Principles of Vibration Control
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Lecture – 06
Energy Dissipation In Structural Materials

Welcome to the second week course of principles of vibration control. So what are the things that we will look you know focus to in this week. We will focus on dynamic properties and selection of materials.

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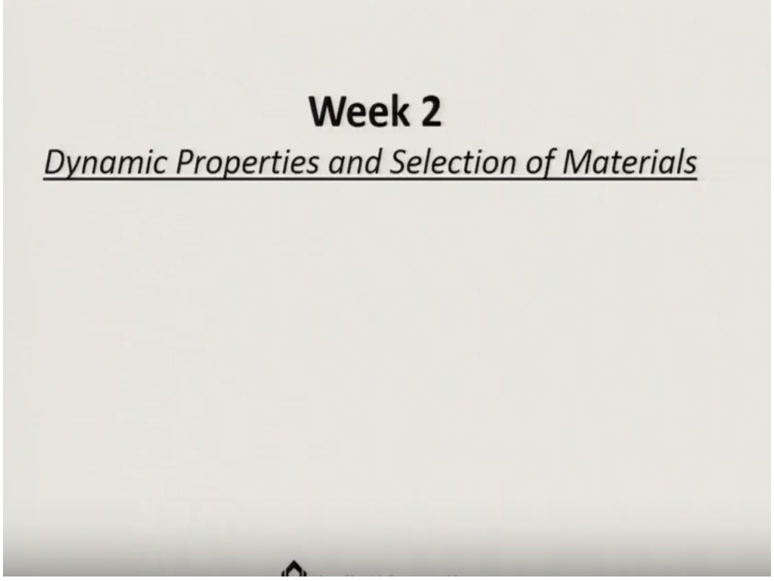

So we now know that, what are the various parameters of the vibrating system. You are also familiar with different strategies of vibration control. So the first important thing now is the that if I have to design a structure which should be safe from vibration point of view what are the dynamic properties that I need to check. And how do I select materials for such structural design. In this week we will focus on that very aspect.

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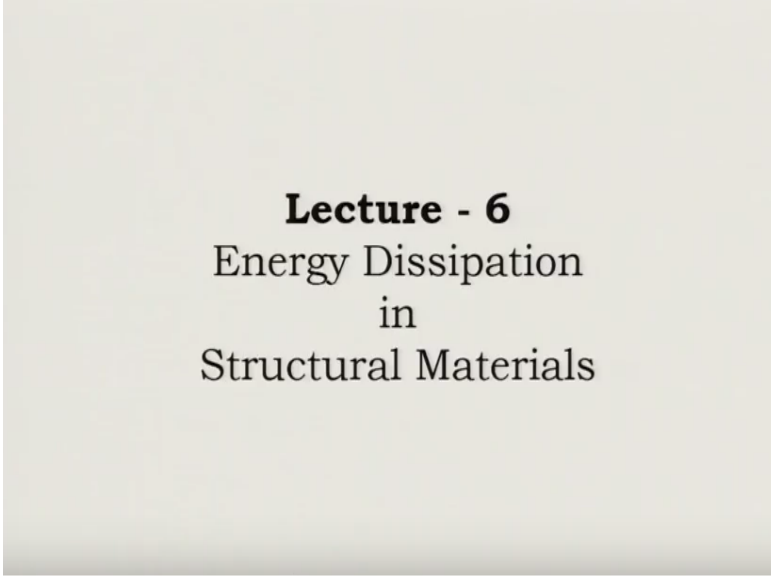

So the first thing that we will see is that if the structure itself can dissipate the vibrational energy that is the best thing because there the vibration can get control very easily inside the structure itself. So we will first see that how the energy dissipation can take place in the structural material itself. So that is what we will first focus on.

Now towards this direction I will talk about quite a few things first of all I will talk about you know how to select a structural material from the vibration control point of view. Because when we generally choose a structural material for a mechanical design.

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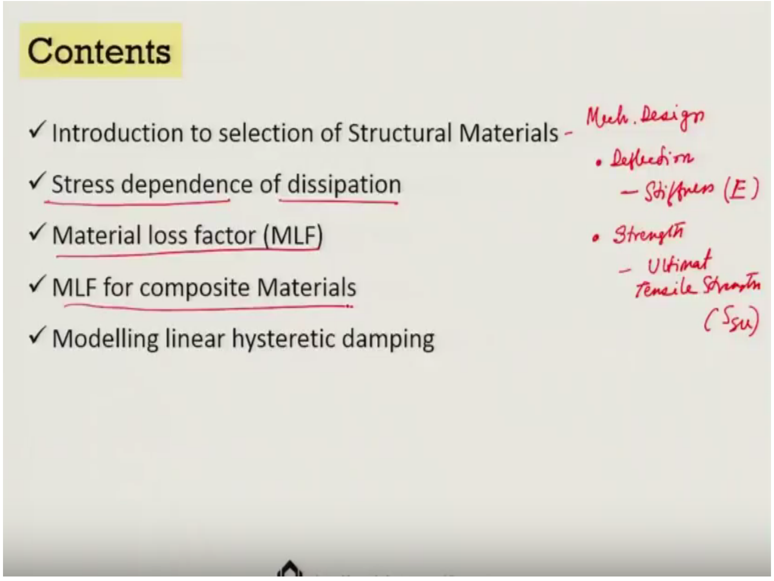

So let us say that this is a mechanical d4esign we are looking for. Then we generally choose it from say deflection point of view where the stiffness becomes important. In other words, this also means that we also check properties like modulus of elasticity and there is another mechanical design which is actually strength based design.

So where things like ultimate tensile strength becomes important for us. Ultimate tensile strength, shear strength, etc. Ok. Parameters like ssu that becomes important for us but we really in fact that is very important that we apply regularly structural materials for vibration control point of view. But we really keep that from the vibration control point of view what parameters are important.

Is this modulus of elasticity is important or ssu are anything else that is important. So that is what we are look into today and we are will also see that energy dissipation what is the stress dependence of this energy dissipation. We will see it. There is also something called as material loss factor. I will introduce it today. You can even get it for composite material. So that also we will see how to get it as a special class of material loss factor.

And then we will talk about how we can model linear hysteretic damping. So these are the things we have planned to do in this particular lecture. Ok. Now the selection of the structural material corresponding to high inherent energy dissipation capability they actually farther depend on 3 factors.

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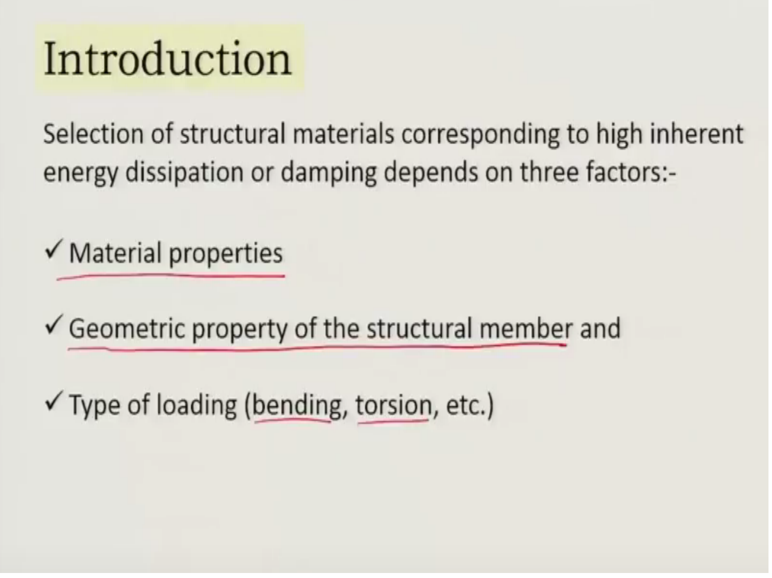

1 factor is called the material property of the structure. So that is what we will be I have just touched on that. So that we have to see. We will also see that the energy dissipation depends many a times on the geometric property of the structure member and.it will also depend on

the type of loading like whether the structure in under bending, torsion or combined bending and torsion or axial loading etc. So in additional to the material property, geometric property and type of loading also come into picture at times in this case.

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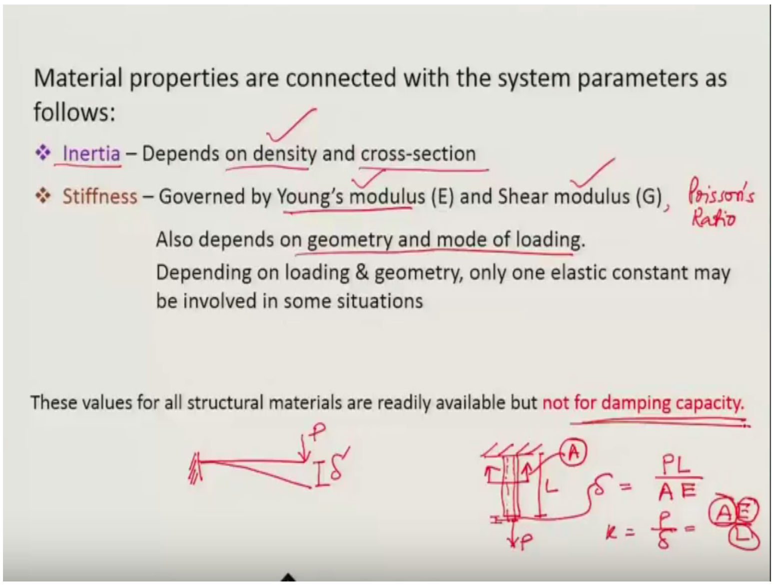

Now the material properties first of all if you look into it because selection of material that is the primary important thing for us. Then in the material property the first important thing for dynamic application is actually the Inertia which in a in an indirect way depend on density and cross section of course is not a material property but a density is a material property.

Similarly, when we talk about stiffness this is governed generally by young's modulus, shear modulus. In some case it will be in addition to that there can be Poisson's ratio. There are some negative Poisson's ratio material which we will call oxidic materials which as the Poisson's ratio variability also coming to picture.

And this stiffness of course it does not only depend on young modulus or shear modulus but also depends on geometry and the mode of loading. For example, you consider a very simple case that you have a kind of cantilever beam which is subjected to axial loading E.

So if this is of length L then we know that the deflection corresponding to this force B the deflection delta that would happen in to it due to the tensile loading. The structure is going to deflect and that is the modified these things and delta is the deflection and that equals to E L over A B where A is the cross sectional area of the systems.

A is the cross sectional area and E is the young's modulus of the system. So that is what is the deflection for the system. So we can see from here that the stiffness if we define that is the P over delta K is P over delta and that is actually A E over N.

So that means the stiffness not only depends on the modulus of elasticity but also depends on the geometric properties like length, area of cross section and also the loading because corresponding to this axial loading its P L over A E and if I consider some other loading let us we have you something like bending loading etc.

So then this is expression is going to get changed. Right. So if we consider the bending deflection then this will be a different deflection, variation with respect to the loading condition and all these things. So these 2 will differ. So thus the stiffness itself will vary depending on not only the material parameters but also geometry and the loading condition.

Now all these is fine as far as the Inertia and stiffness is considered but also the damping is going to change and that also we need to look into it after we totally neglect this part of the parameter that is what we will actually bring today to picture.

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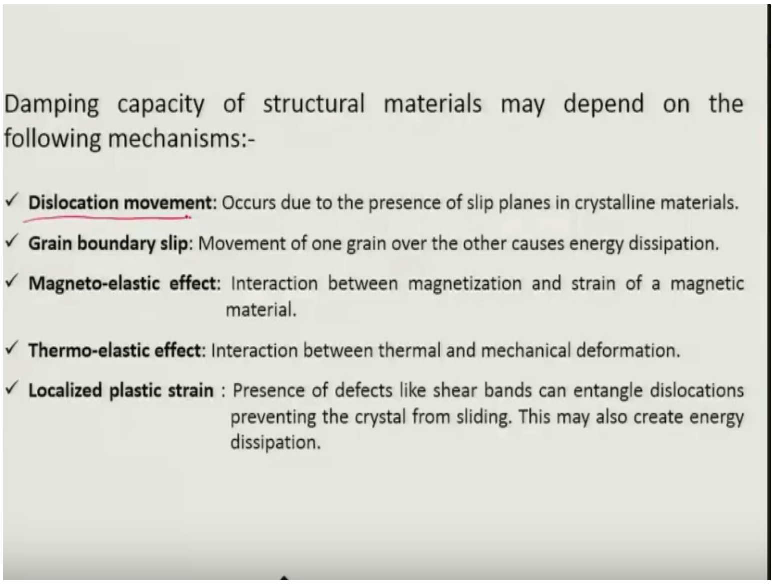

So when we have to think of the damping capacity of a structural material then what is there in the material inherit property that actually gives this nature of damping that we have to think of it. Ok. So there can be many mechanisms inside the material. For example, there can be dislocation movement which generally occurs due to the presence of slip planes in the crystalline materials.

So you know that in any crystalline material there are certain planes inside the material which are actually pr1 for sleeping because they are so high density that. The layer can actually keep away if there is a shear force. So this slip plane shifts and it goes towards the grain bounded. This is what we call as the dislocation movement. Now during the vibration these, energy can be supplied such that his slip plane movements occurs.

And it goes towards the boundary the grain boundary. And hence this process will consume some amount of energy in terms of this location movement and effectively that will be reflected in terms of increasing damping of the system. So thus dislocation movement contributes to damping. Similarly, there can be grain boundary slip.

So if you consider a small you know a structural material and if you look at it lets say using a microscope with a hundred x kind of magnification. And if it is a metal, for example, ok what you will see is that there are various such grains and the grains are actually you know the crystal structures which has 1 single direction of orientation.

May be 1 in this way another in this way etc. So there are directions of crystal orientation. Now in the first case the dislocation movement if there is any failure inside this grain then during the movement it goes to the boundary that is what is our dislocation movement. Second case the grain boundary slip, the grains themselves particularly wherever there is high angle of attack.

Let us say, these grain has the crystal axis in this manner and this grain has the crystal axis in a you know, the relative angle is very high between the 2. So then there will be a continuous sleep between the grain boundary and that will also consume lot of energy and hence the energy dissipation will increase also there can be magneto elastic effect. This I will explain in a much in a much better manner later on.

But right now we can think of it that the magnetisation and strain actually are coupled in a magnetic material. This was shown by joule long before. That in a magnetic material its depending on the grain structure, the grain boundaries etc the magnetisation actually changes.

So some part of the mechanical strain is converted in terms of magnetisation. Then that will also dissipate energy which what is our magneto elastic effect. Similarly, you know about phase transformation and other you know expansions anomaly expansions.

So that keeps lies to the thermo elastic effect of the system and finally there can be some localised plastic strains for some presence of defects like shear bands which can entangle the dislocations preventing the crystal from sliding or may be that you know if you consider a particularly grain and then inside the grain suppose you have the dislocation here and you have some pining by the interstitial position.

So these are the pinning, the pinning you call it the interstitial. atoms. So if that come into picture so there will be some localised plastic strain that will happen. And that will also contribute to the damping of the system. Thus there are several kind of mechanisms may happen that only 1 them responsible.

It may happen that there are many computing mechanisms happening inside the structure which will all contribute to the structural damping of the system.

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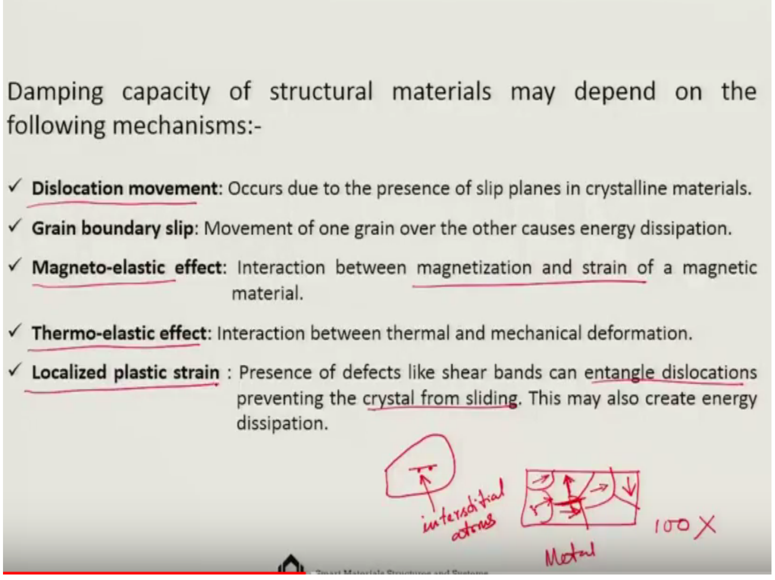

Now be it what may. Whatever is happening is inside. If i take a structural material and let us say I subjected to harmonic loading and then I look into the stress strain plot particularly at a low stress level, then what i will see is that the stress strain diagram actually goes somewhat like this.

It starts from here and with the reversal of stress the strain comes out but there is some you know plastic strain here. You reverse it further then it goes the other side. And then it continuing and this cycle continue on and on. So that is how you get this and this area under itis actually the energy dissipated due to hysteresis we call it.

The reason for hysteresis could be all of those kind of internal dissipation that is taking inside the system. Now suppose we define the energy dissipated per unit volume of a structural material per unit cycle by dn, which is actually given by the area of the hysteresis loop then this is also the mechanical hysteresis loop.

Then this dm, these has been found out experimentally mostly that these dm actually varies with the applied stress sigma in a manner that dm equals to j times sigma to the power n where the j is known as the damping coefficient and n is known as the damping index.

Usually it varies from 2 to 3 in the working stress range. So if n equals to 2 generally for very low stress regime, n equals to 2 so that means the energy dissipation is varying with the square of the stress it is subjected to and if it is at a higher stress range it may vary with the q of the stress.

Actually there is another competing you know theory which says that you can in fact express this dm as j cylon to the power n bar instead of sigma to the power n. So this is a strain dependent coupling. And you can put a different constant here. So this is a better way is because here you know this j is not actually stress dependent like in this case there is a unit dependency that will come here.

But in this case it is not there. But somehow this particular form is more accepted due to hysterical reasons and will continue with these particular descriptions. So at very low stress level if i carry out so that means suppose I take a structural material and with respect to time I am applying a stress sigma. Now suppose this is sigma is an alternating stress, right. That is what is giving us the hysteresis.

If that amplitude of the stress is quiet low, then we are going to see that this what will be the energy dissipation per cycle. Ok. Whereas if we increase this amplitude to a higher 1 of course it should not fail then you would see this particular thing.

That means that this elliptic shape will go out it will vanish and it will become more like a pointed shape at the two ends. And that what happens as the stress amplitude increases and also the expression then becomes instead of sigma to the power s sigma square it would become sigma cube.

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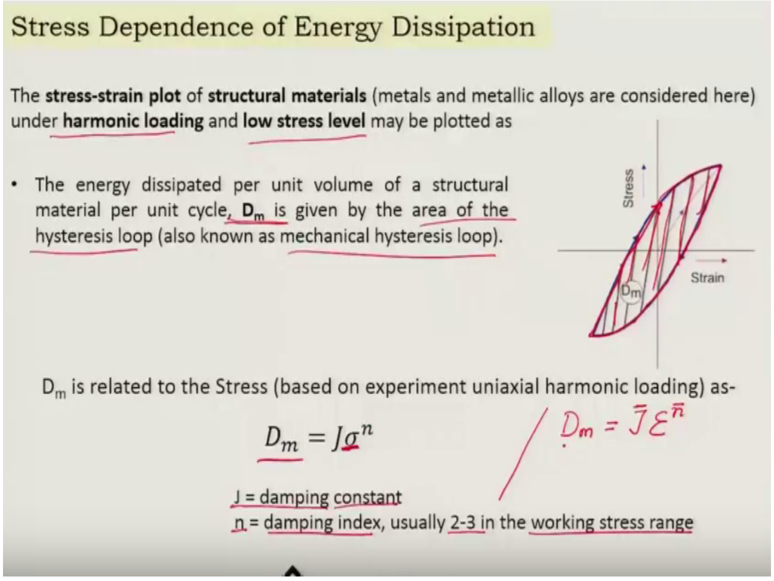

Now 1 important thing however in this discussion is that dm, is independent of frequency. So hysteresis loop does not alter with respect to frequency. This is a very important property of structural damping. Why i am telling is that in case of a viscoelastic material or a damping you know a rubber damping etc the excitation frequency matters a lot.

So suppose the stress level in this case even if it is a relatively high frequency or if it is a relatively low frequency in all the cases this energy dissipation there will be no change as long as the amplitude remains constant, there will be no change. Whereas for Viscoelastic material, there will be a tremendous change between and the these low frequency and high frequency.

And I told you earlier that for viscoelastic material if the frequency changes let us say omega, and then you would see that the modulus of elasticity for example changes very much and low frequency remember is always like a high temperature situation and high frequency always behave like a low temperature situation.

So that is the way it is going to behave. And since the modulus of elasticity is changing so corresponding to that everything will be changing. Now if you compare the performance between the viscous damping and this damping then energy dissipated per cycle here is proportional to the square of the amplitude.

So, in viscous damping, energy dissipation increases linearly with frequency. So it is frequency dependent. It is actually pi c omega x square and in this case it is not frequency dependent. It is it only is proportional to the square of the amplitude. So that is the difference between the 2 systems and over that you can actually find an equivalent viscous damping expression.

So keep in mind that for viscous damping it is pi c omega x square where that means it is dependent on the square of the amplitude. It is dependent on the frequency. It is dependent on the damping constant whereas for this type of structural damping material you would see that it is not the same. Here, the d, I will come to it how we can get the equivalent damping but before that if the n is greater than 2 then for the stress dependence.

Some there are some other representations also which is for the sake of completion we should keep in our mind that you can actually represent it by using 2 damping constants. 1 is j1 sigma square and another is j2 sigma to the power n. So basically 1 is related to the elliptical area of energy dissipation and the rest part that takes is taking care of by the j2 sigma to the power n. Now if the stress becomes multiaxial because there are many cases where the stress may remain actually uniaxial.

In such a case if the stress becomes multiaxial then this sigma is to be actually replaced by sigma equivalent square otherwise if you look at it that our expression is ame. It is again j sigma equivalent to the power n. So only thing we are now writing it as a equal to j sigma equivalent square to the power n by 2. So basically it is j sigma equivalent to the power n and where the sigma equivalent square is actually that depends on the stress constants.

lamda1, the lame's constants and also the stress invariants. I1 nad I2 so these are 3 parameters actually you know if you know you will be able to find out the equivalent stress in case of a multiaxial loading where I1 as the stress invariant. This is sigma1 plus sigma2 plus sigma3 and I2 is sigma1, sigma2, sigma2, sigma3 and sigma3, sigma1 where sigma1, sigma2, sigma3 are the principal stress amplitudes.

So for multi axial case you can handle the situation by know all these principal stress amplitudes then finding out I 1 and I 2 and also we know this lame's constant lamda1 and its value is usually between 0 to 1 in this case and we should be able to find out the sigma equivalent square.

Now the material loss factor could be expressed as eta m that is the material loss factor as the energy dissipated per cycle that is dm divided by the e2 pi wm where wm is the maximum elastic energy per unit volume in the cycle. For you know a very simple case where it is there is only uniaxial loading you know the wm is actually the area under the stress strain curve which is sigma square by 2E.

And that itself becomes lightly more complicated for multi axial loading and it becomes here as I1 square by 2E minus I2 into 1 plus mu over E where mu is the Poisson's ratio. E is the young's modulus I1 and I2 we have already defined here. So you should be able to find out that what is the wm and dm. You can calculate corresponding to this particular formula so we should be able to find out the material loss factor for any particular case

If we know these you know stress invariants and the other mechanical properties of the system. Now so we can calculate the wm and dm. We can actually get the material loss factor.

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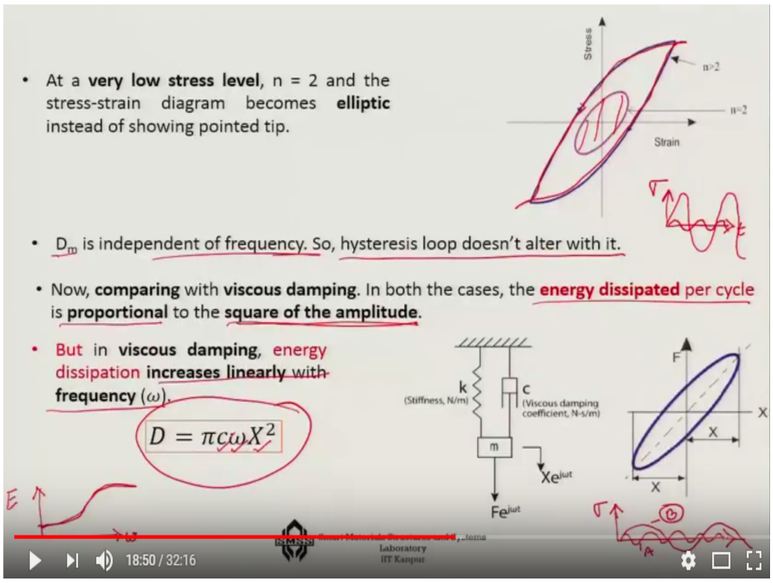

Now if you if the same happens to be a composite specimen. Because composites are generally I will come in to a discussion at a latter stage but they are generally layered structures and they are also used very much for you know many such vibration controlled systems because they have excellent internal damping property.

So this suppose this is like a composite beam and this actually would look very much like your sliced bread. So this slicing can be in different mannered you can you know depending on the situations. It can be either like this. This is 1 way or it can be something if you look at it something that it can be in some other manner.

That means for example it can be in this manner also. So if you have thus various types of this layers then you have to calculate this for a each of the layers. So as the result as you can see here that suppose it has this n layers then you have to calculate the energy that is getting dissipated in each of the layer by this expression.

And then integrating it over the whole length. So first you calculate along this layer so and then integrate it from this to this the whole length and similarly you can actually find out that how much of you know energy is stored and that also you can do over the whole length.

So basically that is the way in which we do it for a composite specimen. As far as the materials are concerned, the kind of you isotropic materials like say aluminum or steel etc for them the order of loss factor I already discussed earlier, that aluminum is possibly the lowest and concrete or cast iron they are possibly the highest in terms of the order of the loss factor.

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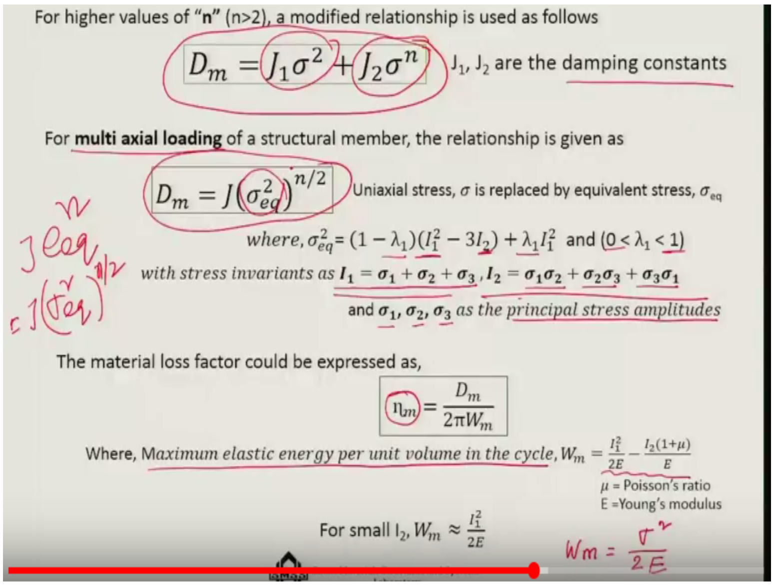

So that is how we can actually calculate the loss factor and compare it with the loss factor of some of the very materials. Now let us come to a situation where we would like to find out hat for a linear hysteric material what are the selection criteria for the system.

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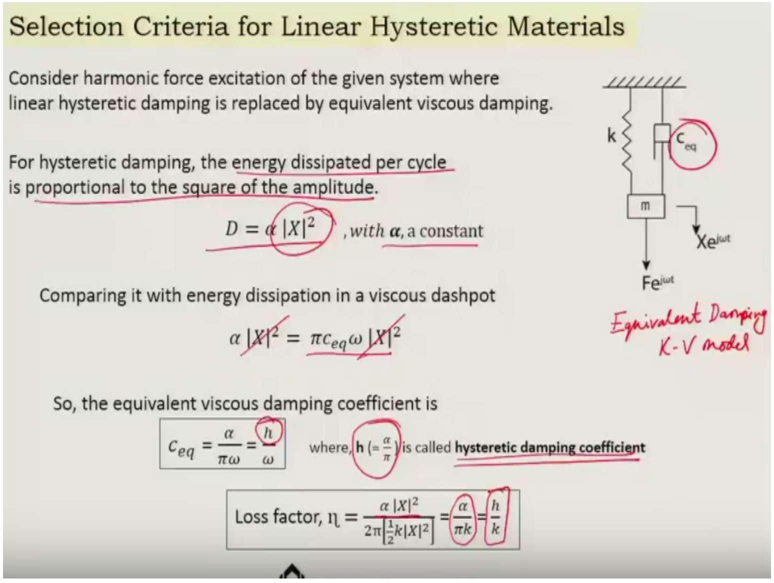

In order to do that what we d1 is that this is an equivalent kelvin Voight representation. Why equivalent it is because simply that that damping parts you know equivalent damping of a kv model. The damping part we have actually replaced by a c equivalent here.

Now how do I know this equivalent. where for the first point is that the energy dissipated per cycle you know that it is proportional to the square of the amplitude. So which means if you use alpha as constant you can write d equals to alpha mode of x square.

Now compare that with the energy dissipation had it been a viscous damping. And you know that for a viscous dashpot. It takes the expression of pi c omega x square. In this case only thing is c is equivalent because we are actually trying to find out what is the equivalent viscous damping constant that can actually replace this.

So we can you know put c equivalent here. And since the x square is going to get cancelled in both the sides we will get c equivalent as alpha over pi omega and these alpha over pi is actually denoted as the hysteric damping coefficient h. So that is how we come as here as the equivalent damping constant which is actually h over omega where h is the hysteric damping coefficient.

Now with this you can also calculate the loss factor remember that the loss factor expression has energy dissipation which you can write F as m alpha x square proportional to you know square of the amplitude as we said earlier and the work d1 is actually 2pi times half k x squares that is you know energy that is stored in the system.

So basically you are going to get it as alpha by pi k and since alpha by pi is h your hysteretic damping coefficient so you can write this as h over k. That means the loss factor depends on the hysteretic damping coefficient h in this case and also on the stiffness of the material.

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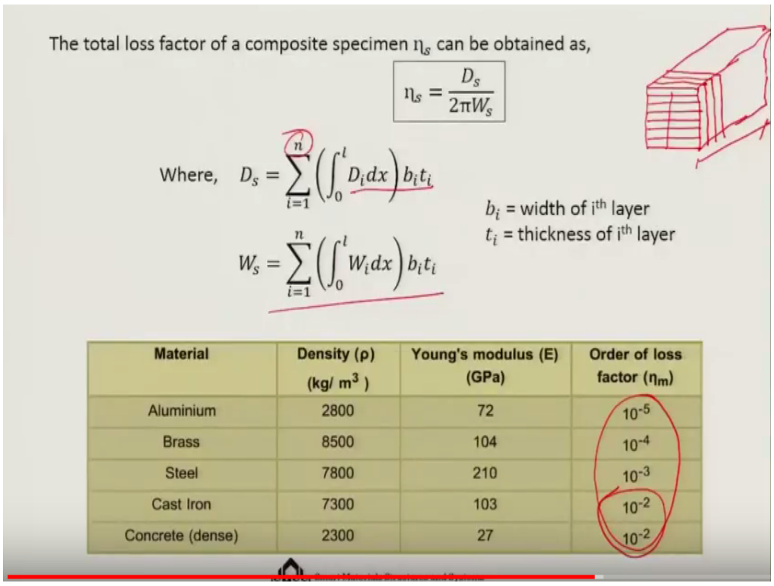

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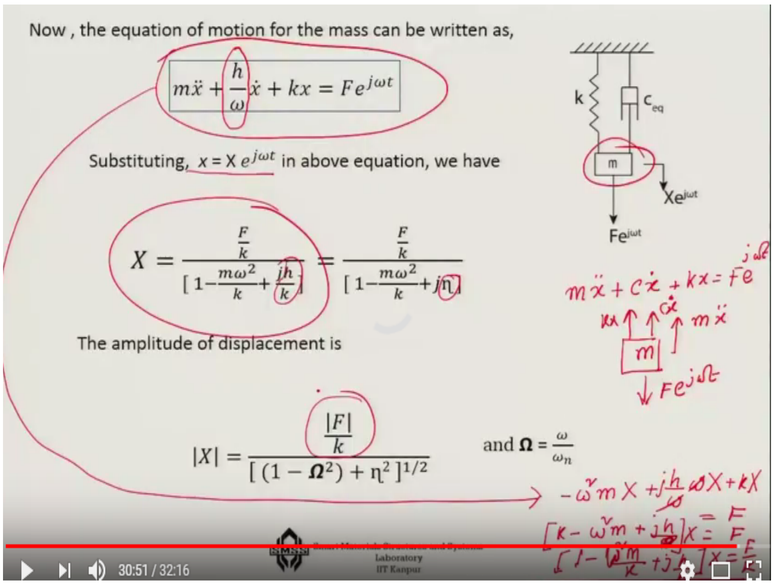

Now let us look in to the equation of motion of the system with this change. So we know that in case of a I have earlier shown you that in case of a conventional damping system, the equation motion is mx double dot plus c x dot plus kx equals to fe raised to the power j omega t. This simply comes if you consider the equilibrium of this mass m, this is your mass m and.

Then this is subject ted to f e raised to the power j omega t, harmonic excitation and because of that there is a inertia force mx double dot and there is kx and there is cx dot. So basically all these things together you know if i make the equilibrium equation I am going to ge this particular equation where the only thing that I have d1 is that this c now i have replaced it here by h over omega by using the equivalent damping constant.

Now here if we are only interested on the steady state solution then i can substitute this x as capital x e raised to the power j omega t and then this capital x will become f by k over if you just do this everything. So we can just try for this particular case for example, if I just write it somewhere her for your ready reference, so we have d1 it earlier also.

That means it will become minus omega square m x ok e raised to power j omega t will get cancelled from both the lass sides plus h by omega and this is again j omega x plus k x which will be f itself and then if i take all of these things together I can write it as k minus omega square m plus j h over the omega gets cancelled

So plus j h times x equals to f. And then if i take the k if i divide it by the k all along then this will become 1 minus omega square m by k plus j h by.k. ok. and that times x will be f over k. If i divide everywhere by k. And then I can get this expression because just a simple you algebraic extension will give u this particular expression where x is f by k1 1 minus m omega square by k plus j h by k. And I can now also write this h by k as the material loss factor j eta.

So the amplitude of x will be what the amplitude of x will be the amplitude of the numerator that is amplitude of force over k and denominator what we are going to get is amplitude of 1 minus m omega square by k square. So it has to be 1 minus omega square square plus eta square and the whole thing will thing will have a square root.

This omega of course omega over omega m so a little bit of simplification will give you this non dimensional excitation parameter. So this is what will be our amplitude of the displacement for the system. So thus we can get n expression for the whole system.

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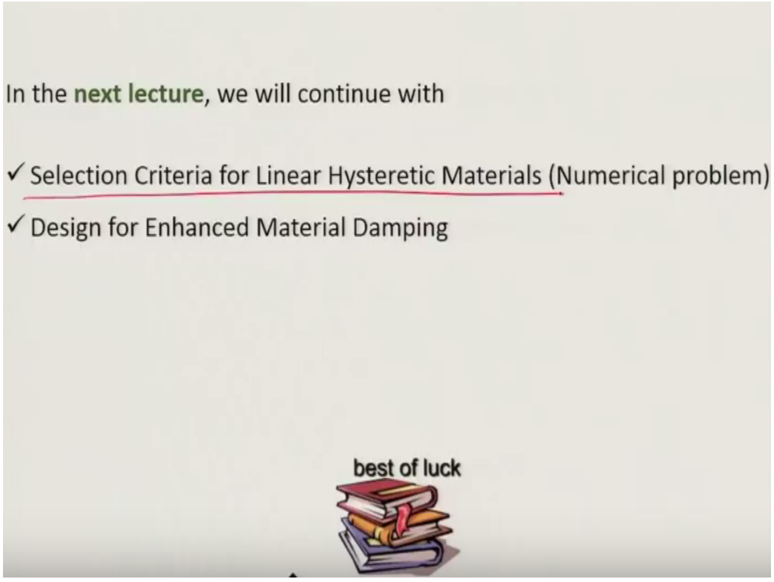

And in the next lecture I am going to talk about the selection criteria now corresponding to this linear hysteretic material. So we will discuss this with respect to a numerical problem, also we will talk about how to design this for enhanced material damping. Thank you.