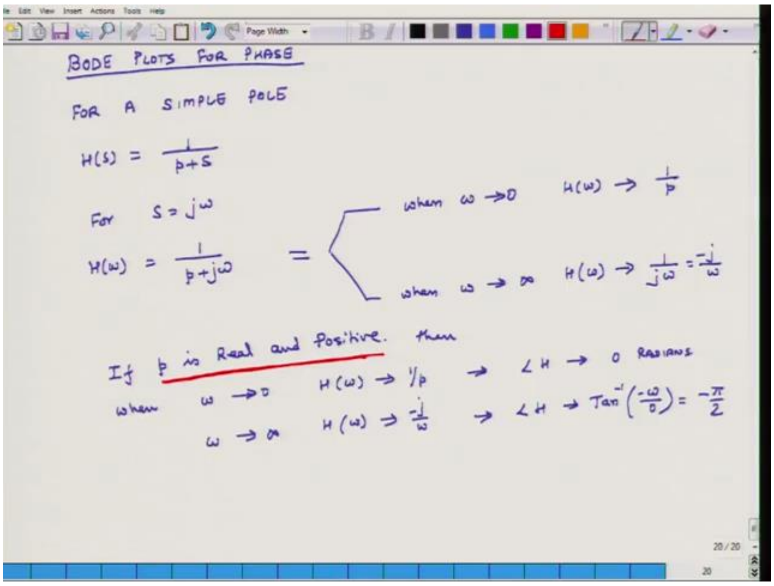

Bode plots for phase. So, the first thing we will do is for a simple zero or for we will start with for S simple pole. So, H of S for a simple pole, the transfer function is 1 over p plus S. Now in the phase plot, we do not take log and 20 all we have to do is compute the phase under 2 conditions, when omega is going to zero and when omega is going to infinity. So this thing is equal to first thing is we have to do is that we do all these discussion on Bode plots when S equals j omega. So, for S equals j omega H of omega equals 1 by p plus j omega and this can take 2 asymptotic values when omega approaches 0 then H of omega equals 1 over p.

And when omega approaches infinity then H of omega becomes actually they should not be equal, this is in approaching value. So, this approaches 1 over p in this case, it H of omega approaches 1 over j omega or it becomes j over omega minus j over omega. Now here we make an assumption that if p is real and positive, if p is real and positive then when omega approaches 0. We know that H of omega approaches 1 over p which means phase of H, it approaches 0 degree, 0 radiance, it approaches 0 radiance, why because the phase is imaginary component divided by the real component, the real component is positive, the imaginary component is 0. So, in the numerator I have 0 in the denominator, I have a positive number and in this phase is 0 radiance.

When omega approaches infinity H of omega approaches minus j over omega. So, what is this? This number is basically pure imaginary and it is negative. So, which means that phase of H is approaching tan inverse of negative omega divided by 0. So, this is equal to minus pi over 2, but remember this is true only if p is real and positive, if p is complex then these things per change we did not have this condition and context of the magnitude plots, but in context of phase plots that value of p whether its real or imaginary and whether it s positive and negative, it has a role.

Now with this understanding, when we develop a phase plot for a simple zero or a simple pole it will look like this.

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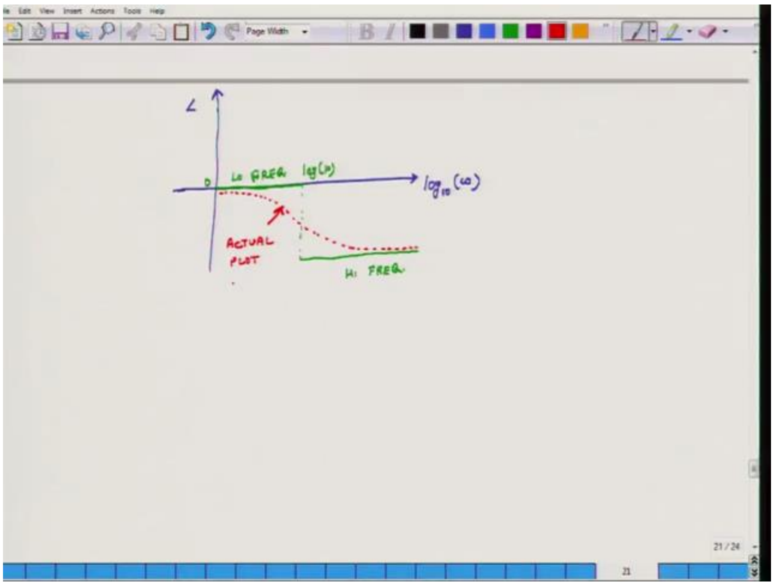

On the x axis, I am going to plot log of omega and once again it will have 2 asymptotes, low frequency and high frequency asymptote and here on the y axis I am going to plot phase in radiance. So, the low frequency asymptote has constant value 0 radiance. So, this is my low frequency asymptote. So, this is 0 and the high frequency asymptote has a value of negative pi over 2 radiances that is my high frequency asymptote. In this low frequency, the cross over point for these 2 will occur when 1 over p equals 1 over omega.

This is here log of p and if you have to plot the actual phase then you have to plot the phase of this function not the asymptotic function. So, in this case, the plot will be tan inverse of omega divided by p negative of that because there is a j in the denominator. So, if that is the case and if you do that plot then the actual phase plot will look something like this. So, this is the actual phase plot and the green line is the asymptotic plot next we will do the phase plot for a simple 0. So, earlier we had done a phase plot for a simple pole now we are going to do a phase plot for a simple 0.

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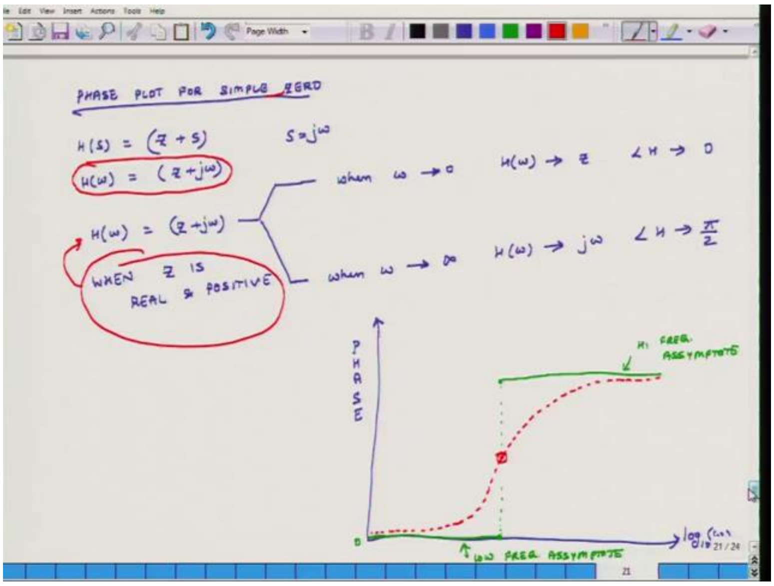

In this case, H of S is z plus S. So, simple 0 and when we are doing Bode plots S equals j omega, H of omega equals z plus j omega. Now H of omega equals z plus j of omega j omega in an asymptotic sense, it can have 2 values 2 functions. So, when omega approaches 0, H of omega approaches z and phase of H approaches zero radiance because omega is tending to 0. So, it has only real component imaginary by. So, this is for the case. So, this is when z is real and positive. So, this is the condition when z is real and then positive then we can make this conclusion when omega approaches a very large number infinity then H of omega approaches j omega and phase of H omega is positive number this approaches pi over 2 radiance.

In this case, when I do a plot for phase then this is my phase in radiance, the low frequency asymptote is a horizontal line and the high frequency asymptote is also horizontal line. So, that is my high frequency asymptote, this is my low frequency asymptote and my 0 is here and the actual response, if I have to plot have to plot the phase of this function assuming z is positive. So, when that is the case and if I plot it, the actual response will look something like this. So, this is a cross over point. So, that is the phase plot for a simple 0. So, what we have discussed till so for is how to construct Bode plots for simple zeros and simple poles.

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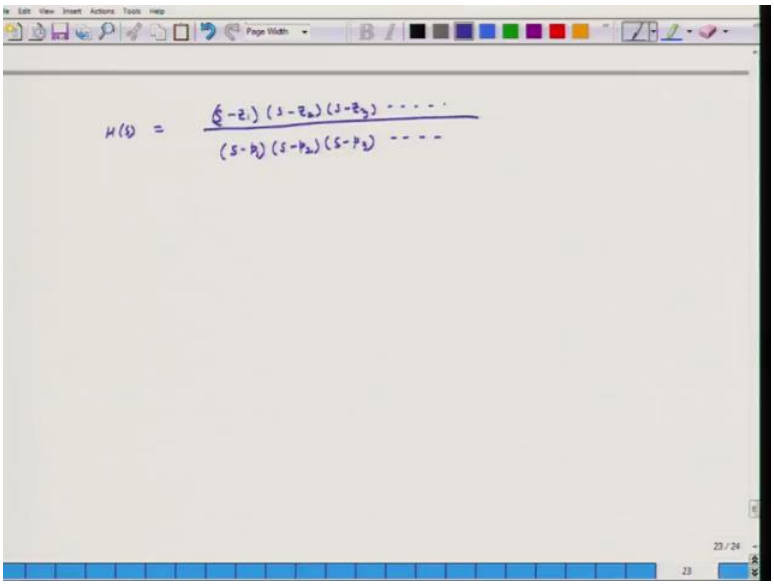

But real transfer functions may not necessarily be a simple zero, simple plot in general it can be represented as a multiple of zeros divided by a function which has several poles. So, now, based on the discussions over large to lectures, we have learnt how to construct phase and magnitude Bode plots for simple zeros and simple poles. So, what we will do next, which is in the next week will be how to construct phase plots for real functions which have combinations of these zeros and poles. So, that concludes the discussion for this week and please checks your assignments and I hope you to do good in your assignments and I look forward to seeing you next week on Monday.

Thank you and have a nice weekend, bye.