Principles of Vibration Control
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Lecture – 09
Linear Viscoelastic Materials & Models

Welcome to the course on principles of vibration control. So now we will be talking about linear viscoelastic materials and their models. So we will be discussing about how to model of viscoelastic materials very specifically in this lecture so the things that we would like to cover is first of all a brief introduction to the viscoelastic materials.

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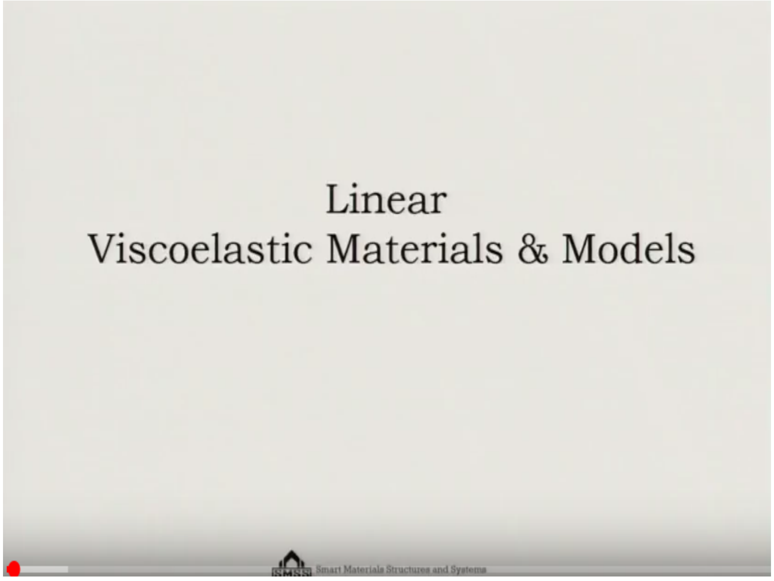

Linear
Viscoelastic Materials & Models

Smart Materials Structures and Systems

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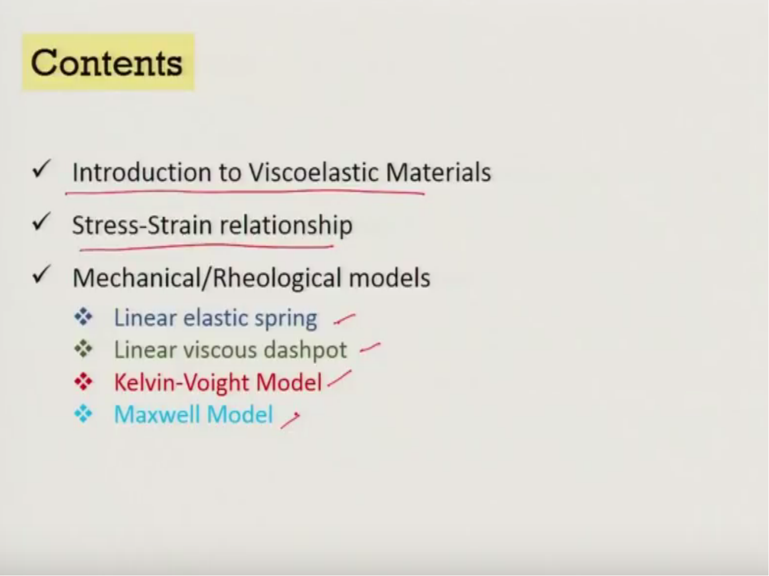

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- ✓ Stress-Strain relationship
- ✓ Mechanical/Rheological models
  - ❖ Linear elastic spring
  - ❖ Linear viscous dashpot
  - ❖ Kelvin-Voight Model
  - ❖ Maxwell Model

And then we will talk about the stress-strain relationship of the viscoelastic material and then we will propose certain models for example we will start with the linear elastic spring and then linear viscous dashpot and something which will be derived out of these 2 by mixing these 2 that is the Kelvin Voigt model and the Maxwell model.

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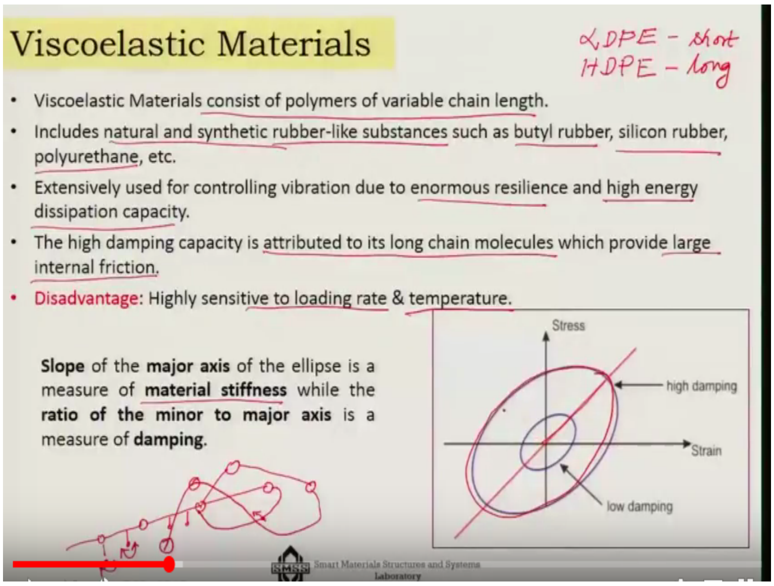

Viscoelastic Materials

LDPE - Short
HDPE - long

- Viscoelastic Materials consist of polymers of variable chain length.
- Includes natural and synthetic rubber-like substances such as butyl rubber, silicon rubber, polyurethane, etc.
- Extensively used for controlling vibration due to enormous resilience and high energy dissipation capacity.
- The high damping capacity is attributed to its long chain molecules which provide large internal friction.
- Disadvantage: Highly sensitive to loading rate & temperature.

Slope of the **major axis** of the ellipse is a measure of **material stiffness** while the **ratio of the minor to major axis** is a measure of **damping**.

So that is what is our learning goal today now first of all we will be giving a brief introduction to the viscoelastic materials here with me we have 2 viscoelastic materials one of them is more lossy that means it will behave in such a manner that it will dissipate more energy and another is more elastic in nature.

So let us see both of them are very much identical in terms of their shape and size etc but if we just drop one of them from a certain height we can see that there is a ball like behavior that means it is actually quite springy that means when I am dropping it. It is not dissipating much of the energy it is getting it stored and that energy is helping it to bounce back and this back and forth is happening for some time.

On the other hand, imagine a material which has lot of loss associated in it like this one from the same height if I drop it you can see that there is almost no bouncing in the material so that would actually help us to do this conjecture that this material even though they are very much identical in nature at room temperature is actually more viscoelastic in comparison to the material.

And this viscoelastic ness of the property of the material in which we have two parts associated in it one is the viscous part and another is the elastic part, the viscous part is actually more predominant in this material which had shown very less amount of actually which has dissipated more energy in the sense that which has shown very less amount of jumping.

Phenomena in comparison to the other one so this with the brief kind of an idea we will keep with us so a very brief introduction about the viscoelastic materials. First of all this viscoelastic materials actually consists of polymers which has variable chain lengths and some of them can be of short chain lengths some of them can be of long chain lengths.

In fact you know that the polyethylene that we use most of the times we divided into 2 categories one is called low-density polyethylene or LDP and another is called a high-density polyethylene HDP. Now low-density polyethylene has actually short chain lengths and it has low molecular weight and high-density poly ethylenes are having long chain length.

And it has high molecular weight so like that in the same polymeric category itself you can have variable chain lengths in it and the viscoelastic materials include natural and synthetic rubber like substances which is more important for us from the vibration control point of view such as butyl rubber silicone rubber poly urethanes etc.

And it is extensively used for controlling vibration because it has enormous resilience as well as high energy dissipation capacity we have just seen shortly before that how high is the energy dissipation capacity that just after the first bouncing it did not bounce back at all.

So that is what is the tell-tale sign of a good viscoelastic material now where from this high damping capacity comes it is generally attributed to the in molecules which actually provide internal frictions so imagine you have a long chain so suppose we consider a viscoelastic material of this type a quite a long chain.

Now what will happen it will not remain straight in between sometimes there will be entanglements of it and then the chain will continue so there are many such entanglements possible inside the system like if it is a very long slender system now naturally there will be internal frictions between these loops.

So the chain to chain internal frictions will take place and not only that these chains also may have lot of pendants in them lot of atoms from the main structure and then there will be degree of freedom of these things as well so they may start to actually you know dissipate energy like little pendulums which are fixed with the system and hence they will also contribute in terms of energy dissipation.

So the very nature the very structure of viscoelastic materials is always in favour of energy dissipation and that is why there are very good use to ask for vibration control however there are some disadvantages of it for example viscoelastic materials are highly sensitive to loading rate how fast I am actually you know loading of his viscoelastic material .

And what is the temperature at which I am working on the viscoelastic material in general we say that a very high loading rate is not very good to see the viscoelastic materials in action and it is also true from the temperature point of view a high temperature is also not very good to see the viscoelastic materials in action neither a very low temperature.

It is somewhere in between we call it to be a glass transition temperature where you see the best properties of the viscoelastic material now in the stress-strain curve I have already earlier told you that actually the you know the elliptical form comes up in the which also depicts the area under the ellipse also depicts the hysteresis or the energy that is lost per cycle.

Now this particular ellipse the major axis of the ellipse is actually a measure of the material stiffness while the ratio of the major to minor so if we know the major we know that how much is this actually stiffness but the ratio of the major to minor which in some sense tells us that what is the you know ellipsoid nature that would actually give us a measure of the damping of the material.

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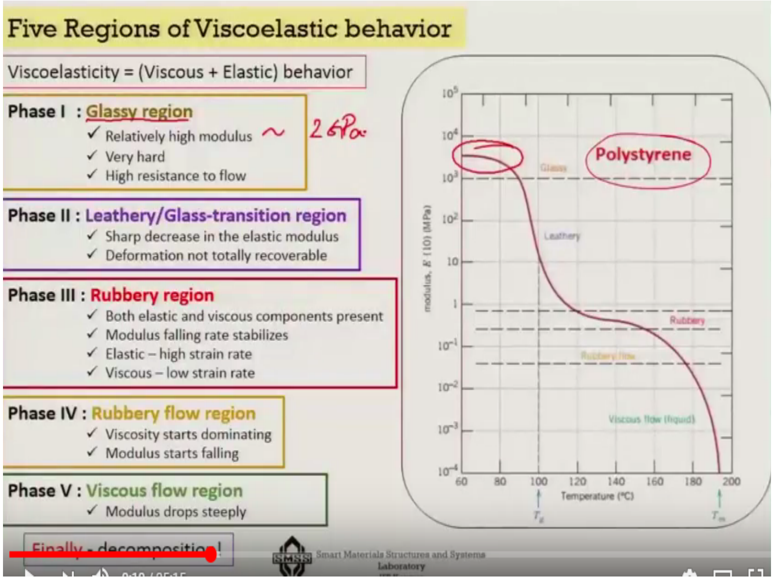

Five Regions of Viscoelastic behavior

Viscoelasticity = (Viscous + Elastic) behavior

Phase I : Glassy region
- ✓ Relatively high modulus ~ 2 GPa
- ✓ Very hard
- ✓ High resistance to flow

Phase II : Leathery/Glass-transition region
- ✓ Sharp decrease in the elastic modulus
- ✓ Deformation not totally recoverable

Phase III : Rubbery region
- ✓ Both elastic and viscous components present
- ✓ Modulus falling rate stabilizes
- ✓ Elastic – high strain rate
- ✓ Viscous – low strain rate

Phase IV : Rubbery flow region
- ✓ Viscosity starts dominating
- ✓ Modulus starts falling

Phase V : Viscous flow region
- ✓ Modulus drops steeply

Finally - decomposition!

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Now with this little background of his viscoelastic materials we would also like to see that how these viscoelastic materials behave with respect to temperature suppose I take this material called polystyrene which shows very good viscoelastic properties close to room temperature now this particular material or such types of viscoelastic materials actually shows five phases in terms of their physical nature.

So much unlike the metals which may in the solid state itself in a metal or a metallic or a kind of a crystal will show at me at most one or 2 phases in it or maybe there will be some changes in crystal structures which may be 2 or 3 times you know the change of crystal structure will happen.

But here for this kind of viscoelastic materials there are total changes in terms of the nature of the material itself for example for this polystyrene if you keep it at a very low temperature it will be like in a glassy region so this is the phase one or the glassy region which will be here at low temperature at this point.

You will see that the modulus of elasticity is relatively high means what is going to come up at a later stage in comparison to that the modulus of elasticity is relatively high nearly close to we say that to giga pascal or so it cannot be very high for you know polymeric materials but close to that and it is very hard that is true you take any such material.

And you know for example simple polyethylene and then plastic sheet what you call and keep it in the freeze and then you will see that it would become very hard in a low temperature region so with respect to low temperature this behavior also you will see and it will have a high resistance to flow.

So that is your phase one and in fact this happens because all the chains which gives this fantastic mobility to the viscoelastic materials they are even frozen at a low temperature so that it doesn't really get an opportunity to so the viscoelastic behavior.

Now as the temperature is increasing this frozen chains are becoming active they are they. They would start to move against each other so this is where we will get this leathery or glass transition region and this is marked by a sharp decrease in the elastic modulus and they this may also have deformation which will not be totally recoverable like your you know elastic.

You know recovery may not take place and that is what is this transition region or the leathery region now there are several theories for this transition some people say that let us imagine that this is the main chain of a viscoelastic material then it can happen that these main chain the part of the main chain will actually crank itself and will start to rotate with respect to some equilibrium condition.

And thus gives a kind of a you know mobility in the system there can be another mobility also as I told you that suppose if there is a you know a side chain from the main chain then there can be mobility also in terms of the motion of the side chains so all these things basically contribute towards the sharp decline in terms of from glass transition region.

And then we come to the region which is known as the rubbery region in this region both elastic and viscous components are present of course viscous component will be more dominant now and the modulus falling rate will be stabilized now because almost all the links has become active so that will stabilize elastic it behavior will come at a high strain rate and viscous behavior will come at a low strain rate.

So that means that the rubbery state if I apply the deformation at a very high rate then it will not be able to follow it and it will show a kind of elastic behavior resisting that kind of a change but if I do it slowly it will follow it and that is the viscous behavior at the low strain rate.

So this is what is our phase 3 beyond that the viscosity will start to dominate because the temperature has increased now so the rubbery flow will start to take place so and the modulus will start to fall once again and finally when we are at phase 5 then we have the viscous flow region and the modulus will drop very steeply.

Basically we will reach the fluidic region and the melting point is somewhere here so for polystyrene less than 200 degree centigrade same you will see this kind of a change so thus in a nutshell once again we are going to see five regions of viscoelastic behavior very low temperature region one which is glassy at that time the modulus of elasticity is high.

And then the modulus of elasticity starts to decrease that is the leathery or the transition region or glass transition region and which happens because of I told you that the mobility of the chains and the modulus of elasticity falls down significantly after that there is a kind of a stabilized region which is marked rubbery region and then from the rubbery region.

Once again the you know beyond a certain temperature the viscosity will start to dominate so we will be having a rubbery flow region and from rubbery flow region still in the rubbery flow region there will be some resistance to deformation that means there will be some kind of a elastic behavior but beyond a certain temperature from the rubbery flow region.

We will come to the viscous flow region where it will be only viscosity which will resist the deformation of the system so basically it will be proportional to the strain rate of deformation and that is how we are going to get the five regions of viscoelastic behavior of course

Finally you know if you increase the temperature much for some of the materials not for all a decomposition can happen that means all these things will break down and the gases will come out of it so that can happen in some of the materials.

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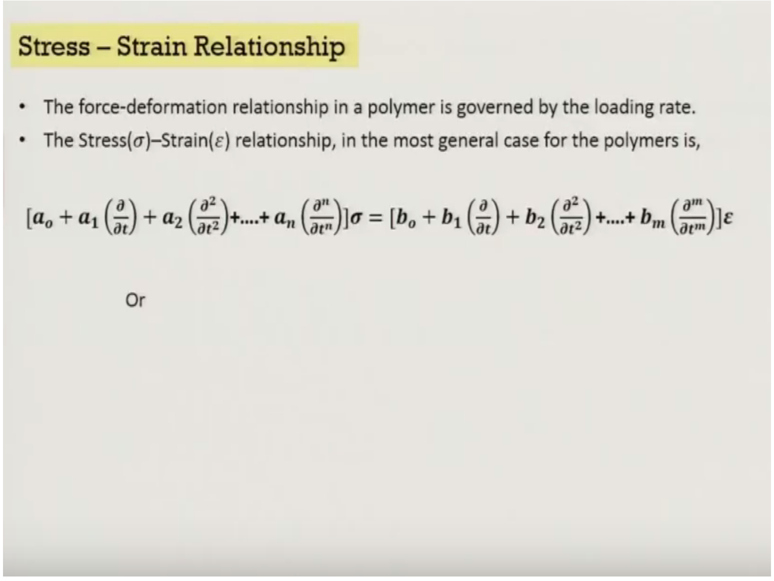

Stress – Strain Relationship

- The force-deformation relationship in a polymer is governed by the loading rate.
- The Stress(σ)–Strain(ε) relationship, in the most general case for the polymers is,

$$[a_o + a_1\left(\frac{\partial}{\partial t}\right) + a_2\left(\frac{\partial^2}{\partial t^2}\right) + \dots + a_n\left(\frac{\partial^n}{\partial t^n}\right)]\sigma = [b_o + b_1\left(\frac{\partial}{\partial t}\right) + b_2\left(\frac{\partial^2}{\partial t^2}\right) + \dots + b_m\left(\frac{\partial^m}{\partial t^m}\right)]\varepsilon$$

Or

Now we come to the whole thing that we have said so far qualitatively we come to a quantitative description of the fact. Now naturally if we want to say that we have to say here that both the stress is proportional to the strain rate and from the another side also the strain is proportional to the rate of change of stress so what it means is that we have for example the stress proportional to strain part where A 0 and B 0 will come into picture there is a typical of metals.

But then beyond that also we have this stress rate or strain rate or rate of change of that or even higher order changes appears in the constitutive relationship of the system now this particular thing we can actually sum it up in terms of a generalized Hooke's law as you can see here that we can keep the first term separate A 0 Sigma + I equals to 1 to n.

we have the stress rate relate terms here and that equals to once again in the strain side first term separate there is a reason I will show you because it will be easier for us to understand b0 epsilon and then for J equals to 1 to M we have the strain rate terms that will come into picture in short this is a you know law extension of the Hooke's law in which not only that stress is proportional to strain.

But also many more intricate things will be you know kind of it will be expressed through this relationship where stress and strain rates depending on the type of material will also play an important relationship if all these coefficients A0 to AN and B0 to BN remain constant then we will call the material to be a linear viscoelastic material.

For metals for example a 1 to n is 0 and B 1 to B M is 0 so what it means is that for metals a 0 Sigma equals to B 0 epsilon so if I divide both the sides by a 0 because a 0 is just a constant so Sigma is B 0 over a 0 times epsilon and I can write it as e times epsilon so that is how for a metallic case we do not get any other variation of rate of strains etc.

But simply stress proportional to strain which is our very simplified form of the Hooke's law whereas in this case you get all these rates that will come into the picture now we will talk about the testing of the viscoelastic materials so based on the stress strain relationships we will show this testing of his viscoelastic materials.

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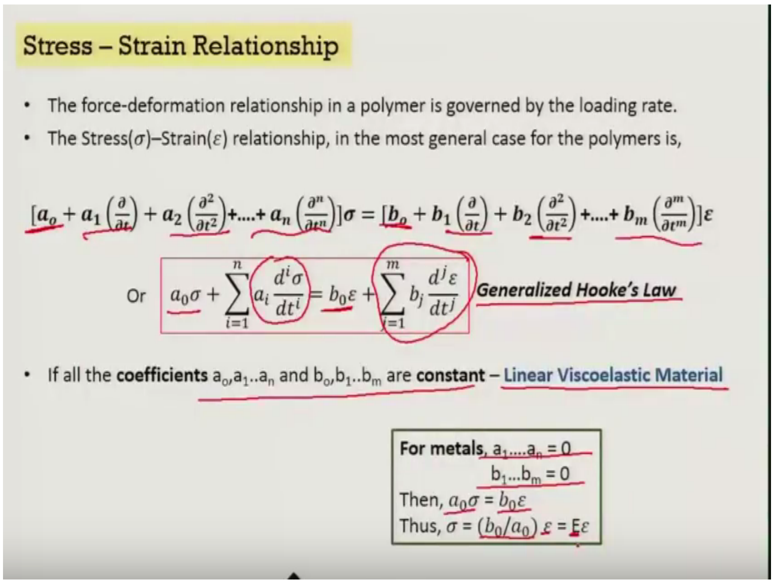

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Or $\boxed{a_0\sigma + \sum_{i=1}^{n} a_i \frac{d^i\sigma}{dt^i} = b_0\varepsilon + \sum_{j=1}^{m} b_j \frac{d^j\varepsilon}{dt^j}}$ *Generalized Hooke's Law*

- If all the **coefficients** $a_o, a_1..a_n$ and $b_o, b_1..b_m$ are **constant** – Linear Viscoelastic Material

For metals, $a_1 \ldots a_n = 0$
$b_1 \ldots b_m = 0$
Then, $a_0\sigma = b_0\varepsilon$
Thus, $\sigma = (b_0/a_0)\,\varepsilon = E\varepsilon$

Now it involves loading a material either at constant stress and holding that stress for some length of time and then removing the load that means at a particular time I have started the load constant it is not changing holding it and removing it so this is what we say as a creep recovery test.

So if we do this test on a viscoelastic material what we will find is that as soon as I apply this trace the strain will start and then my stress is constant but the strain will not stop so this is the instantaneous part of the strain after that my strain will start to show the secondary part or tertiary part which is also known as the creep strain.

So if I keep the load fixed or in other words the stress fixed then we will fast see the instantaneous strain followed by the creep strain that will happen to the system what will happen if I remove the stress instantly some part of it will come down and that is what we call it to be elastic recovery.

There is some part of it which will not come down instantly we are going to call it to un elastic recovery and there is some part of it which will not be recovered that means it will not come down to 0 at all so this is the permanent strain that may be fixed inside the system such is the nature of a creep recovery test when we are subjected to the material to this kind of a stressing of the system.

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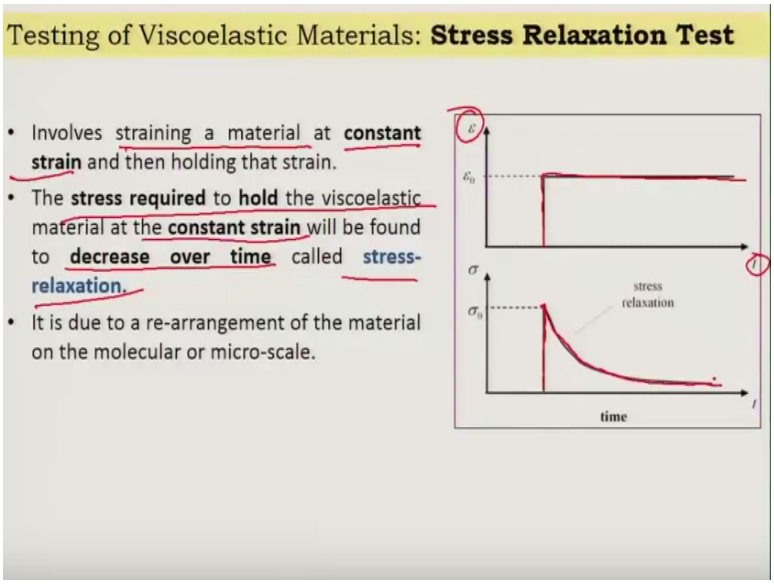

Testing of Viscoelastic Materials: **Stress Relaxation Test**

- Involves straining a material at **constant strain** and then holding that strain.
- The **stress required** to **hold** the viscoelastic material at the **constant strain** will be found to **decrease over time** called **stress-relaxation.**
- It is due to a re-arrangement of the material on the molecular or micro-scale.

Now let us look into the stress relaxation test which is this stress recovery or the creep and the stress relaxation adjust to other types of the coil now in case of these we are going to strain a material at constant strain not constant stress like the last time. So here this is epsilon versus T curve where I am applying an instant strain.

Then holding it constant here what you will see is that the stress required to hold the viscoelastic material the constant strain will actually decrease over time and these phenomena is known as stress relaxation. So you can see here as I have applied the strain I am getting some stress developed in the system and then this stress is actually coming down slowly it may not come totally to 0 but it will come down constantly and this is what is known as the stress relaxation in the system.

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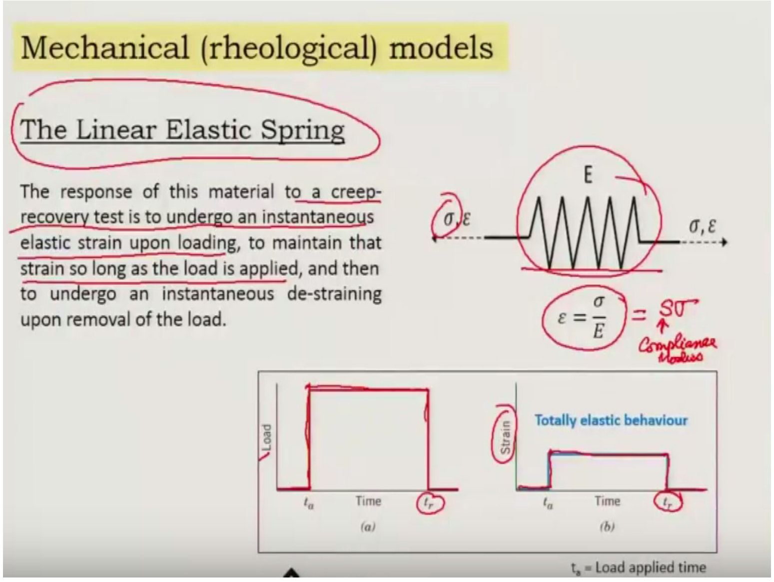

Mechanical (rheological) models

The Linear Elastic Spring

The response of this material to a creep-recovery test is to undergo an instantaneous elastic strain upon loading, to maintain that strain so long as the load is applied, and then to undergo an instantaneous de-straining upon removal of the load.

$\varepsilon = \frac{\sigma}{E} = S\sigma$

Compliance Modulus

t_a = Load applied time

Now let us look into some of the models that is a basically mechanical models we also call them to be Rheological model that we actually use in terms of describing the behavior of the system and also this models are some of the very simple form of the generic Hooke's law that we had for discussed for standard linear solid.

So the first model towards this direction is the linear elastic spring model so in this case we have a spring here it is a linear and elastic spring and I am applying stress to it and I am getting instantaneous strain into the system so the response of this material to a creep recovery test let us say to undergo an instantaneous elastic strain upon loading to maintain that strain.

So long as the load is applied and then to undergo and instantaneous this straining up and removal of the load so if you look at it that here we are applying the stress and then I am getting an instantaneous strain so this is the load or the stress this is the strain so you are getting some instantaneous strain.

We are holding the load here we are having the strain also to be constant because this is spring this is not showing any viscosity and then I am removing the load at that same time TR time of relaxation the strain also will come down and it will come to the best state.

So that is how an elastic spring should behave and that means I can defeat it by a very simple relationship where the stress is proportional to strain epsilon is Sigma over E or you also call it as s times Sigma where s is also known as the compliance model of the system.

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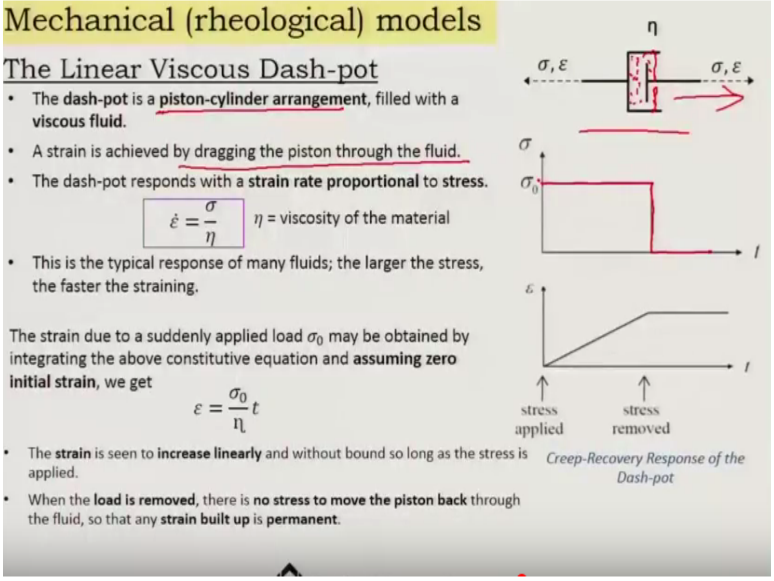

Mechanical (rheological) models

The Linear Viscous Dash-pot

- The **dash-pot** is a piston-cylinder arrangement, filled with a **viscous fluid.**
- A strain is achieved by dragging the piston through the fluid.
- The dash-pot responds with a **strain rate proportional** to **stress.**

$$\boxed{\dot{\varepsilon} = \frac{\sigma}{\eta}}$$ η = viscosity of the material

- This is the typical response of many fluids; the larger the stress, the faster the straining.

The strain due to a suddenly applied load σ_0 may be obtained by integrating the above constitutive equation and **assuming zero initial strain,** we get

$$\varepsilon = \frac{\sigma_0}{\eta} t$$

- The **strain** is seen to **increase linearly** and without bound so long as the stress is applied.
- When the **load is removed,** there is **no stress to move the piston back** through the fluid, so that any **strain built up** is **permanent.**

Creep-Recovery Response of the Dash-pot

So this is you're very much simplified Hooke's law that a spring shows now if I go from these to a viscous dashpot how the scenario is going to get changed well I told you earlier that a dashpot is a piston cylinder arrangement as you can see here this is a piston cylinder arrangement and which is filled up with some kind of it discussed it.

We have not shown it here because there's just a symbol but this is the way you can consider that the dashpot is filled up now a strain is achieved by dragging the piston through the fluid this we can easily understand that if I try to pull it this way it will come out and that means a strain will be achieved.

So if I subject these to a test of something like that I am applying a stress to the system keeping the stress fixed for some time then removing the stress and bringing it to 0 how will this fellow behave well at the very first instant when I have applied the stress I want the dashpot to move.

But have you ever seen a door closer which you can instantly close the door you cannot so there it is this very same thing you apply the stress but the strain is 0 initially that means the dashpot will refused to move at the very fast instant because it takes some time for this molecules to rearrange themselves.

So that refusal will come and then gradually it will give up so gradually you will see that the strain is happening in the system and it will come to a point of maximum strain when I am having this stress constant beyond that point even if I remove the stress the deformation is going to stay there why because there is no springiness in the system.

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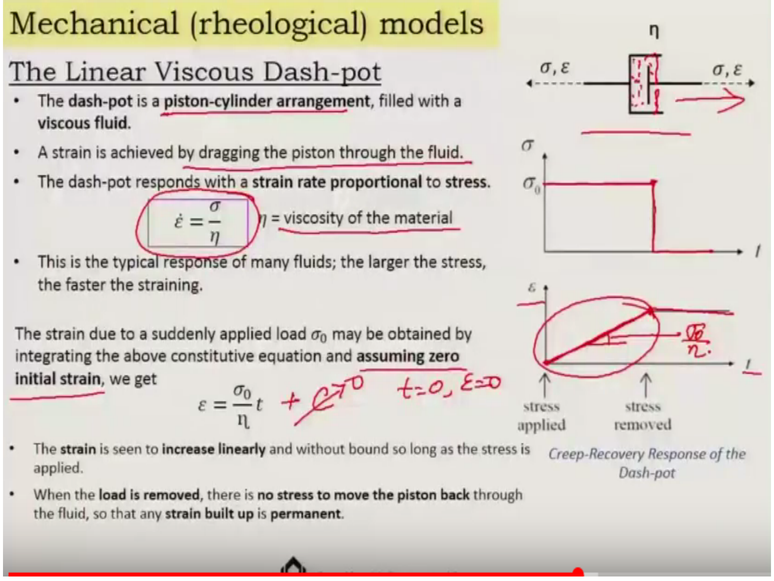

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The strain due to a suddenly applied load σ_0 may be obtained by integrating the above constitutive equation and **assuming zero initial strain**, we get

$$\varepsilon = \frac{\sigma_0}{\eta} t \quad + \cancel{C}^{\to 0} \quad t=0,\ \varepsilon=0$$

- The **strain** is seen to **increase linearly** and without bound so long as the stress is applied.
- When the **load is removed**, there is **no stress to move the piston back** through the fluid, so that any **strain built up** is **permanent**.

Creep-Recovery Response of the Dash-pot

There is nothing through which the system will recover itself that now naturally as you can see here that in this particular region it is actually the strain rate which is actually constant and hence we can actually express it with respect to this kind of a formulation that the stress is proportional to the strain rate.

So that means epsilon dot rate of change of strain with respect to time that equals to Sigma ETA where eta is not actually the loss factor here you have to keep in mind there is some kind of a damping coefficient or viscosity of the material so epsilon equals 2 so if epsilon dot equals to sigma over ETA what happens if I integrate it.

I will get it as epsilon equals to Sigma over ETA T + some constant now if I assume 0 initial strain condition that means at T equals to 0 T equals to 0 if I assume there was no press strain in the system that means this constant will be 0.

So that means with respect to strain the relationship would look like if epsilon equals to Sigma 0 over ETA times T so the strain is seen to increase linearly that is what we have seen here that with respect to the strain temperature behaviour strain is increasing linearly so that the slope of this curve is actually Sigma 0 over ETA.

So that is how it is going to behave and when the load is removed there is no stress to move the piston back so the piston will actually remain at that state and the strain built up will be permanent so that is what is our linear viscous dashpot model now what if i mix that that means what if I make a Kelvin Voight.

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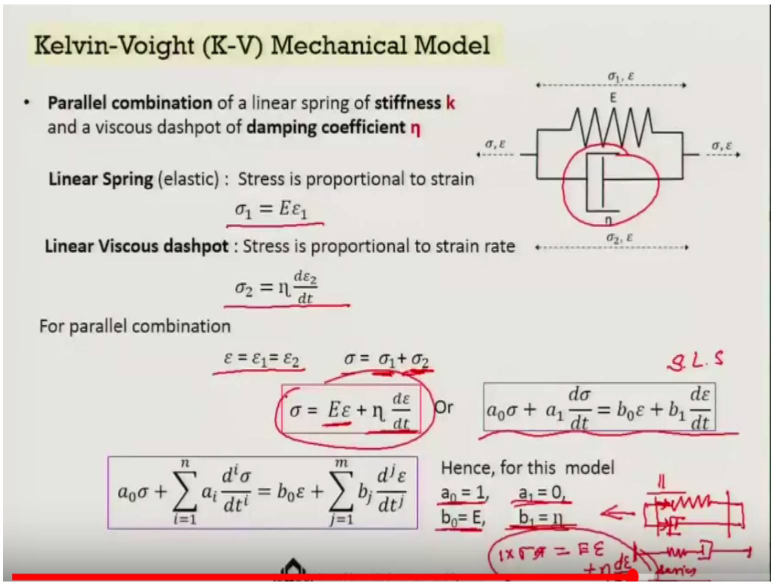

Kelvin-Voight (K-V) Mechanical Model

- **Parallel combination** of a linear spring of **stiffness** k and a viscous dashpot of **damping coefficient** η

Linear Spring (elastic) : Stress is proportional to strain

$$\sigma_1 = E\varepsilon_1$$

Linear Viscous dashpot : Stress is proportional to strain rate

$$\sigma_2 = \eta \frac{d\varepsilon_2}{dt}$$

For parallel combination

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \qquad \sigma = \sigma_1 + \sigma_2$$

$$\boxed{\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}} \quad \text{Or} \quad \boxed{a_0\sigma + a_1 \frac{d\sigma}{dt} = b_0\varepsilon + b_1 \frac{d\varepsilon}{dt}}$$

S.L.S

$$\boxed{a_0\sigma + \sum_{i=1}^{n} a_i \frac{d^i\sigma}{dt^i} = b_0\varepsilon + \sum_{j=1}^{m} b_j \frac{d^j\varepsilon}{dt^j}}$$

Hence, for this model
$a_0 = 1$, $a_1 = 0$,
$b_0 = E$, $b_1 = \eta$

$1 \times \sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}$

There are 2 ways in which I can mix the spring and the dashpot one is that a spring and a dashpot as has been shown here as parallel and another possibility is that the spring here and in the dust potties in the same line itself so this is a parallel circuit and this is a series circuit.

So I can build up these 2 models now we have to see that what is the you know kind of implication of building up these 2 models so in the first case then see the spring part Sigma one is e epsilon one and stress in the second part in these part is Sigma 2 as ETA D epsilon 2 D T.

Now because it is a parallel combination that means whatever suffering or whatever good thing or bad thing that is going to happen it has to happen together so that means the strain that it will be subjected to from both the sides it is same both for the spring as well as for the dashboard.

So epsilon is epsilon 1 equals to epsilon 2 on the other hand the load is actually shared by both of them so if I try to pull it then the resistance there will be some resistance here some here it will be shared by both of them so that means the stress is actually partly the stress in the spring and partly this stress in the damper.

So I already know that Sigma 1 is proportional to strain so epsilon and Sigma 2 is proportional to strain rate so it is ETA D epsilon DT so if I compare that with my you know a 0 to n model and B 0 to b n model of a standard linear solid model you will see that this implies that for this model ao has to be unity.

B 0 has to beE only. A1 is to be 0 and Bi is to ETA. So if you put all those things is 0 as 1 so that means it is 1 times Sigma + a 1 is 0 so that second term does not come into picture at all that equals to B 0 is e so it is e times epsilon + V 1 is ETA which is the damping coefficient.

So ETA times D epsilon DT and we are going to get back the same equation here so it is a special case of the generic Hooks model where these are the parametric values of the system so that is what is our Kelvin Voigt mechanical model of the system now if I look at the response of that Kelvin Voight model.

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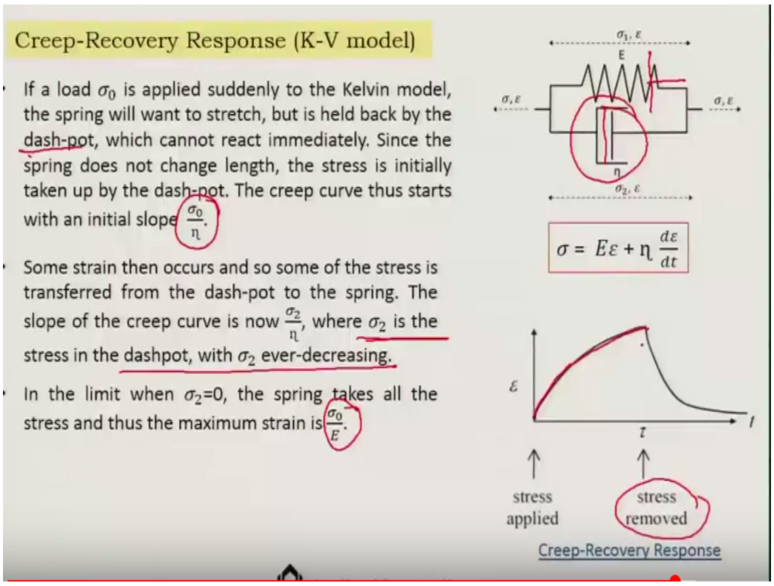

Creep-Recovery Response (K-V model)

- If a load σ_0 is applied suddenly to the Kelvin model, the spring will want to stretch, but is held back by the dash-pot, which cannot react immediately. Since the spring does not change length, the stress is initially taken up by the dash-pot. The creep curve thus starts with an initial slope $\frac{\sigma_0}{\eta}$.
- Some strain then occurs and so some of the stress is transferred from the dash-pot to the spring. The slope of the creep curve is now $\frac{\sigma_2}{\eta}$, where σ_2 is the stress in the dashpot, with σ_2 ever-decreasing.
- In the limit when $\sigma_2=0$, the spring takes all the stress and thus the maximum strain is $\frac{\sigma_0}{E}$.

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}$$

Creep-Recovery Response

There are 2 response that I will discuss today one is if a load Sigma 0 is applied suddenly to the Kelvin Voight model and then what will happen then the spring will want to stretch but it cannot stretch because the dashpot is there it whatever has to happen I told you in a parallel model has to happen together

So since the dashpot will not allow it to get strained so that means the dashpot will take the load so that means in the very fast part the creep car will start with an initial slope of Sigma 0 over ETA because this is where the dashpot will first be prominent it will come into picture.

Now as some strain will be occurring to the system so that is your Sigma 0 over it apart some strain is occurring to this system and then what will happen some stress will be then transferred from the dashpot to the spring the spring will have now some space to deform

So now the spring can actually squeeze because the dashpot has moved forward so that it will that squeeze will take place and then that means that the second part of the stress will now start to show its effect so this will have the Sigma 2 part that will come into the picture of the system and where so Sigma 1 part will come into the picture.

And Sigma 2 the dashpot part will be actually ever decreasing on the other hand the Sigma 1 part will become gradually Sigma 0 over heat so that is what is happening here that the spring has taken the domination and that is why if I remove this trace then the spring starts to show the initial deformation.

And then the rest of the deformations gradually it is coming down to 0 so this part is actually dashpot dominated initially and this part is actually spring dominated so with us in the creep recovery response we get the manifestation of both the properties of the system.

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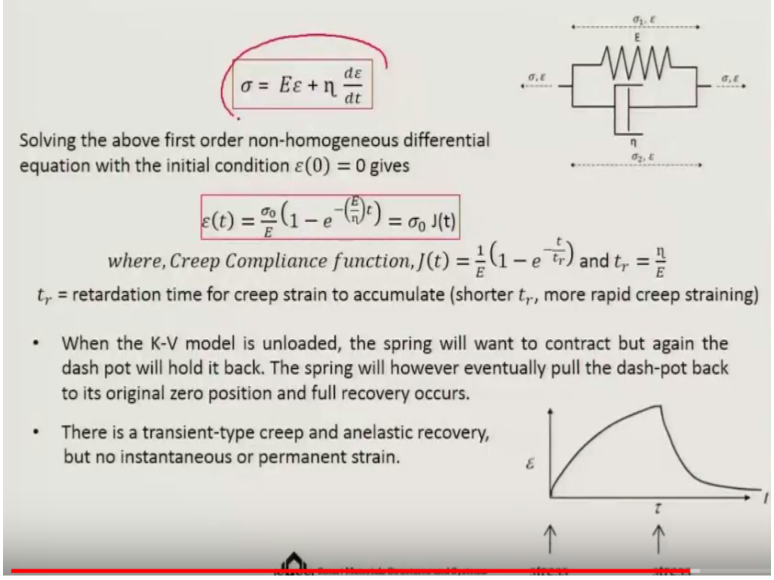

$$\boxed{\sigma = E\varepsilon + \eta\frac{d\varepsilon}{dt}}$$

Solving the above first order non-homogeneous differential equation with the initial condition $\varepsilon(0) = 0$ gives

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\left(\frac{E}{\eta}\right)t}\right) = \sigma_0\,\mathrm{J(t)}}$$

$where, Creep\ Compliance\ function, J(t) = \frac{1}{E}\left(1 - e^{-\frac{t}{t_r}}\right)$ and $t_r = \frac{\eta}{E}$

t_r = retardation time for creep strain to accumulate (shorter t_r, more rapid creep straining)

- When the K-V model is unloaded, the spring will want to contract but again the dash pot will hold it back. The spring will however eventually pull the dash-pot back to its original zero position and full recovery occurs.
- There is a transient-type creep and anelastic recovery, but no instantaneous or permanent strain.

So we have the relationship that is e epsilon + ETA times D epsilon DT now if I try to solve this with an initial condition that there is no pre strain in the system you will see that at any point of time you will get epsilon which is we defeated by this kind of a relationship where there is Sigma 0 over E.

And then there is an exponential part 1 minus e to the power minus e by ETA T which you can write it as Sigma 0 times JT and JT is known as the creep compliance function which is 1 over e 1 minus e to the power minus ey ETA T you can even write it as the ratio of the time the time of relaxation where TR is the ETA over e and this is also known as the retardation time for creep strain to accumulate.

So when the kv model is actually unloaded the spring will want to contract but again the dashpot will hold it back the spring will however eventually pull the dashpot back to its original 0 position and pool recovery will start to occur. So there is a transient type creep and an elastic recovery in the system but no instantaneous or permanent strain in the system so this is what is the Kelvin Voigt model system.

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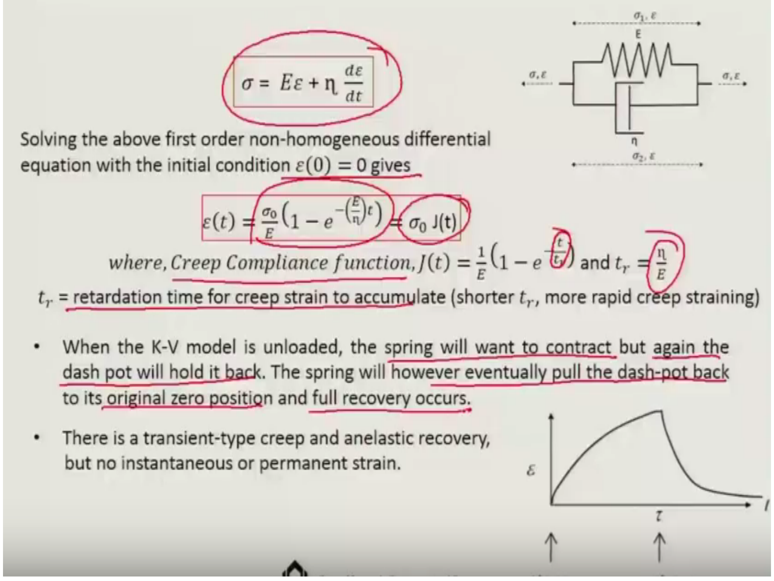

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}$$

Solving the above first order non-homogeneous differential equation with the initial condition $\varepsilon(0) = 0$ gives

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\left(\frac{E}{\eta}\right)t}\right) = \sigma_0 J(t)$$

where, Creep Compliance function, $J(t) = \frac{1}{E}\left(1 - e^{-\frac{t}{t_r}}\right)$ and $t_r = \frac{\eta}{E}$

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- When the K-V model is unloaded, the spring will want to contract but again the dash pot will hold it back. The spring will however eventually pull the dash-pot back to its original zero position and full recovery occurs.
- There is a transient-type creep and anelastic recovery, but no instantaneous or permanent strain.

And if you look at the stress relaxation in the Kelvin Voigt model then in this case you are subjecting the material to a constant strain that means epsilon 0 at T equals to 0 the you know the constant strain is coming to picture. So d epsilon DT is 0 and then you get the relationship as Sigma equals to epsilon 0 from this relationship if I give you know put D epsilon DT to be 0 this term vanishes.

And he is this so Sigma equals to e epsilon 0 thus the stress is taken up by the spring and is constant. So there is in fact it shows according to this model that there is no stress relaxing over time but actually this Kelvin Voight model undergoes an instantaneous strain of epsilon and an infinite stress needs to be applied to do this so in practice this would never happen because in order to show an instantaneous strain like this relationship shows that.

And instantaneous deformation is possible in the system this is only possible if infinite stress is applied to the system otherwise this will not happen and hence we can say that the model is good with modelling creep which we have shown in the last present part of it that this is good but it is not good in terms of showing the instantaneous strain.

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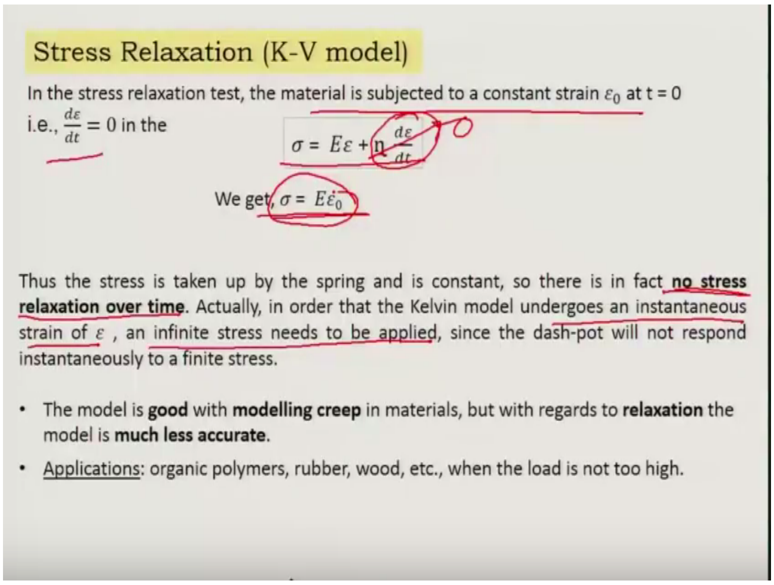

Stress Relaxation (K-V model)

In the stress relaxation test, the material is subjected to a constant strain ε_0 at t = 0

i.e., $\frac{d\varepsilon}{dt} = 0$ in the

$$\sigma = E\varepsilon + \eta \cancel{\frac{d\varepsilon}{dt}} \to 0$$

We get, $\sigma = E\varepsilon_0$

Thus the stress is taken up by the spring and is constant, so there is in fact **no stress relaxation over time**. Actually, in order that the Kelvin model undergoes an instantaneous strain of ε , an infinite stress needs to be applied, since the dash-pot will not respond instantaneously to a finite stress.

- The model is **good** with **modelling creep** in materials, but with regards to **relaxation** the model is **much less accurate**.
- Applications: organic polymers, rubber, wood, etc., when the load is not too high.

So with regards to relaxation the model is much less accurate and its application is in organic polymers in rubber woods when the load is not too high so this is where we will put an end to this lecture because here we have only talked about Kelvin Voigt model in the next lecture we will talk about the Maxwell model.