Principles of Vibration Control
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Lecture – 07
Material Selection Criterion Against Damping

Welcome to the course on principles of vibration control. We have discussed about vibration damping models and on that I told you that there are some methods other than whatever we have discussed in the last lectures in which are using which we can also obtain the damping ratio and the damping coefficient. So that is what I would like to first elaborate today before going to the next points so this is like the damping in which we can actually find out using a percentage overshoot technique.

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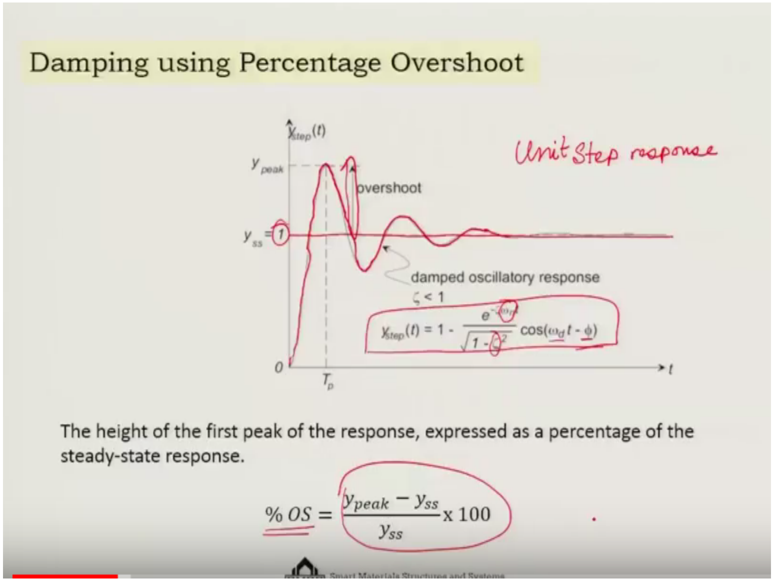

Now in this technique what we do is that we apply a reference signal. So this is like a unit step signal or a reference signal that we have you know excited the system with ok. Let's say you know we are just if you consider that there is a structure for which you are trying to find out the response of the system and you are just placing like this is a load and you are just placing the load on the structure very gently and they are on it continues to remain there till T tends to infinity.

If we do like that and then if you look into the response of the structure what you will see is that in the response this structure will first go and I am just approximately following the line there can be a little bit of breaking but it will simply go to the overshoot region then it will come down and then gradually the vibration will get settle down until and unless that response will follow the static deflection pattern of the system.

So this is called step response of a system this is a step response and since we have used a unity value in this step so it is called unit step response of the system. Now in this unit step response the moment you obtain such unit step response Experiment then you can actually try to characterize it because mathematically if it is a single degree of freedom system then it is depicted with respect to the damping ratio.

Zeta with respect to the natural frequency Omega N and the damped natural frequency Omega D and the phase difference Phi by using this kind of a relationship. Now since we know this relationship mathematically. So we can also find out that what is the overshoot that has happened in the system and from that we can also find out the difference between the peak response and the steady-state response as a ratio and in the terms of the percentage of that. This is what is known as percentage overshoot.

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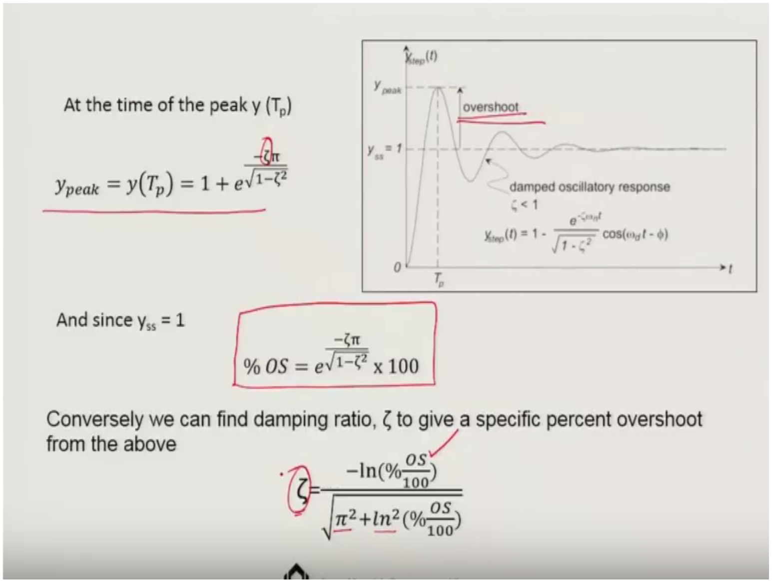

Once we know the percentage overshoot then actually we can use these same formulations in terms of obtaining the damping ratio of the system. So for example, here we know what is the peak response of the system and we see it here that this is a function of the damping ratio and hence we can also find out the percentage overshoot and we can find out that the percentage ratio is a nonlinear function of the damping ratio of the system.

Now you can also reverse the ratio that means if I know the percentage overshoot if this is known to me through the experiment I can get the damping ratio by actually reversing this what you will see is that the damping ratio is depending on the ratio of two logarithms one is this - logarithm of percentage overshoot and another is square root of pi square + logarithm square of percentage overshoot.

So if the percentage overshoot is known to us we should be able to find out these ETA using this kind of a relationship and you know then once you know these Zeta which is the damping ratio you should be able to find out the structural Loss Factor or the you know loss factor itself we should be able to find out from the damping ratio itself. Now this is like how to obtain the damping constant using another root that is what I first discussed.

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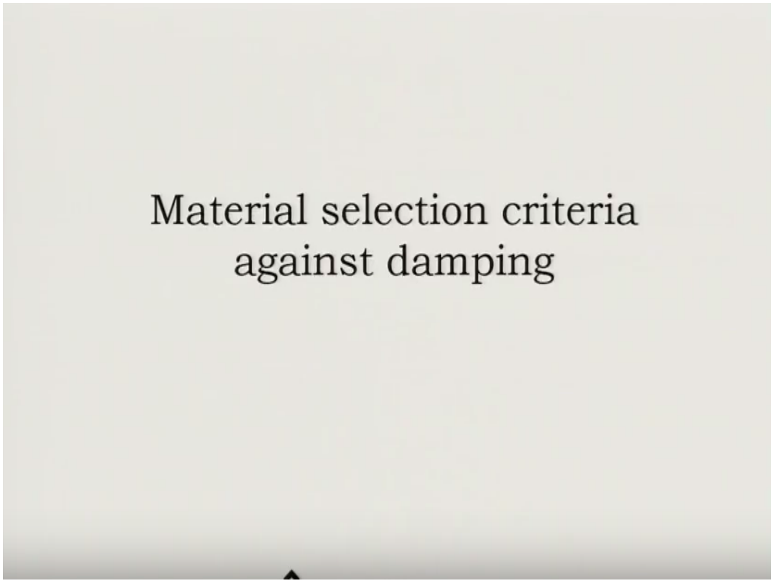

Now we want to talk about the main theme of today's lecture that is the material selection criteria against damping. How do we select a material using what mechanical properties and using what kind of a figure of Merit or a material property index in terms of actually choosing a material which will be good against damping so that is what we want to discuss today with the help of some examples?

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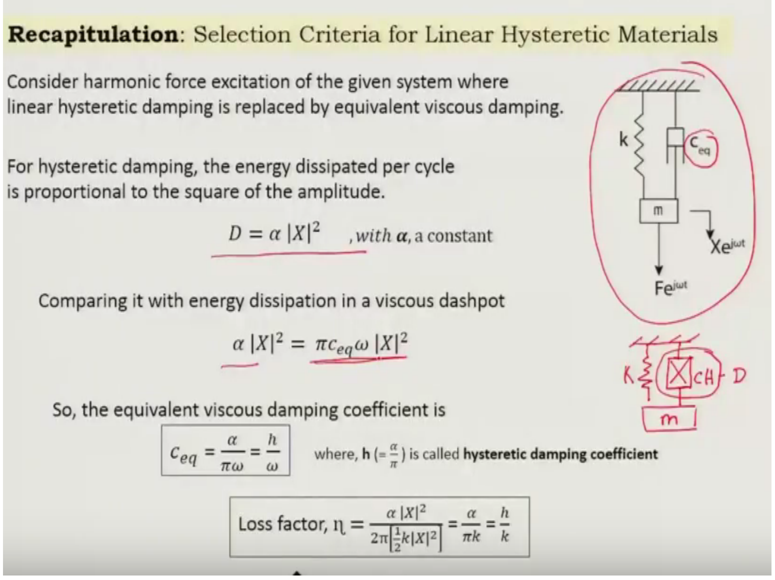

To start with this example let us just recapitulate what we have discussed on structural damping so I have told you that this is what is the equivalent representation of a structural damping which means if you remember those symbols then the structural damping is this is the K and then you have the structural damping system and this is associated with a mass M M K and this is the structural damping ok.

So damping due to hysteresis and this part we said that you can actually model it as an equivalent viscous damper this is the C equivalent once you compare the energy that is dissipated through these you know particular structural damper compare that with a viscous damping energy dissipation then you can actually write this kind of a you know symbol and with the C equivalent.

So the energy that is dissipated in the case of structural damping I have already said that that is alpha mod of X square where alpha is a constant and mod of X is simply the amplitude of the response of the system. Now this is actually equated with the energy dissipated for an equivalent viscous damping which is PI C Equivalent Omega mod of X square note that in the left hand side case that is the case with structural damping the energy dissipation is not a function of frequency whereas in the viscous dashpot case it is a function of frequency.

So this is like it is not a fair comparison between the two but it is some sort of a equivalent damping that we are trying to figure out through this exercise now the equivalent damping of course will be then if I compare the two C equivalent will be alpha over PI Omega and by denoting HS alpha by PI which is known as the hysteretic damping coefficient we get C equivalent as H by Omega that is first thing.

That this C equivalent has H by Omega and then you also consider our definition of the loss factor where the numerator is the energy dissipated per cycle alpha mod of X square and the denominator is the so alpha mod X square by 2 pi so to say and denominator is the energy stored in the system so these 2 pi actually goes here that is hop KX square.

So if I do all the same you know simplifications then it will come out to be H over K that means H as the hysteretic damping coefficient and K as the stiffness of the spring these two will be responsible for obtaining the Loss Factor ETA of the system so this the details we have discussed I am just recapitulating as this will be of very good use in today's discussion.

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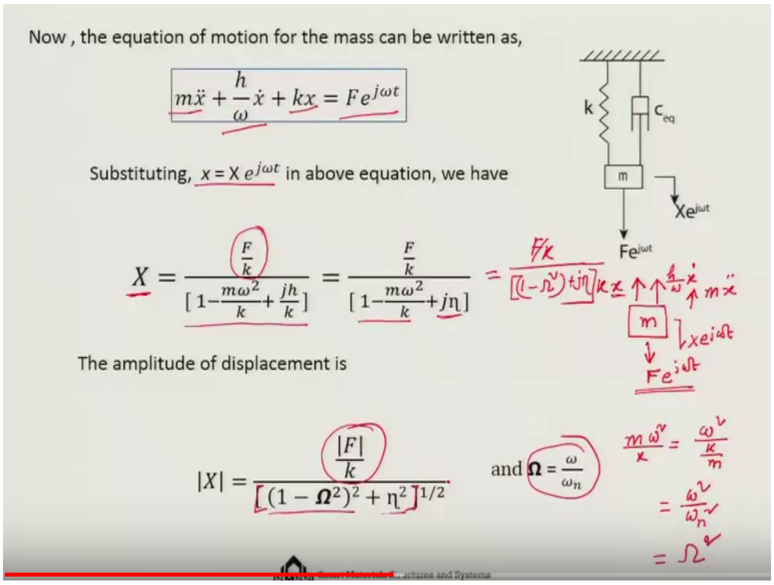

So now for this system which is having a single degree of freedom and it is having structural damping the equation of motion will be MX double dot that is the inertia force. So you remember once again the <free body> diagram that I keep on discussing which is important for us that this is the direction of motion that is XE raised to the power J Omega T.

So that means these are the forces working on the system FE raise to the power J Omega T and then you have the spring resistance force which is your KX and then you have the resistive force due to the equivalent viscous damping so that is essentially your H by Omega X dot and also we have the inertia force for the system which is MX double dot.

So you have these three forces MX double dot H by Omega X dot and KX and these three forces are balanced with respect to the F E to the power J Omega T that is what is going to give us the governing equation of motion of the system. Now let us say we are only interested in the steady state solution of the system as usual so let us put this X here this small X as something which is also the harmonic response with the same frequency as the excitation frequency because that is what we find in steady state response.

So X as capital X raised to the power J Omega T if I substitute that I am going to get an algebraic expression all these other things will get cancelled E raised to the power J Omega T will get cancelled from both the sides as we are considering from T equals to 0 to infinity and it is not a non trivial so we can cancel it from the both the sides so we are going to get a relationship between X and your force over K F PI K which is like static deflection of the system.

And we have these 1 - in the denominator M Omega square by K + J H by K also keep in mind that J H by K H by K is nothing but ETA so we can rather write it as F PI K divided by 1 - M Omega square by K + J ETA. We can further simplify it if you just keep in mind that Omega is actually Omega over Omega in so basically you can write this M Omega square over K as Omega square divided by K by M as Omega square divided by Omega n square and that is nothing but Omega square.

So I can write this as something like if by k1 - Omega square there's a real part + J ETA I can write it in this manner if I do that then the amplitude of X will be because this is complex so the amplitude of X will be this mod of F over K in the numerator and here we have one - omega square square + ETA square and the whole thing with a square root.

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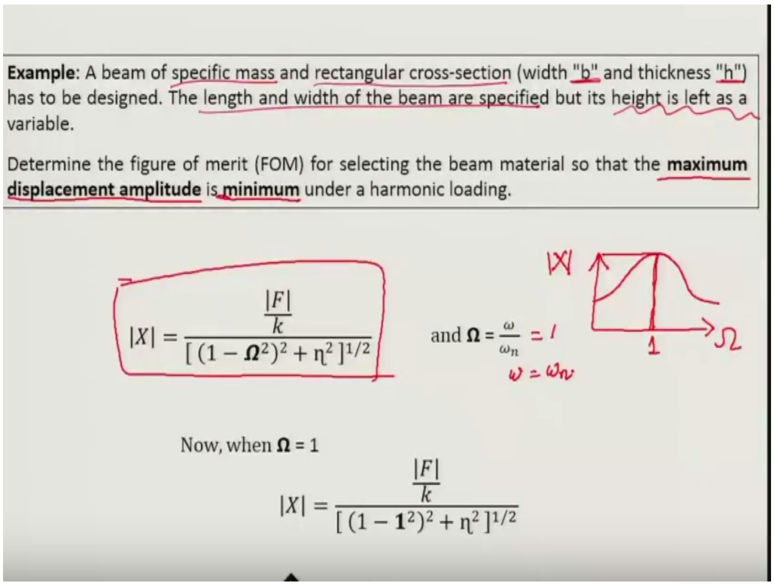

So that is what is going to give us the you know amplitude of displacement of the system now with these we would like to find out that how the material properties actually pitch into it so let us look into that so let us take one concrete example let's say I have a beam of specific mass that means mass is specified 5 gram or 5 kg it is specified and rectangular cross section so that it has a width which can be something like B and thickness which is H this is to be designed and the length and width of the beam are actually specified.

So this is also known so mass is known to us length is known width is known but what is left as a free variable is height. Height is left as a free variable in the system now we have to determine the figure of Merit I will just explain when we will talk about the figure of Merit for selecting the beep material so that the maximum displacement amplitude no doubt that the maximum displacement amplitude is actually minimum under a harmonic loading.

So you want to minimize the maximum displacement amplitude of the system. So what is the displacement amplitude of this system this we have already talked about that this is governed by this particular equation which is mod of x equals to mod of F over K divided by 1 - Omega square square + ETA square and the whole thing in a square root now when you think that in this displacement maximum displacement will occur because if you actually look at it this is a single degree of freedom system.

Right and this is supposed to oh make a plot and this is suppose the mod of X plot with respect to Omega we can also plot it as mod of X divided by mod of F by K to make it non dimensionalize but whatever suppose it is a mod of X over Omega plot so it is clear that at

Omega equals to unity you are going to get the peak so it would look something like this so that means maximum displacement is going to occur at Omega equals to unity.

Why because Omega equals to unity means this equals to unity which means Omega equals to Omega n which means the beam is vibrating at its natural frequency and as a result you expect maximum in resonance from the system now this beam can be having many degrees of freedom eventually that would mean that it would have many natural frequencies not just one.

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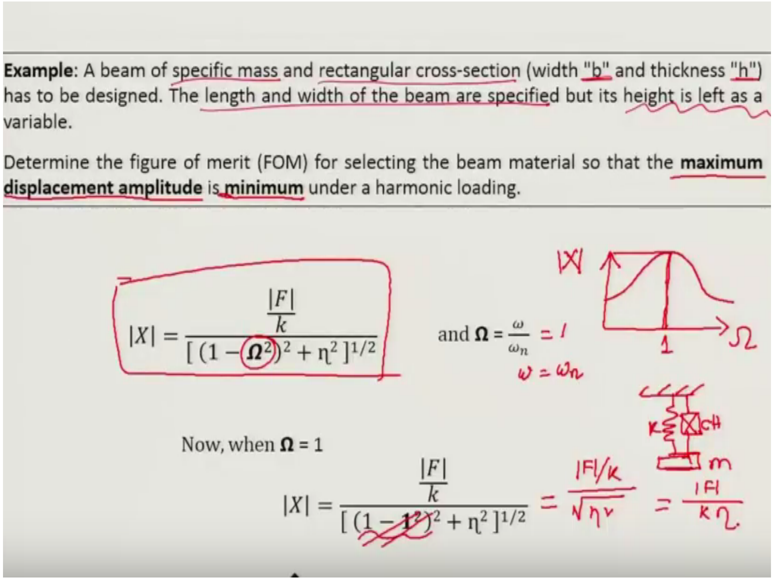

But in this case we are focusing on the fundamental frequency of the beam and representing this is a mathematical abstraction that representing the beam as a single degree of freedom system ok so that is what is the basic representation that we are carrying out for the beam. So it's a single degree of freedom system MK and the you know hysteretic damping of the system.

Now so Omega I have to put equals to unity so in this expression I can put Omega equals to unity so Omega Square will be unity so that means this part will get cancelled out it will be zero. So that means what we are left with maximum displacement is actually F over K divided by square root of ETA square in other word you can write it as F over K ETA that is what is the maximum displacement amplitude with respect to the non-dimensional frequency ratio Omega for the system.

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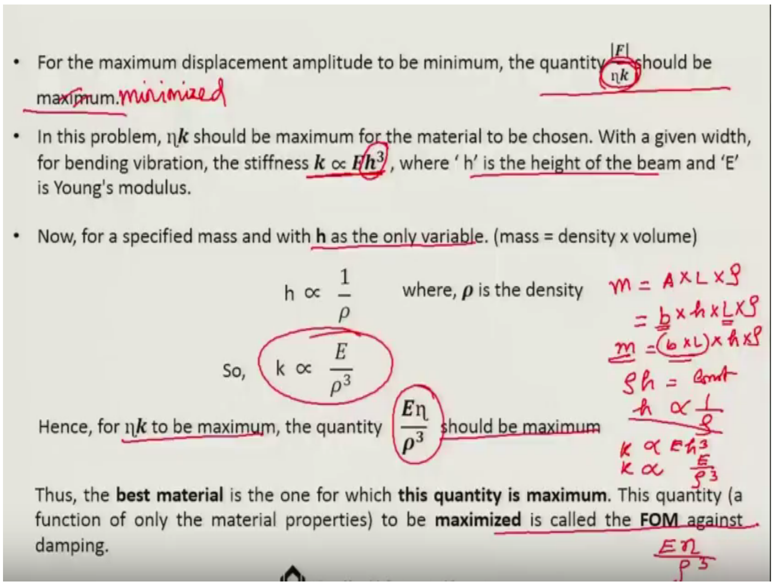

Now you have to sooner is choose the material such that this F by K ETA gets minimized that is the point we have to keep in our mind so the quantity F by ETA K should be actually you know that has to be minimized basically ok so for the maximum displacement amplitude to be minimum this should be in fact I'd say this should be minimized finally because that is the maximum amplitude and we need to minimize this.

Now naturally we still do not have any with us a clear material property we know a derived material property that means if ETA is large or it K is large then we are going to minimize the maximum displacement but what is K now if you look at this K for bending vibration you would see I do not go into the details right now but K is actually proportional to the modulus of elasticity of the system E and the cube of the thickness H cube where H is the height of the beam and E is the Young's modulus.

So for a specified mass if the mass is specified and H is kept as the free variable then we know that mass equals to what area of cross section so I can even write that area of cross section so a-anyway area of cross section times length times density and this area of cross section can be written in terms of B as the width times H as the thickness L times Rho L and B are specified for us.

So that means mass is actually you can write that B Times L that is separate which is constant times H times Rho and since mass is also fixed this is also fixed so what it means is that Rho H is actually constant that is why I said that H actually varies inversely with the density because RHO H is to be kept constant so if you increase Rho H decreases and vice versa.

Now where exactly we are in terms of the stiffness K Stiffness K if you look at it that that is now coming out in the system ok because we have this E H cube right so K varies with EH cube so let us write K varies with E H cube and H varies inversely with Rho that means K varies with E over Rho Q right.

That is what we have written here that K varies with E over Rho Q so if I have to you know maximize if I have to minimize the a by ETA K then I have to maximize ETA K so it occurs to be maximum that would mean that this ETA times this quantity that means in terms of material property E ETA times Rho Q that is the quantity here that should be maximum because if that is maximum then these denominator is maximum and that means the overall maximum displacement will be minimized.

So thus the best material for which this quantity is maximum will be where we get a high E high ETA and sorry yes High E high ETA and low density of the system so that will help us in terms of finding out the figure of Merit against damping.

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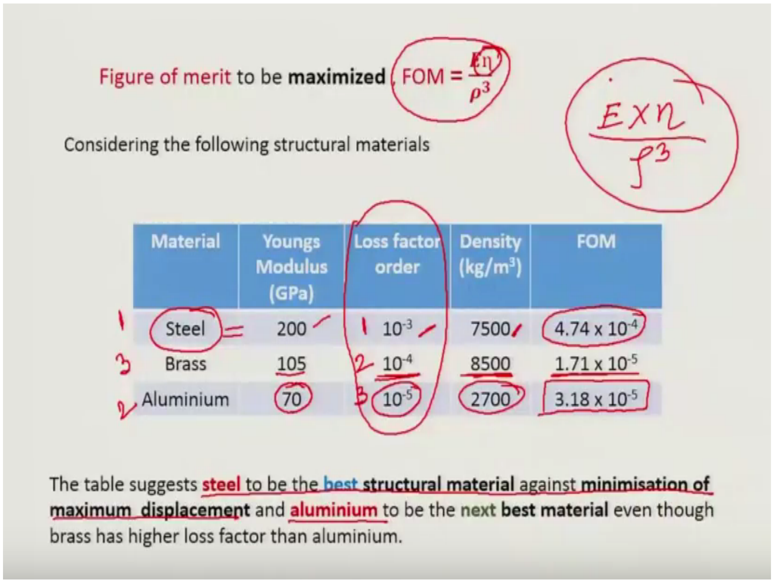

So let us look into some example of this so our figure of Merit is E ETA over Rho cube and we want to maximize this ETA bar ok Let us take three materials that you know still you know the modulus of elasticity has 200 giga pascal steels loss factor is about 10 raised to the power - 3 range. This actually the most poor in terms of Loss Factor in this group and density is 7500 kg per meter cube, average density.

So if you use these values you will get a figure of Merit which is 4.74 into 10 to the power - 4.On the other hand if you use brush with 105 Giga Pascal Loss Factor is 10 raised to the power - 4 so it is actually better than steel in terms of the you can say that it has a lower Loss Factor.

So it has a better resonating property for example in that sense it is better but not of course in the sense of damping and then density is about 8500 which is not good and as the result brass will give you something like 1 point 7 1 into 10 to the power - 5 so that is quite low in comparison to steel Let's now look into aluminum 70 Giga Pascal modulus of elasticity aluminium has a even worse loss factor 10 to the power - 5.

So if you would have only considered the loss factor that is the point that I want to make through this exercise then you would have definitely gone for aluminium you know because the loss factor of aluminium is very low you would not have gone for it you would have directly gone for only the steel which has a very high gloss factor but in this case it is not directly you are looking at only ETA but you have to look at E and density also.

So that is what we are looking at and hence you have a modulus of elasticity 70 GPA density 2700. You are going to get a figure which is not bad at all it is 3.18 into ten to the power - 5 at least it is better than the brush. So we can see that in this case steel will be the best structural material against minimization of maximum displacement and the next best will be actually aluminium and the next to next based that is the third in this figure.

So this is first this is second and brass will be third so if you consider the figure of Merit then this is what will be your choice of material that first you should consider steel because that gives you the maximum figure of Merit. Second you should consider aluminum and third you should consider brass if you would have gone for the Loss Factor order only then you would have thought that first is steel there is no doubt in it.

But second place so this one was first second place would have been gone to brush and third would have been aluminium but because it is not only the loss factor but it is E times ETA Rho cube you have to consider this composite you know figure of Merit so the picture got changed that is very important you know because we are getting an insight into the response of a system we are getting this new figure of Merit which will help us to take a better judgment in the system.