Fundamentals of Acoustics
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Lecture – 11
Important Mathematical Concepts-Bode Plot for Simple Pole

Hello, welcome to Fundamentals of Acoustics. Today is the fifth day of the second week of this course, today and tomorrow we will be discussing all about Bode plots. So, what are Bode plots? Well there are 2 types of Bode plots.

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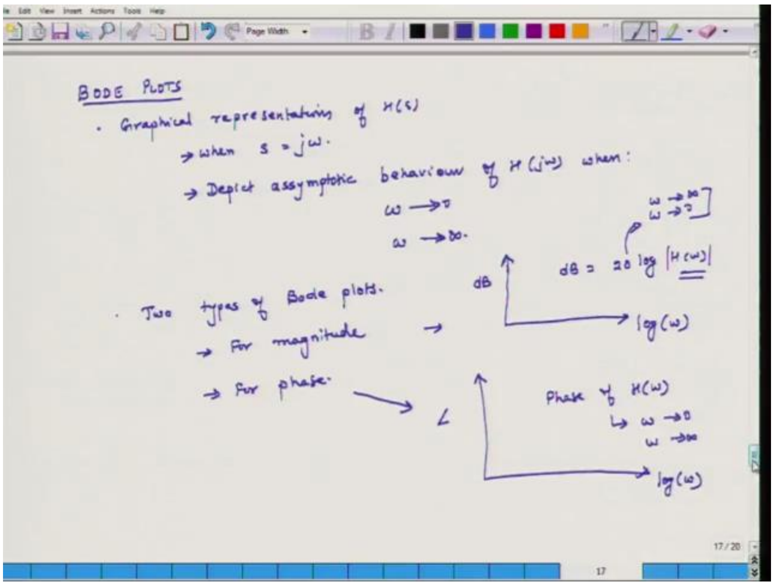

And we will explain them, there is one Bode plot for magnitude and one Bode plot for phase and these plots; Bode plots; the first thing is that they are graphical representations of the transfer function, but this as do not represent the transfer function, they represent transfer function when S equals j omega. So, when we equate S to j omega then the plots we get; we get Bode plots.

The second thing about these Bode plots is they depict asymptotic behavior of H of o j omega when in 2 conditions, when omega approaches 0 and then when omega approaches infinity. So, Bode plots represent H S, when S equals j omega in the special condition, when omega approaches 0 or when omega approaches infinity because they represent asymptotic behavior then there are 2 types of Bode plots. So, we know that H which is a transfer function, it is a complex function. So, its numerator can be a complex function and denominator can be a complex function. So, this complex function will have a magnitude and it will also have a phase.

There are 2 types of Bode plots, one is for magnitude and then the other one is for phase, when we do the magnitude plot, we on the x axis, we plot log of omega. So, we just do not plot omega, but log of omega and on the y axis, we plot decibels. Now the definition of these decibels is that d B equals 20 log H of omega and its magnitude. So, because we are plotting the log of the magnitude of the transfer function, this is a Bode plot for magnitude for phase on the x axis, we plot log omega and on the y axis, we plot phase and this could be in radiance. So, this is phase of H of w, but it is not just H of w under 2 conditions, when omega approaches 0 and when omega approaches infinity asymptotic behavior.

Same thing here, for magnitude plot when omega approaches 0 and when omega approaches infinity then the asymptotic behavior of the magnitude is plotted on the Bode plot and that is what we look at. So, this is the overview. Now we will look at some simple examples which will make this concept much more clear.

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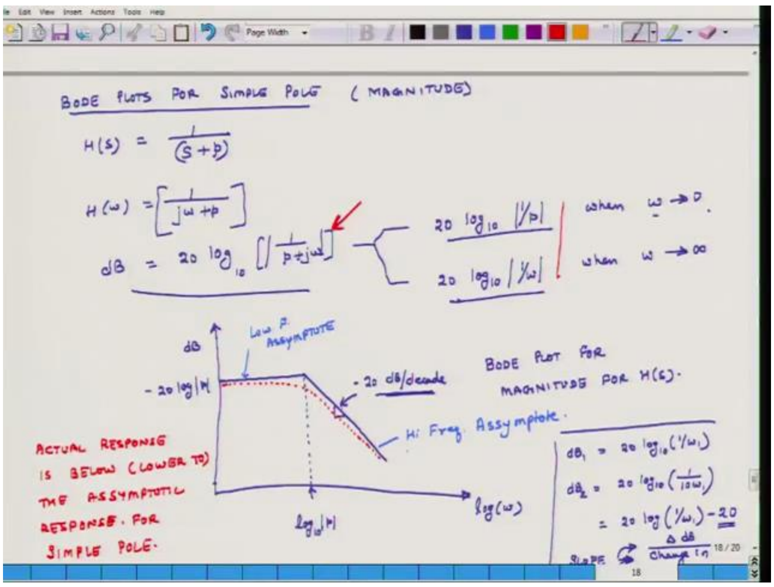

Bode plot, for simple pole and right now, we will just worry about the magnitude plot, we will talk about phase plot later for a simple pool. So, if there is a simple pool, suppose there is a transfer function H of S and it can be represented just by a single pole, when it is the transfer function is nothing, but one single simple pole and it can be expressed as S plus P.

This is a simple pole. So, the first thing is said is that when we are talking about Bode plot, we equate S to be S j omega. So, in this case H of j omega or j i, just are regular number. So, I can just write it as omega is equal to 1 over j omega plus p. So, the transfer function for a system which can be represented just by a simple pole is 1 over S plus p and when complex frequency equals omega then H of omega is 1 over j omega plus t, the next thing is that I have to calculate the value of transfer function in decibels. So, d B equals 20 log and please remember that H of omega is a complex number. So, I cannot take a log of complex number and to take its magnitude in base 10. So, it is 1 over p plus j omega and I have to take its magnitude.

Now, what did we say? We said that a Bode plot is a representation of the magnitude of the transfer function. So, in decibel units, this is the magnitude of the transfer function, but we evaluate this magnitude under 2 conditions as omega approaches 0 and as omega approaches infinity. So, this d B, this becomes when omega approaches 0, then this becomes 20 log 10 of what? 1 over p, when omega approaches 0 and the magnitude approaches 20 log 10, 1 over omega, when omega becomes when becomes very large then it becomes very large compare to p. So, I do not have to worry about p.

Now I am going to plot this. So, I said that the x axis. So, this is Bode plot for magnitude for H of S. So, on the x axis, I will plot log of omega and on the y axis I am plotting decibels. Now I am going to plot 2 lines, one line will correspond to the condition when omega approaches 0. So, that line is called the low frequency asymptote and the other line, I will plot where omega approaches infinity and that is called the high frequency asymptote. So, the low frequency asymptote, its value does not change with omega it is just constant 20 log of 1 over p or it is because 1 over p it is log is minus of log of p. So, it is this thing.

This value is minus 20 log p, I should be a little more mathematically precise. So, I will put p in modulus because p may or may not be positive number or it may not even be real it may be complex. So, I have to take it is modulus. So, this is the low frequency asymptote and the high frequency asymptote is 20 log 1 over omega. So, what happens as omega increases? The value of this term, it falls. So the high frequency asymptote looks like this, it is a straight line on log decibel axis.

As omega in this increasing, its log decreases and I have a negatively sloped line. So, this is my higher frequency asymptote, this is the high frequency asymptote, the third thing about this graph is to figure out where do these 2 asymptotes meet. So, these 2 asymptotes are going to meet when 20 log of 1 over p equals 20 log of 1 over omega that is when these 2 lines are going to meet. So, these 2 lines are going to meet when 20 log of 1 over p equals 20 log of 1 over omega.

Now, at that condition, 20 will cancel away and 1 over p will have to be same as 1 over omega this means when p equals omega then these 2 lines will meet.

The point of intersection of these 2 lines is when the on the log axis on the frequency axis, this value is log of p, actually once again p may or may not be real. So, I will put log of k. So, this is the asymptotic response. So, the board magnitude plot for H of S as a low frequency asymptote, which is the horizontal line and it cuts the y axis at the location minus 20 log of p and then the high frequency asymptote is a negatively sloped line and it meets the low frequency asymptote when p becomes equal to omega.

The last thing in this is that the slope of this line is 20 decibels per decade or actually negative 20 decibels per decade. Why is it negative 20 of decibels per decade? We will look at it. So, consider in the case, d B 1 when omega is equal to omega 1 is 20 log 10, 1 by omega 1. Now I am going to increase my omega by a factor of 10 which means that the frequency has gone up by a decade. So, d B 2 is equal to 20 log of 10 1 over 10 omega 1 because I want to see what is the slope of this line. So, I am multiplying by omega by a factor of 1, I may not, we will calculate d B 2. So, this is equal to 20 log of 1 over omega 1 and then minus 20 because log of 1 times 10 omega 1 is log of 1 over omega 1 minus log of 1 over 10. So, that is there which means that slope. So, what I see is that if I increase the factor by a decade slope is change in log omega by change in d B.

That is the slope. So, this is minus 20, change in decibels is minus 20, change in log of omega initially it is log of 1 over omega. Now it is minus 1 log of 1 over omega. So it has gone up. So, it is minus 20 decibels per decade. So, once again to recap the Bode plot for a simple pole has 2 lines asymptotic lines, low frequency asymptote, a higher frequency asymptote. The low frequency asymptote cuts the d B axis or the vertical axis at minus 20 log p position and it intersects the high frequency asymptote when p becomes equal to when omega becomes equals to p at that point and the slope of high frequency asymptote is 20 log negative 20 decibels per decade. Now this is the asymptotic response. If I have to plot the actual response then I will actually plot this curve then I will actually plot this curve.

These are asymptotic responses, I will actually plot this curve and I will keep on increasing omega, I will start omega from 0. So, what I will see is that the curve will look something like this. So, the actual curves are below or lower the asymptotic response. Actually response is below the asymptotic response for a simple pole. So, this is the overview for a simple pole and what this shows is that what that the Bode plot for a simple pole at least the magnitude component as a low frequency and a high frequency asymptote and the asymptotic response is a little higher than the actual response, this is little higher than the actual response.

This is the conclusion of our discussion for today and tomorrow we will develop a similar plot for a simple 0 and then we will start them figuring out, how to construct Bode plot, Bode magnitude plots for complex functions.

Thank you and have a great day, bye.