Fundamentals of Acoustics
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Lecture – 10
Important Mathematical Concepts-Pole Zero Plot

Hello, welcome to Fundamental of Acoustics. Today is the 4th day of this current week and today we will start discussion on poles zeros and their plots and also bode plots which are very helpful in understanding the overall synthetic response of linear time invariant systems and all these concepts will be heavily used as we delve deeper into the realm of Acoustics. So, it is important that we understand these concepts, review them and from next week onwards we will be actually starting to get into concepts related to Acoustics and all these concepts which we have been reviewing over this week and also the last week will be very useful for our for developing a detailed understanding of Acoustics.

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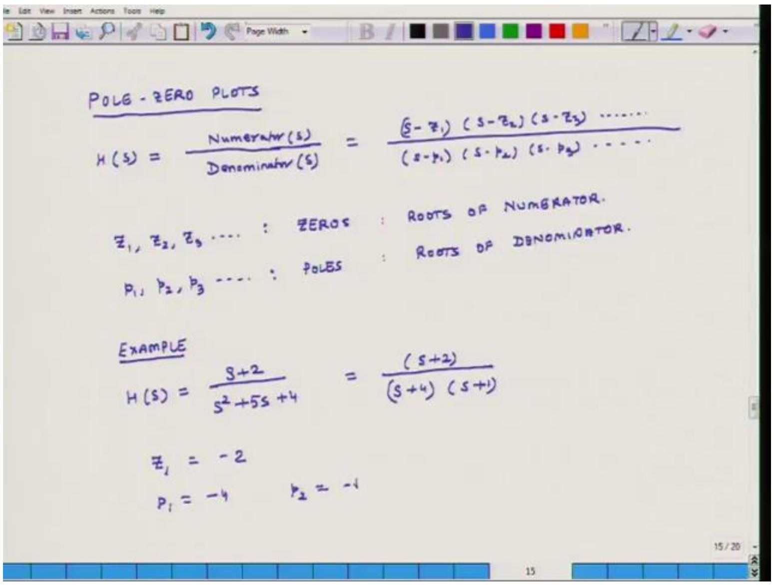

What I plan to start discussing today is pole zero plot. So, we have seen that a linear time invariant system; it will have a transfer function and this transfer function will have a numerator and the numerator can depend on complex frequency as and it will also have a denominator which will also change with frequency complex frequency as so both the numerator as well as the denominator, they could be functions of complex frequencies.

So, both the numerator as well as the denominator, they are algebraic expressions in complex frequency. So, what that means, is that I can factorize the numerator into its linear factors and I can also factorize the denominator into its linear factors.

Let us say the roots of the numerator we call them Z. So, the roots, the linear factors of the numerator could be S minus Z 1 S minus Z 2 S minus Z 3 and it can have a lot of linear factors, it depends on how complex the numerator is, if it is just a linear term then it will have 1 root, if it is quadratic term, it will have 2 roots and so on and so forth and similarly the denominator will have roots and its linear factors will be S minus P 1 S minus P 2 S minus P 3 and so on and so forth. So, these zees, zee 1 zee 2 or Z 1, Z 2, Z 3 and so on and so forth, they are called zeros, they are called zeros and they are roots of numerator.

When complex frequency approaches or becomes equal to 1 of these roots corresponding to that complex frequency, the value of transfer function becomes 0, unless it becomes 0 similarly the poles, the roots of the denominator P 1, P 2, P 3 and so on and so forth, these are called poles. So, these are roots of denominator, roots of denominator. So, when the complex frequency becomes equal to at least one of these roots of the denominator then the value of transfer function at that complex frequency becomes infinite. So, I poles the transfer function becomes very large at zeros, the complex function, the transfer function becomes very small, we can approach 0.

So, let us do one example. So, H S is equal to S plus 2 divided by S square plus 5 S plus 4. So, the numerator is linearly dependent on S and the denominator is a quadratic function of S. So, now, what I can do is I can factorize the numerator. So, numerator is just S plus 2 and the denominator are S plus 4 times S plus 1. So, what that means is that this transfer function has one 0 zee 1 equals minus 2 and it has 2 poles P 1 equals minus 4 P 2 equals minus 1.

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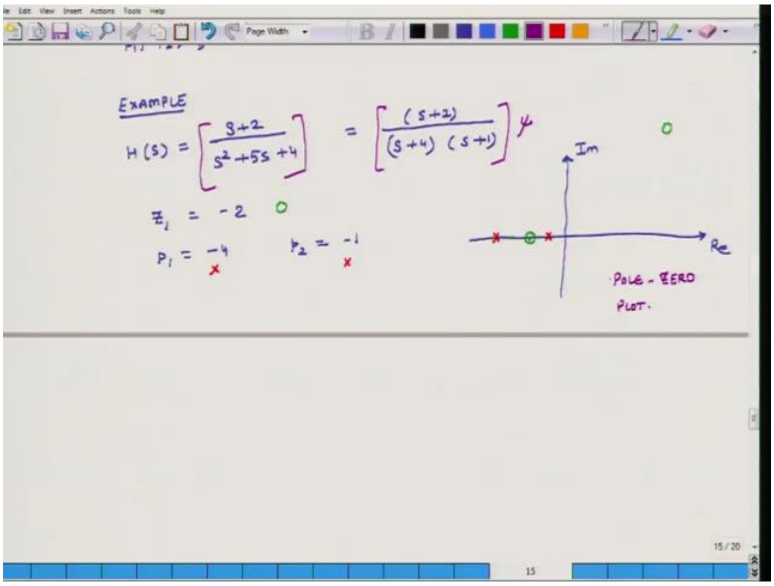

I can plot these poles and zeros on a pole zero plots. So, this is again a complex plane. So, real axis, imaginary axis and 1 minus 1 minus 2 minus 3 minus 4, I have a 0 at minus 2. So, I designate 0 by this circle and then I have 2 poles, 1 pole is at minus 1 and another pole is at minus 4. So, this is a pole. So, this is my symbol for pole.

This is a pole 0 plot, what is the purpose of this pole zero plot? If I have a pole 0 plot for a system then from that pole zero plot I can very easily construct the transfer function. I can very easily construct the transfer function of course, there may be a scalar factor some phi that phi I cannot estimate from a pole zero plot you know, but whatever is inside the brackets that can be reduced from the pole zero plot and once I have the transfer function, I know how the system is going to behave as its frequency changes for a given input. Now what is the output going to be? I can compute from this transfer function. So, just one graph can give us the whole information about the entire system. So, this is the importance of pole zero plot.

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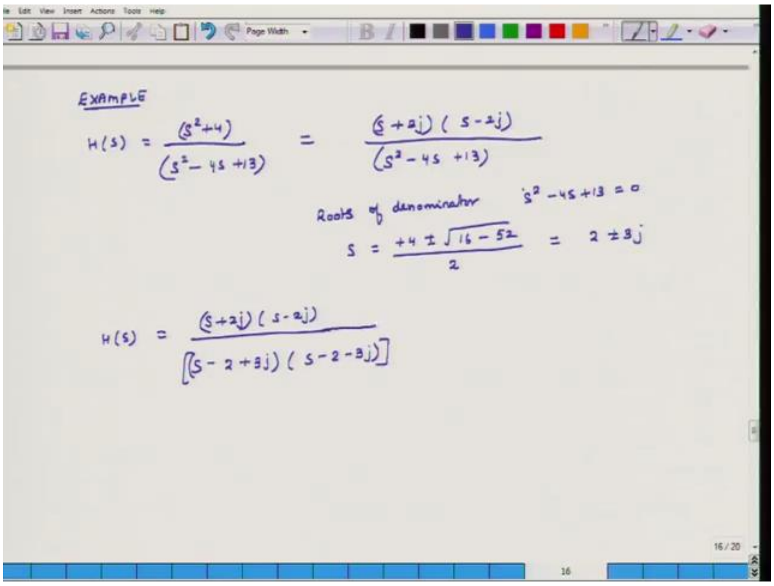

Let us do another example. So, in the first case, all the poles and all the zeros, they were looking on the real axis only, but that may not necessarily be the case always. So, let us consider this system H S equals S square plus 4 divided by S square minus 4 S plus 13. So, the roots of the numerator are S plus 2 J and S minus 2 J and for the denominator we will compute that. So, for computing we do roots of denominator. So, this is a quadratic expression in X. So, S equals minus. So, the thing is S square minus 4 S plus 13, if I have to find out its roots, I have to equate it to 0 and then I compute its roots. So, this is equal to plus 4 plus minus D square minus 4 A C. So, 16 minus A C minus 52 divided by 2 A 2. So, I get 2 plus minus 3 J. So, the function becomes H of S equals S plus 2 J and S minus 2 J divided by S minus 2 plus 3 J and S minus 2 minus 3 J.

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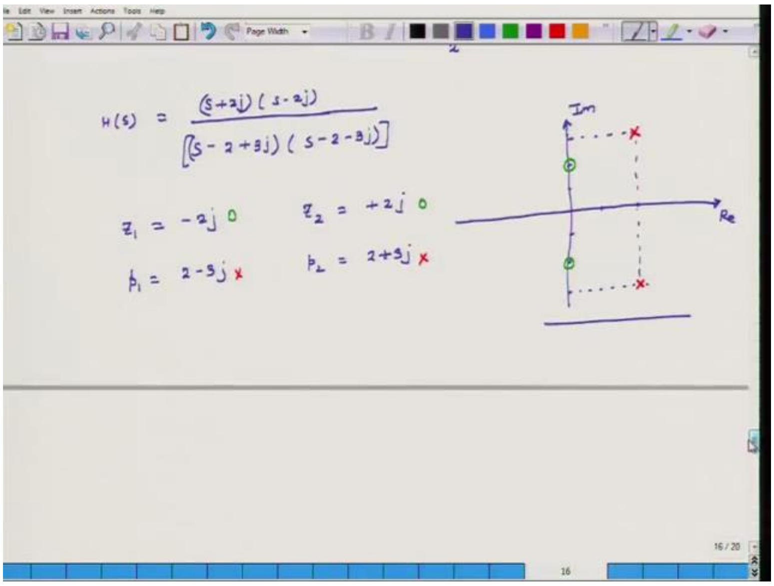

Now we write down the roots of numerator and denominator. So, zee 1 equals minus 2 J, zee 2 equals plus 2 J, P 1 equals 2 minus 3 J and P 2 equals 2 plus 3 J. This system has 2 poles and 2 roots and we will plot them on the complex plane. So, that is my real axis that is my imaginary axis. So, this is 1, 2, minus 1, minus 2. So, first I am going to plot the poles or the roots. So, 1 root is at plus 2 J other root is at minus 2 J and so, these are zeros. So, these are the 2 zeros and now I am going to plot the roots for the denominator which are poles. So, it is 1 plus minus 3J. So, 1 2 and this is plus 3 J, this is minus 3 J. So, this is 1pole and this is the other pole. So, this is one case where the poles and zeros are not necessary on the real axis, but they are somewhere in the complex plane. So, that completes our discussion on pole zero plots. The next thing we are going to discuss is bode plots and I think it will be better that we start it tomorrow.

With this we conclude the discussion for today and tomorrow and day after tomorrow in the next 2 classes, we will cover information, I mean cover the concepts of bode plots.

Thank you and have a great day, bye.