Principles of Vibration Control
Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology - Kanpur

Lecture – 05
Coulomb & Hysteretic Damping Model

We will discuss today about the few of the damping models. So the damping models of course we will discuss today one of course that I have already discussed that is the viscous damping. Just for continuation, I am keeping that number 2 we will talk about coulomb damping.

We will also talk about next tension of coulomb damping which is known as nth power damping and we will talk about hysteretic damping which is sometimes also referred as structural damping.

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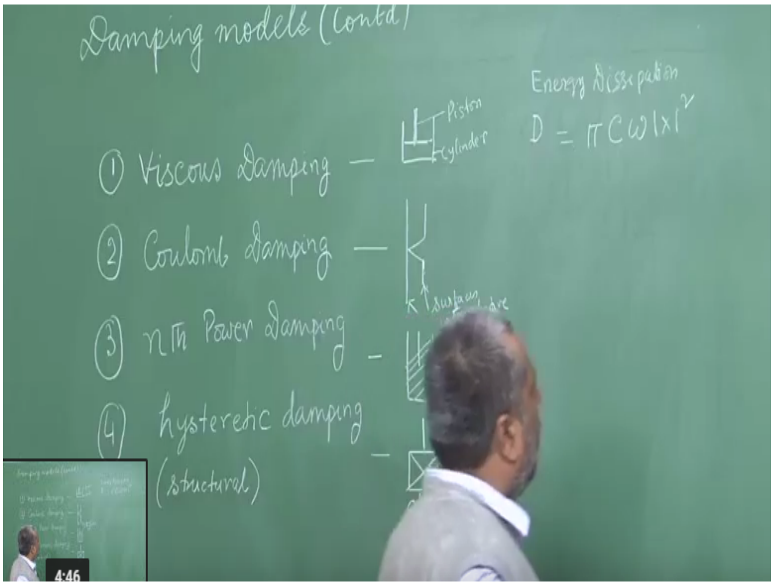

So these are the four dampings that we will discuss. Now first, of all about the viscous damping. I have already discussed that this is the symbol of viscous damping. A typical piston and you know a combination of a piston and then we will have the coulomb damping. Coulomb damping is typically shown as 1 solid surface for which we have another surface which is rubbing over it.

So, that is what the coulomb damping is. So here we have piston, cylinder, this is the cylinder and this is the piston and here we have 2 surfaces which are relative velocities with each other, surfaces with relative velocity. There also we have a special case of nth power damping which looks very close to the piston damping but actually in this case the piston usually will be having an orifice.

So hence we actually represent it in this manner. So essentially whenever the piston is having small orifice in it then you know we will get this power damping in to picture. And then we have the hysteric damping. This is also very much similar to the other damping system except that we will put a block here. So it will be something like this.

And we generally call it as Small C, capital H as the hysteric damping System. So there are the four usual symbols of the various damping systems. Now we have talked about viscous damping. So we may be one important results f viscous damping will keep here now and that is what the energy dissipation per cycle.

So that is the energy dissipation. And that we already know, that this is pi c omega x square where x is the amplitude of the vibration system. So this energy dissipation for the viscous damping we have already derived and we have known that this is it. Now for coulomb damping we would also like to find out what is the energy dissipated and then we would like to see that you know how we can get an equivalent damping constant.

Also we will look into nth power damping and hysteric damping. So, keeping number 2 first in mind that whenever two surfaces are in relative velocities with each other this is the case of friction actually. This occurs as I have very in a very generalised way I have said if that the surfaces with relative velocities with each other so this occurs in very many case where the friction induced oscillation comes to picture.

For example, if you look at the tower bridge. The best (())(05:24) bridges where the entire load of the bridge comes down to the bearing switch hold the bridge and here, you will see that if there is a friction in these bearings then the entire bridge vibrates. In fact, the first failure of the tower bridge happens because of this kind of a bearing failure. So friction induced vibration is very common

And a very simple way of representing the friction induced vibration is that we always say that the frictional force can be represented in terms of the constant fc and signam of the velocity of the system. So that means if you consider to be your 2 systems which are in relative velocity with each other and if this is our direction of movement let us say x. And also this instant my direction of velocity is x dot.

So depending on the sign of this x dot, I am going to have the frictional force, the resisting force which would be positive or negative. So that means you consider that there is a bit of a harmonic motion happening in the system. Let us say, with respect to the mean position.

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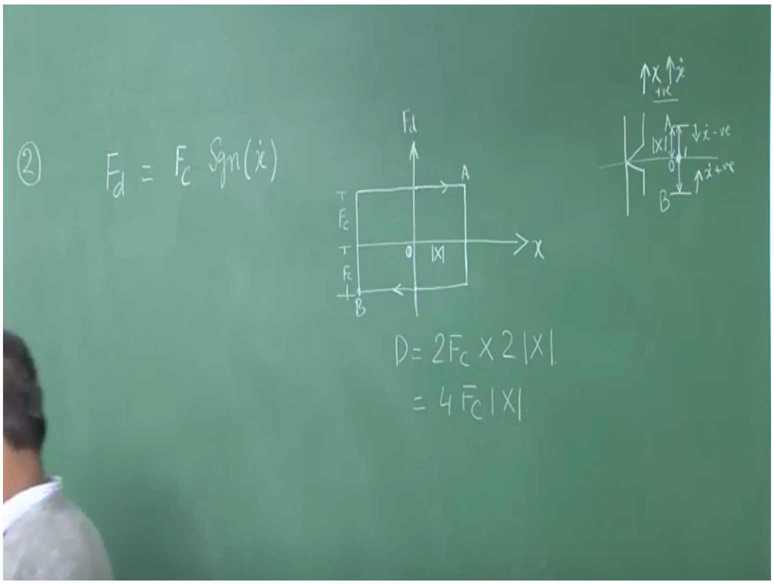

Suppose this is what our mean position and against this mean position we are going up and then down. So up to a particular limit, this friction is what happening to the system and this is what our mean point, O. And this is what is my next point, A and this is what is my point B. So if i try to point this the frictional force with respect to the displacement because that is what is important for me.

That is what is going to give me the total work d1 and that is what is going to give me the what, you call the energy that is dissipated in to the system. Suppose I plot tad ifer as x, so when I am at this point then my displacement is 0 Sign is not defined but the moment I will start to move to the positive direction I am going to get a force here. So suddenly my force will come in to picture and these force the magnitude of the force will remain constant.

So these force remain constant until I go to the point A. So from O we are going to the point A and then the whole system is starling back so as the whole system is starling back, my displacement is vanishing we are going to get immediately because from this point onwards the displacement is vanishing but immediately the direction of velocity is changing.

So this is becoming negative the reverse direction. Now this is my positive direction as we have defined here. So immediately it becomes negative. So the constant remains the same but it becomes negative. And the displacement vanishes at this point, so when I am reaching point O my displacement is vanishing again and then.

I am going to the other extreme here, there is no change in amplitude but only thing is that this is changing. This direction so from point B agin there is a direction change now. From here again the velocity is becoming positive so there is a change now. It has become positive and it has reached it neutral position and thus this is happening again and again. So, this is what, is my force for this displacement relationship while in friction.

So this is in each and every case this maximum value fc which is remaining constant except for the change of sign that is what is going to define for us that. What will be total work d1 in the system so if we say that this particular amplitude is actually a mode of x so if I say be capital X actually and denote it as mode of x this distance from here to here so then this is what our mode of x so this is what our x here.

Mode of x, so we can very easily write down the total area that means the energy dissipated as 2 fc times 2 mode of x that it is fc that is the energy dissipated in this particular case. So we can write here that this is four fc mode of x. So what is you know if I say suppose as you know that this is signam function is a linear function.

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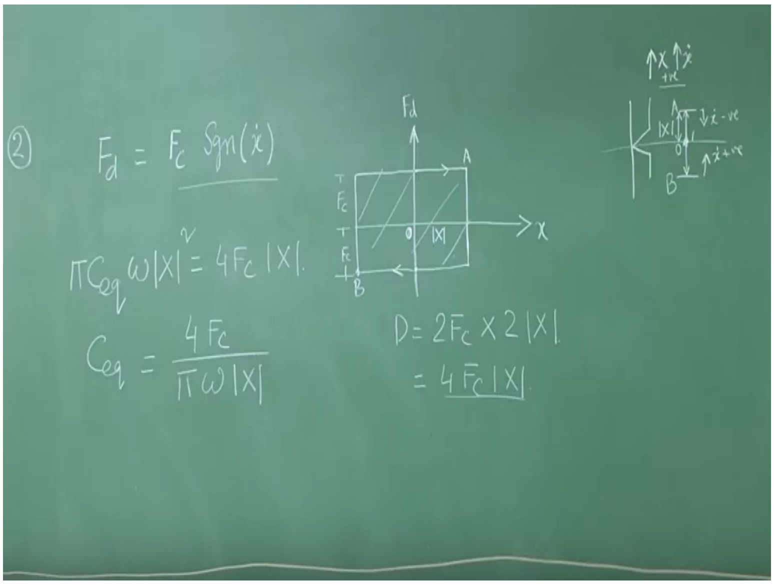

So modelling sometimes in all linear function is difficult. So suppose I want to keep my life simple, sometimes I want to use still the relationship of the viscoelastic system. So I just want to find out what is the equivalent damping constant. In that case what I will do is that I will find out what is this area which is this four fc x

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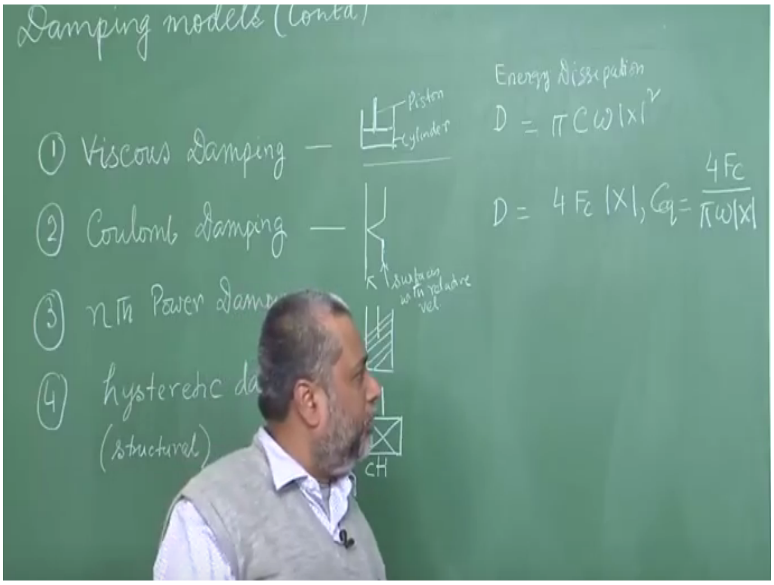

And that let our pi c equivalent viscoelastic damper dispel the same amount of energy per cycle. So pi c omega x equals to now c is equivalent this equals to our four fc x So if with the case, then I can very easily find out that our c equivalent will be four fc over pi omega x. Right. The square x is going to get cancelled X is not trivial. So this is what will be my equivalent damping constant for the system.

So, I can write that c equivalent or my system is actually four fc by pi omega x. So that is what is about the frictional damping. Right now we will keep this much only with us. Later on we will try to solve a governing equation with frictional damping as fc signam x dot and we will also see that what is its application but this is a simpler way of tackling that whenever the damping comes into picture we do not going to the non linear part.

We will simply find out what is the equivalent damping constant and then from we actually post the problem as a viscous damping problem and then solve the system. Now, that is as far as the coulomb damping goes. The next is nth power damping. This is a very special case. This happens particularly in recoil mechanisms.

For example, all the big guns whenever you actually you know discharge a canon shot from a gun, then you must have seen that the canon actually recoils back. Now, this recoiling distance we sometimes try to keep it as small as possible.

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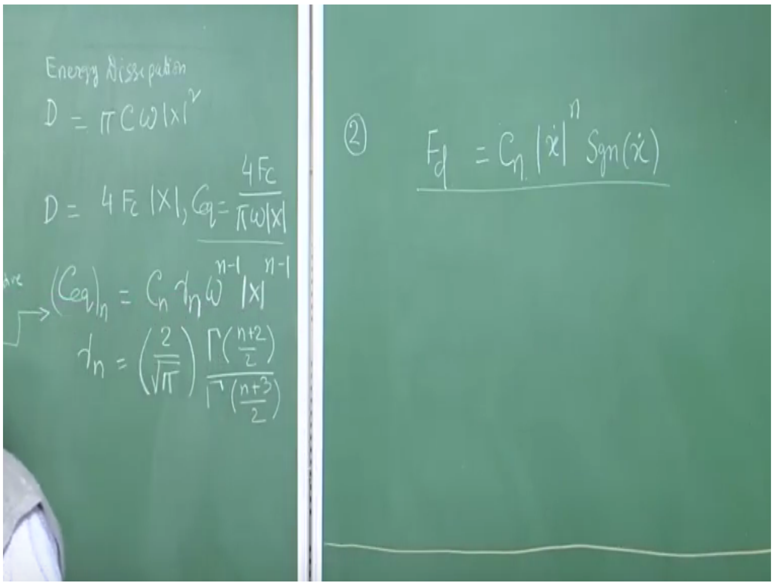

So in such a case this is a very special case where we will get actually the damping force fd as a nth power damping and in this case we write as fd as Cn mode of x dot to the power n. So this is the power of the amplitude times signam x dot. So this is a velocity powered damping. And the value of N is generally greater than 1.

It can be 2, 3 you know cube damping or any other high values are possible with it. Now, in this case if I use this formulation then I will simply write that the C equivalent in this case involves a bit of mathematically treatment but the c equivalent nth power is actually cn gamma n omega to power n - 1 mode of x to power n - 1. Where gamma n is 2 by square root of pi times a gamma function of n + by 2 by 2 divide by a gamma function of n + 3 by 2. I will be not going to the derivation. This is just simply for your reference that corresponding to the nth power damping.

This will be our equivalent damping system. Just like we have d1 the equivalent damping in the other case here also this is the you know, equivalent. This is the actually resistive force and this is the equivalent damping that we will get in this kind of a case.

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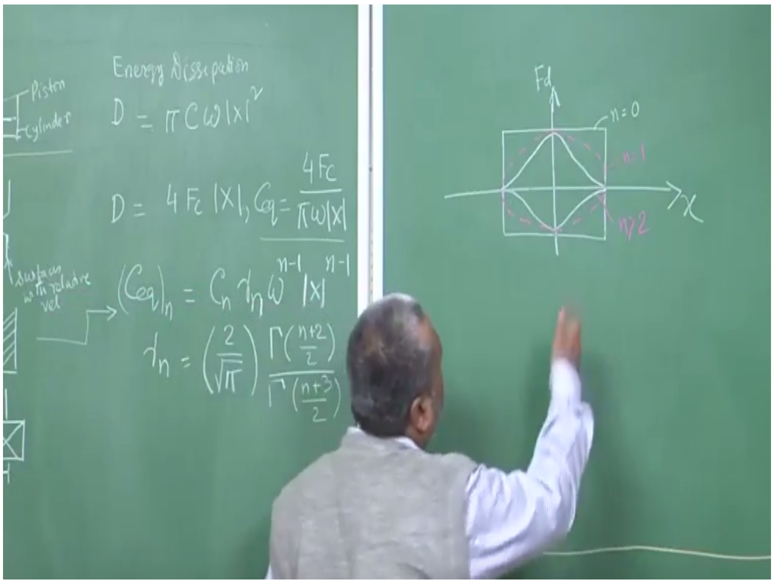

Now, actually all these first 3 things together you can plot them in fact. You can plot them in terms of the shape of the resistive force. So all these first 3 cases together, if you actually plot the shape of the force verses displacement so fd verses x the damping force verse displacement then as we just shown that the very constant case is the case for coulomb damping.

So this is what our coulomb damping case and that is for n equals to 0. This is a special case of the power law when the n equals to 0 so that is what the coulomb damping case. Now if we want to plot the equivalent m viscous damping case then the equivalent viscous damping is actually having an inscribed ellipse inside it.

That is what will be a shape of it. This is the case of viscous damping. So in this n equals to unity. And if I go for higher damping cases then we get actually flattening here and again a sharpening here and again it happens like this flattening here and a sharpening here. So this is what n is greater 2 etc.

That is what is my actual power damping cases that will come in to the picture. Now, of course a you can also get a viscous damping case in terms of that you know a complete ellipse the equivalent viscous damping where the area has to be equal to the area of the rectangle.

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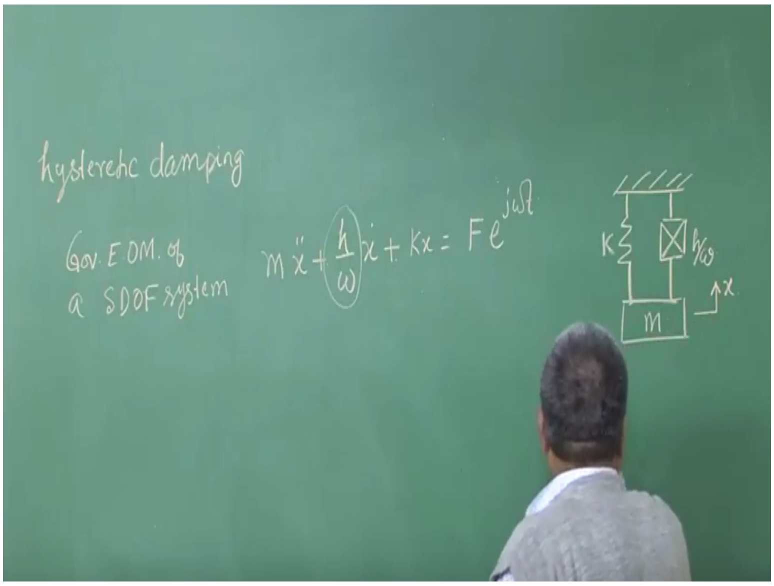

So accordingly we have to adjust the ellipse size but that is how you know if we have to find out the equivalent 1 that is how you have to do it. So this is more or less a graphic representation of the first 3 damping case that we have you know just now discussed. The last 1 is the hysteretic or the structural damping.

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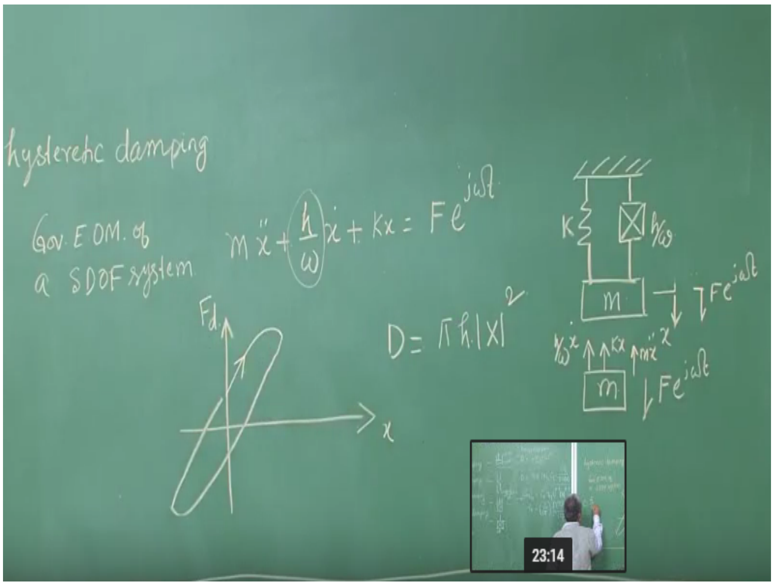

Now for this a bit of derivation can be d1 And I will cover it up in the next lecture but in this lecture I will actually take meme results of that and we will try to construct the governing equation corresponding to this particular case of hysteretic damping. So, for, hysteretic damping which also called the structural damping

The governing equation of motion of a single degree of freedom system this will be an x double dot + the effect of the hysteresis will come. So this will H over omega X dot + KX equals to some harmonic excitation. Let's say fe to the power j omega t. So that these damping kind of constant coefficient that is in this case that is actually frequency independent coefficient and this is h over omega and this is what you know I will be discussing in the next class.

But if this is the model that we actually consider for this single degree of freedom system then for these model. So equivalent representation, if I try to draw it would look something like this that and. I have a K here and I have a block here. Right. That is the structural damping part and that what is why h by omega and then I have the mass here, the inertia of the system and the x is the displacement of the system.

So this is what and the force that is working on the system in this case oppose fe raised to the power j omega t. Ok. So depending on the direction of the force actually we will get the resistive displacement.

So let us say that right now we will keep x in this direction. So that there resistive forces work If you look at the (( ))(21:30) diagram of the system as you have d1 in the earlier cases then you have a KX and you have a resistive force which is h by omega x dot and you have an inertia force which is opposite to this .

So this is m X double dot. And you have the force fa e raised to the power j omega t. So that equilibrium is going to give you this equation of motion of the system. So this is what is our case hysteric damping and how will it look like in this case.

If I actually try to just plot the damping force with respect to displacement, then it will look something like this. That means it will go straight and come here and then it will be going to continue. So, that is what is our structural or the hysteretic damping case. In this case, the energy that will be dissipated is actually pi h x square. In fact, what we have d1 is that simply the c omega if you look at it.

This c is now c equals to h by omega in this case. Correct. C equivalent is actually h by omega in this case. So in our expression of this where we had this c equivalent omega that is simply h. So that is why I said that this is actually h. So in this case energy dissipation is actually pi h x square. That is what is the energy that will be dissipated in this particular you know the case of hysteretic damping model. or the structural damping model.

So these are the four important damping models that come in to the picture. Now along with that I will like to explain a few of the basic terms which are related to damping which we have to use in this particular course again and again. So that is what I want to discuss now.

Some of the very common terms and how these very common terms will be actually used. What is the expression of them? That is what we would like to discuss. So, first of the most important term that will come in to picture is called loss factor. As the name suggests, that this is a factor.

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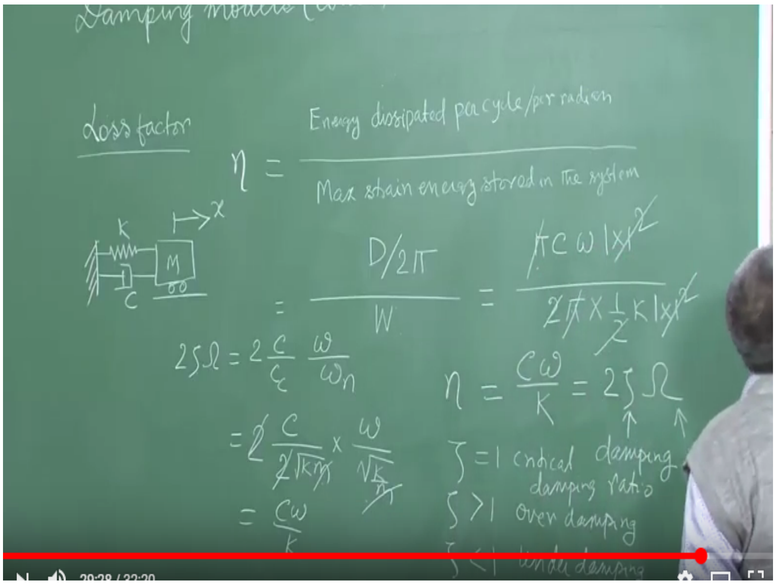

And this factor which is generally shown as eta and this eta is actually energy dissipated per cycle, per radian. Let us make it proper dimensionless and then we have the maximum strain energy stored in the system. That is what is my loss factor, eta.

The energy dissipated per cycle per radian and maximum store energy stored in the system so for example for simple viscous case you can do it for yourself. You have the energy dissipated suppose it id D. Then it is per radial so D by 2 pi. Because is the energy dissipated per cycle that means for 2 pi radian.

And then the work d1 let us call it W in face we can work it out for a single degree of freedom system which is something like this you have K here. You have damper here and you have a mass here which is on a frictionless system and for this kind of a case we can very easily evaluate it.

Our D is pi c omega x square. So X is the displacement and capital x is the amplitude of it. SO pi c omega x square divided by 2 pi times work d1 which is half k x square. So that is, what is my total expression. So, if I cancel these 2's pi's they will get cancelled.

Of course x square so what I am going to get is c omega by k. So this is what is my eta which c omega by k. Now this is also I can write it terms of the damping ratios. Because if you try to work it out that if you take 2 zeta omega that is 2 c over cc times omega over omega L.

And just work it out that this is nothing but 2 c over 2 root km times omega over root k over m. So this m should cancel. 2 will cancel for us. So what we are going to get is actually c omega by k. So that means I can write this eta, loss factor as 2 zeta omega where this is damping ratio and this is the non dimensional lap frequency ratio.

We have to use this terms again and again. So I am deriving it here itself. Keep in mind that damping ratio equal to unity for critical damping. It is greater than 1, it is over damping. And if it is less than 1 which is the usual case, then it is under damping. So these are the things that we have to keep in our mind.

Then there are various ways in which we can actually find out these of course there is 1 way is if you the damping coefficient, you know omega k or if you know the damping ratio and the non dimensional frequency ratio.

You can find out the eta. Another way of just doing this whole thing which I will just very quickly talk about is with the help of what we call the up power point technique. So if i use this power point technique then it would look like we have to draw what we call a frequency response function for the system.

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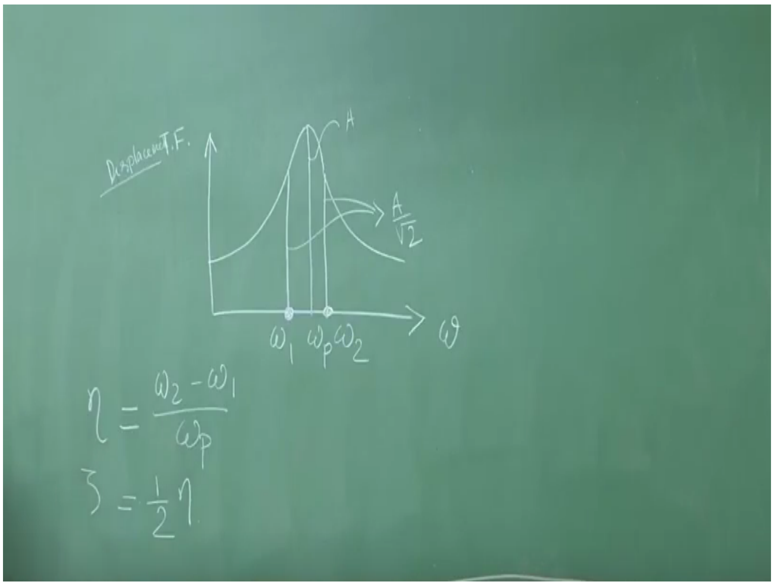

So we have to draw a omega verses the transfer function of the system. Now this transfer function could be F over X or you know there are many transfer functions which I will discuss. It can be a displacement transfer function. It can be a force transfer function. Essentially, it is a ratio of output to the input of the system.

So if I draw that then this for single degree of freedom system. It will come like this. If it is a linear 1 it will be perfectly symmetric and this is the resonating point we have to note it down. That this is the resonating point and 2 other points also we need with us and this 2 points if this is having an amplitude A, these 2 points will have an amplitude of A by u2. So if omega 1 and omega 2 are the 2 frequencies at these 2 points.

I can get an expression of ets from this experiment in which it will be omega 2 - omega 1 by omega p and of course I can get the damping ratio from this experiment which will be half of eta is 2 zi. So zi will be half of this eta. So you can get from this kind of an experiment. From the transfer function plot with respect to omega, you can get the entire system and you can get the loss factor from it.

So this is where we will put an end to the system and in the next class we will more about structural damping. Thank you.