Principles of Vibration Control
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Lecture – 08
Design For Enhanced Material Damping

Today we will talk about the structural damping with respect to some more complex conditions so let us look into that and towards the end we will also show you how we can demonstrate the same thing experimentally so what we are going to focus today on how to design or the structural damping such that we get enhanced material damping.

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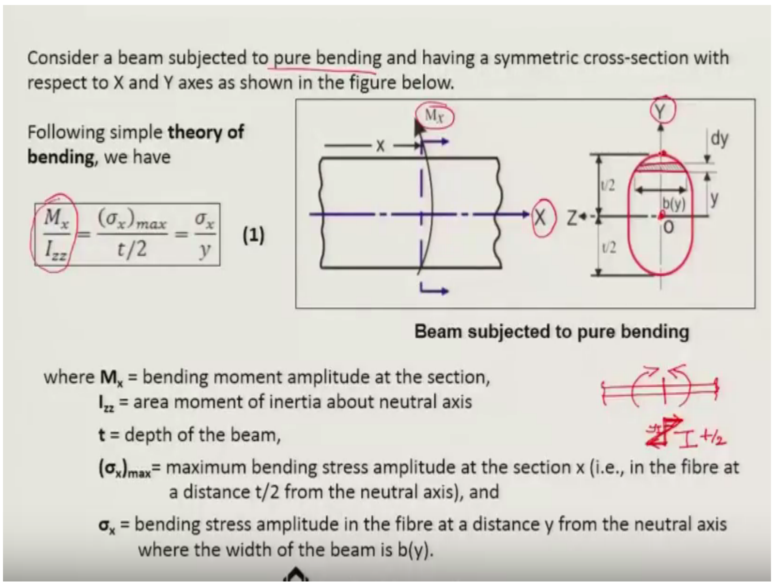

So that we can extract maximum from a structure the structural level itself. So let us consider this simple problem that this is a beam which is subjected to pure bending as you can see here that the beam is subjected to bending here and you may consider that it is something like a bending case like this where it is subjected to pure bending.

And having a symmetric cross section with respect to X and Y axis. So this is one axis that is the longitudinal axis with respect to which it has a symmetric cross section that means up and down or symmetric as well as with respect to the y axis it has a symmetric cross section both the sides are symmetric so that is a simple case.

Now for this type of a case the simple theory of bending we have to keep in our mind and that is M X the M is the bending so M X over IZZ that is the area moment of inertia about the zz, z axis which is actually perpendicular to the plane here so that MX over izz set is actually nothing but sigma x max over the distance.

So basically you some cross-section is subjected to pure bending like say this is the beam that we are considering and this is the section then in that section you will see that a completely equal and opposite stress distribution about the neutral axis will take place and this distance is actually half of the thickness so that is why in any of the extreme fibers you are going to see the maximum stress which is Sigma x max and the corresponding distance from the neutral axis is T / 2.

And that is also equal to the stress at any other location let us say I choose some interim location the stress at that location divided by the distance from the neutral axis which is y in this case ok so that means if I consider a small cross section here then this is at a distance Y and for this if the stress here is Sigma X this Sigma X by Y is actually equal to Sigma X max at this point divided by T / 2.

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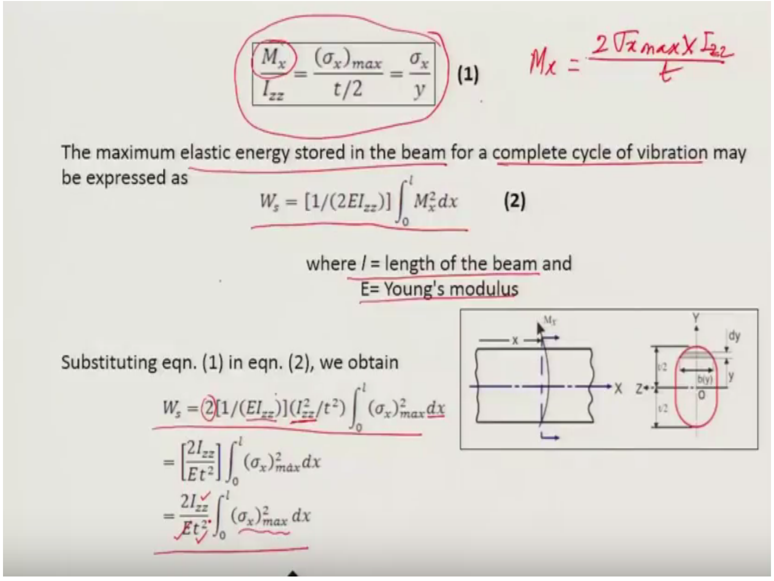

So that is the simple theory of bending so with this kind of conditions if we next actually look into it that our relationship the first equation is MX over izz as Sigma X max over T / 2 which is equals to Sigma X over Y that is what is our first equation.

Now let us find out also we will use this equation but let us find out that what is the maximum elastic energy that is stored in the beam for a complete cycle of vibration and with some assumptions like L as the length of the beam and E as the Young's modulus.

Now we can actually get this equation here as W S as half one over 2E IZZ times integration 0 to m mx square dx. So that is the total energy elastic energy that is stored in the beam because we are considering it as a conservative system. Now if I use this equation 1 and equation 2 together then what we are going to see is that we will find that this relationship will come now in which this MX square we are replacing by this equation and that means MX square can be represented as Sigma.

So MX as Sigma x max times IZZ divided by T by 2 so 2 goes to the top so that is what you know we are having here that is where the 2 is appearing for us and this is what is Sigma X max square is coming and IZZ square is not a function of DX because the cross section actually remains constant.

That is what we have assumed in this case so that is why IZZ square is out of the integration itself and of course the 2E IZZ is there so this E IZZ is actually remaining again modulus of elasticity and IZZ remaining constant in this particular equation.

So with this you know we can actually simplify it and we will get a relationship of WS as we have seen here in which will be having variables like ZZ WS as a function of IZZ the Sigma X max square integration 0 to LDX and E and D Square so some geometric parameters and some material parameters of the system.

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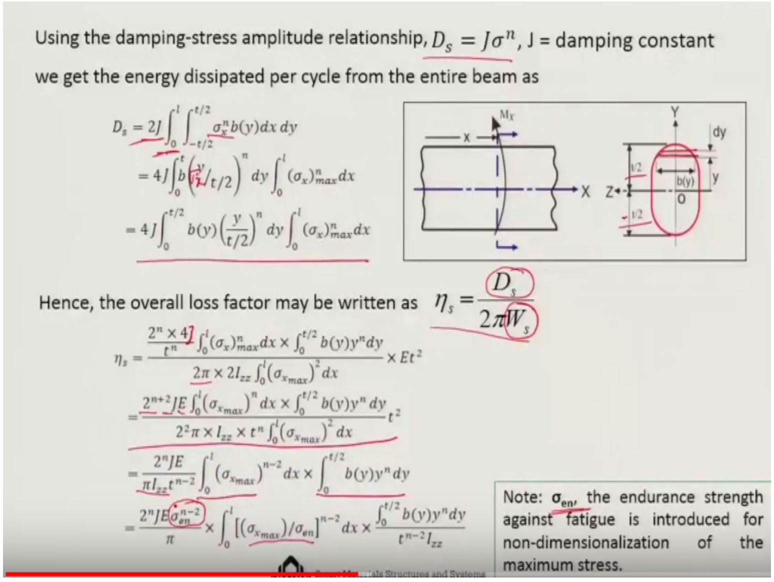

Now we also know that the damping stress amplitude relationship which I said in the very beginning that is what we are heading to is that a Loss Factor definition so in which we need the damping the energy that is dissipated which is J Sigma to the power n so the energy that is dissipated per cycle is J Sigma to the power n by 2 pi and the ratio of Plodder loss factor is this divided by w s.

So this is what we are finding in this expression. Now d s we can try to integrate it because J Sigma to the power n is actually per unit volume so we need to find out that if B Y is at any point the width of the system the strip then so B Y and dy is the area of the system and that from - T by 2 to T by 2.

So this is - T by 2 to T by 2 we are integrating this thing so that is kind of giving us the area and then Sigma X to the power n comes into the picture because that is also varying with respect to Y the stress is not going the same so this thing together from - T by 2 to T by 2 and then from the length of the entire you know sample the area which is subjected to the length that is subjected to pure bending say it is 0 to L.

So that we need to integrate that is the second integration for and that is multiplied by 2 J that is how the 2 J the 2 term is coming because we are doing it from - T 1 to 2 T by 2 or otherwise we have to do it from 0 to T so that is why the 2 is coming and J is of course the damping constant that we had earlier discussed for structural damping.

So with all these definition of the terms now we can put Sigma X to the power n as actually Sigma X y at such TY to the whole to the power N and then dy dx and we can find out that what is this you know the final explanation and now we can use this expression in D S and also we have already found out what is WS.

So we apply all these things together the terms look little bit algebraically involved but it is as such it is very innocuous because it is just simply that we have several integrations to carry out that song so what you will find is that this is how the entire you know system would look like that 2 to the power n into 4 J this J of course will be capital divided by T to the power n times this integration 0 to L Sigma X max to the power M DX.

And the other integration is 0 to t by 2 BY Y to the power n dy so that is what is my top part and in the bottom part it is d by 2 pi w so 2pi is here and then the work done which is to IZZ 0 to L Sigma X max squared DX so if we now you know simplify all the things and we can get the 2 term out the J out modulus of elasticity out.

Basically those terms we are taking it out which are not going to change either with respect to X or with respect to Y so that is how our this expression will be coming up and which can be further simplified and clubbing all these terms together all the constants and all the variables in one side then the integrations one by one that 2 integrations are there.

Now in this integration there is this term sigma x max which will be coming out and we would like to normalize it we would like to normalize it with respect to the endurance strength which is the strength against fatigue so instead of sigma x max we would like to put it Sigma X max divided by Sigma E N and then this is how it will come into picture.

Now that you are normalizing it here it should also come out as a particular property here you are pre multiplying the same so that the effect will not be altered only thing we are normalizing the Sigma X max so this will be our final normalized relationship of the system

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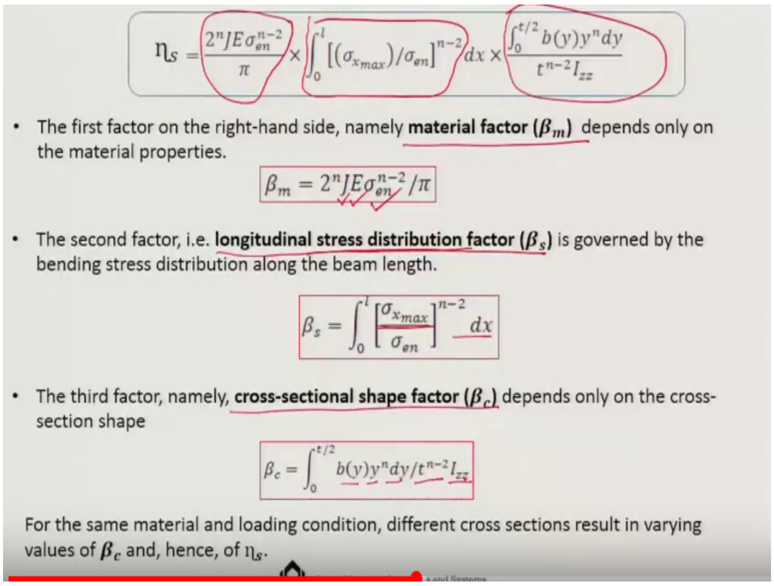

Now with this if we actually evaluate ETA S what we are going to see is that ETA S is going to have 3 parts in it 3 clear parts.

The first part here and that this part is if you look at it that the first part here with respect to the ETA S components that defines ETA S then the first part has all the material properties in it then we have a stress related part which is the loading related part you may say and then we have a third part here which is actually the geometry related part.

So we have the first factor we will call it as the material factor B time even though there are some constants in it but the most important we 3 material factors are there J E and Sigma Y n to the power n - 2 the second factor is actually the stress distribution factor because that is talking about how Sigma x max is distributed all over the length of the system

So that is the longitudinal stress distribution factor and the third factor if you look at it that is a cross sectional shape factor and that talks about that how these cross section is changing so you have BY Y to the power n dy / T to the power n - 2 times IZZ. So clearly speaking our loss factor has actually dependence on 3 factors now.

Material factor longitudinal, stress distribution factor and cross sectional shape factor which we will be calling in smooth as beta M Beta S and beta C now let us look at it that for the same material and loading condition how different cross sections that means beta C how that will create a variation in ETA S itself.

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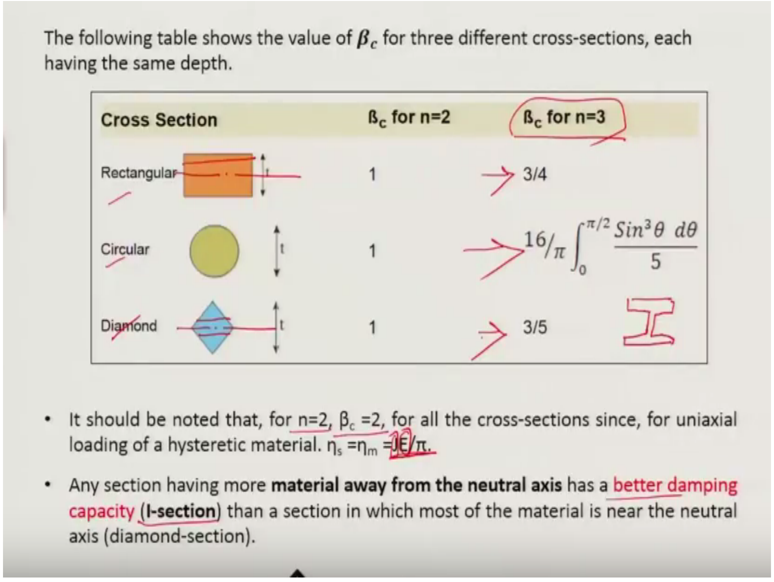

Let us see that type of a case so here we have considered 3 cross sections one is a rectangular cross section another is a circular one and the third one is a diamond cross section so rectangular circular and diamond cross section and their thickness is the same in all the 3 cases now the beta C for N equals to 2 is actually also same if you try to evaluate we will see beta C for N equals to 2 is not changing but Beta C will change for N greater than 2 like for N equals to 3.

Here you can see that rectangular section is going to give you about 0.75 diamond is going to give you about 0.6 and circular of course you have to carry out this integration but if you do all these things then what you will find is that among this particular cases of course rectangular is definitely better than the Diamond cross section.

In fact, the reason why it is better is because with respect to the you know neutral axis if you compare between the 2 in the rectangular cross section there are relatively more materials which are away whereas here diamond cross section more materials are close to the neutral axis and less materials are having. So as a result the beta C is actually more for the rectangular cross section and less for diamond.

Now if N equals to 2 if you keep and beta C also equals to 2 then for all the cross sections the ETA S would become the same as je over PI that will system the geometry dependence will not be there the last factor will only depend on J and modulus of elasticity. On the other hand for other values like for beta C for N equals to 3 you will see that this separation is coming out.

Also keeping in mind that it is the best if you go away from the neutral axis possibly this type of configuration where you are maximum away from the neutral axis will be always good for this system so these are actually called I sections and these are found out to be having better damping capacity than Diamond or circular or rectangular section.

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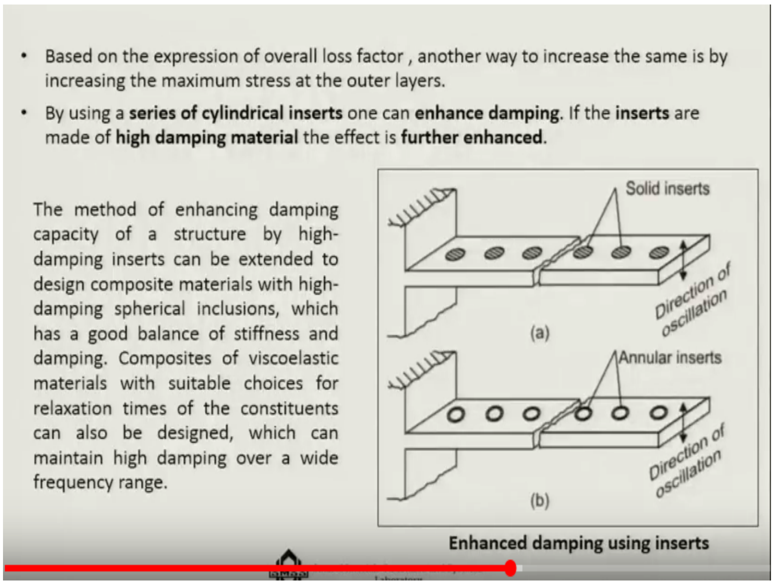

Now based on the expressions of the overall Loss Factor we have talked about how you can use beta C but we can also see the effect of this term that is the you know the stress concentration part of it ok so the stress distribution factors the beta S itself. So we can have a look at this particular term

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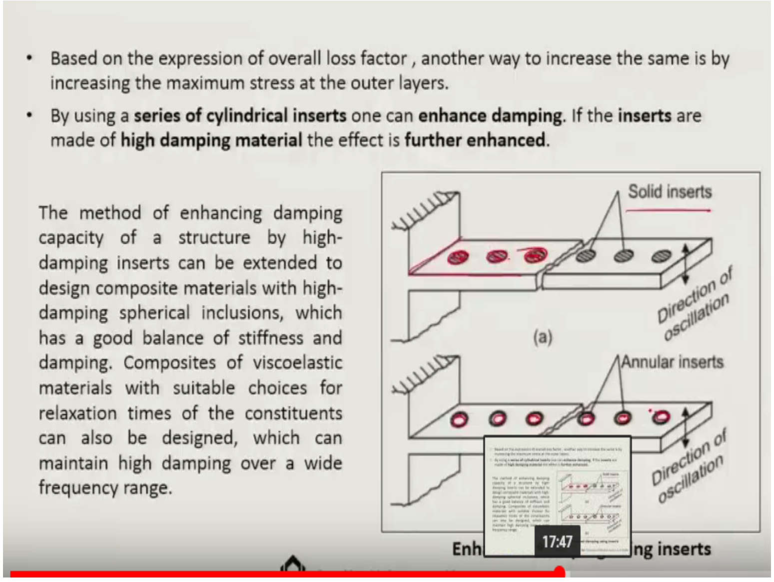

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So towards that direction if you look at it you will see that you have some options there for example this is what I am going to experimentally prove unit on that for example instead of having a single structural material what if we actually put groups here the groups will increase the stress concentration and in these groups what if we fill it up with the high damping material.

So tentatively we are going to get 2 advantages because we have the groups so we have higher stress so there will be stress concentration so the cross-sectional property related damping would increase and also because we have replaced this material by some solid viscoelastic inserts which is itself a high damping material so my damping is going to be doubly benefited by this type of system.

Now in some cases they have also tried another variation of it that means they have used a annular one instead of having full area they have used an annular one this is also a good way of doing the things because at least you know you are using the partly the material and stress concentration you are generating anyway because of this particular you know use of this what particular cross-section.

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I told you that the structural damping can be actually improved if we can control the stress in the cross section of any structural member and further to that in the region where you are actually concentrating the stress if you can have viscoelastic material then the loss factor will be even more in those region and as a result by actually altering the load path and as well as by applying damping materials.

We can enhance the structural damping enormously now today I am going to show you one experiment to which we can demonstrate it so this one is an aluminium bar as you can see it's a solid aluminium bar and this aluminium bar we are going to put it on this particular test setup and we are going to heat this aluminium bar with a small impact hammer.

This entire test procedure is actually used to find out the modulus of elasticity and the velocity of sound in a solid and this follows a particular ASTM code which is known as ASTM e one eight seven six now according to that code we have designed the supporting system and there are where's there and all we have to do is to actually keep this system here nicely suspended on these and then we have to heat it with this impact hammer.

And this sound system is going to pick up the sound that will be generated from such a system and it is we are going to show it to you the data that will be captured in a in the computer and we will show you that the structural insert what is the level of damping and if we change the structure inside if we is for solid aluminium what is the level of damping and if we change the solid aluminium by something which has actually the structural inserts how this is to get changed.

So we actually take this solid and we actually hit it so you can see this clear and loud sound which aluminium having quite no damping you know you get this so these 3 recordings we have done with this solid insert now we are going to replace this solid insert where this solid gel aluminium bar by another aluminium bar and you should look at it very carefully that this aluminium bar has actually this small holes here.

These are not through holes only up to a certain level the hole done and the reason is that in the top level then there will be further stress concentration in the system and this small holes we have actually filled it up with viscoelastic material and we are going to put it in the same region here and we are going to carry out the same experiment with this system.

You can see that the sound got changed meaning thereby there is much more energy dissipation in this system so thus we have hit both of these samples now we are going to see the recorded result so now we can focus into the results that we are getting into the system.

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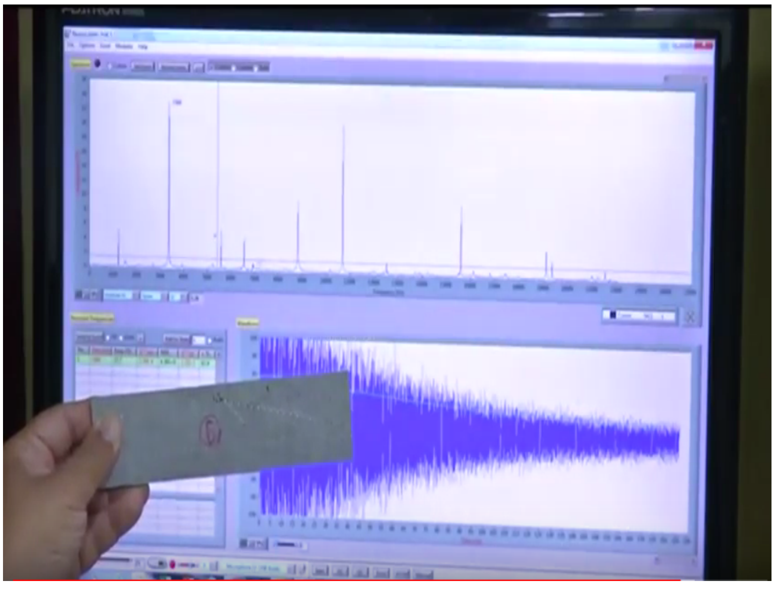

We are now going to show the frequency response function first for this solid aluminium bar and as you can see. Here that there are several frequencies that is appearing as we hit this system and we have chosen this frequency the second frequency which is 3366 harge and its amplitude is 23.7 and the quality factor is 7.79 into10 to the power - 4.

So this is when we are considering a solid aluminum bar and we hit it this is the frequency response function a frame and this is the amplitude that we are finding which is 23.7 and the resonating frequency correspondingly is 3366. Now from there if we go to the other bar where we have developed the stress concentration and where we have applied this viscoelastic material artificially.

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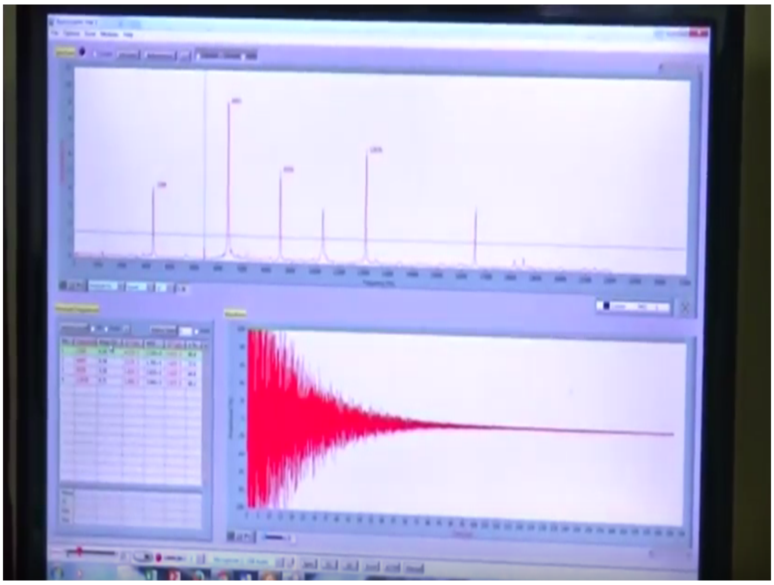

Now if we go to that then we will see that the frequency has actually reduced it is 3289 harge and also the amplitude in comparison to 23.7 now the amplitude is 4.29 so the amplitude has actually come down. One-fifth of the amplitude that it had when there was no such holes and the viscoelastic inserts that was there in the system so that is a fantastic improvement that we can see.

In fact, if we look both of them then you can see that this blue part is actually when it is not damp and the red one is actually when it is damp and we can see that how fast it is damping with the red one and it's this particular frequency are you have seen that the amplitude has come down very sharply.

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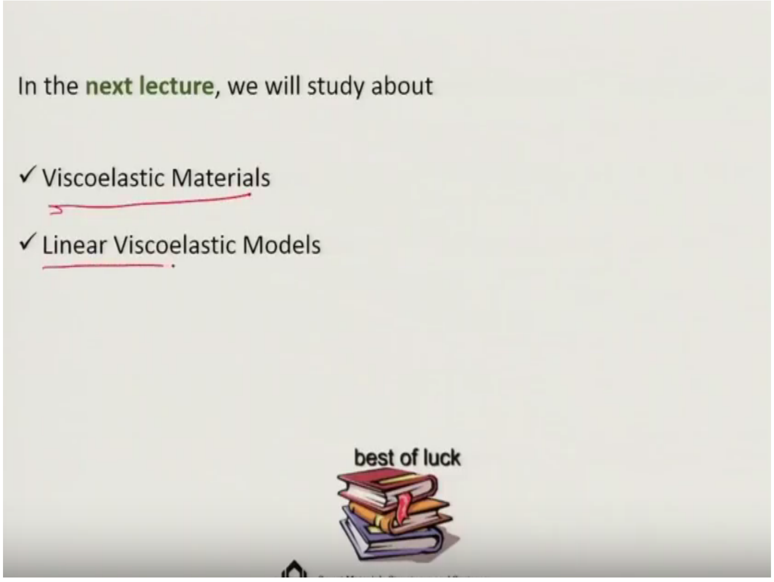

So this is what we can say as a very good strategy in terms of enhancement of structural data for this particular analysis we have used a trial version of buzz or sonic software this we have used and using that we have carried out this particular analysis but we are going to talk about viscoelastic materials and linear viscoelastic models. Thank you.