Principles of Vibration Control
Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology-Kanpur

Lecture – 04
Viscous Damping Model

Welcome to the fourth lecture, of principles of vibration control. In this lecture we will do a bit of board work because I will try to show you first of the viscous damping model and how the energy is dissipated through a viscous damping model. And then we will take up a very simple case in which we will see that you know for a single degree of freedom system, how actually we can get the governing equation motion and.

How we can predict that which part of this frequency domain transfer function of the system is actually stiffness dominated or damming dominated or dominated by the mass of the system. So phase by phase we will go through that. But begin with first we will do the first viscous damping model of a system. So when we talk about viscous damping model, here we are talking about the damping force which can be modelled in terms of a piston and a cylinder.

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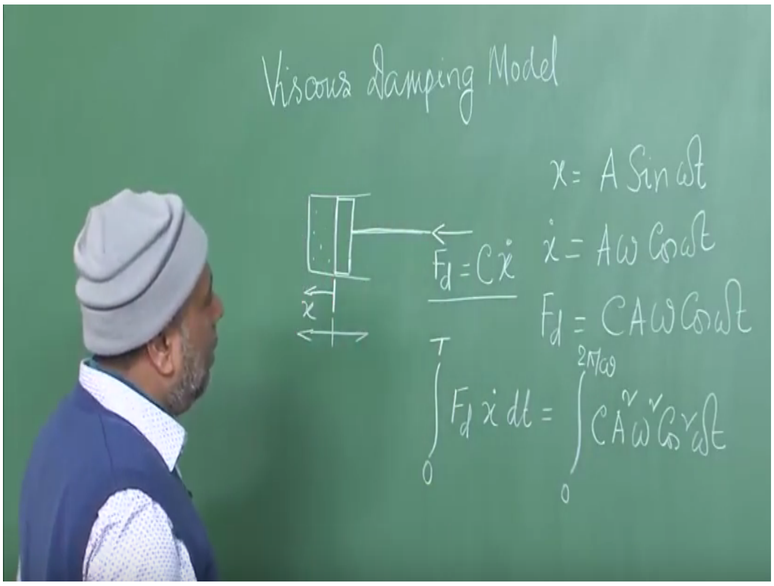

And the cylinder you know whenever you are trying to give a drive force to the cylinder you are going to get a resistance you know the inside the viscous fluid and this viscous force will be proportional to the velocity. So if this fellow moves at a speed x, at a displacement is x and the velocity is x dot. So the damping force is modelled as it is proportional to the velocity of the system and hence the fd equals to cx dot. And this is the conventional representation of the system.

Now let us say that this happening in a manner to and fro that means this system is going you know back and forth you know and we want to find out that what the energy that is dissipated in 1 cycle. So x in that case can be represented by a periodic signal. So we can write it as something like x equals to a sine omega t so that it is periodic.

If I try to visualise x with respect to time t, x versus t then it will come something like this where a is the amplitude. Of that is what is my displacement the periodic displacement that I am giving to the piston and I am trying to find the what the viscoelastic energy dissipation per cycle. So x dot will be a omega cos omega t and let us say the force is c a omega cos omega and the energy that is dissipated per cycle if we try to find that out that means for a time period of 0 to t where this t is 2 pi by omega. Ok.

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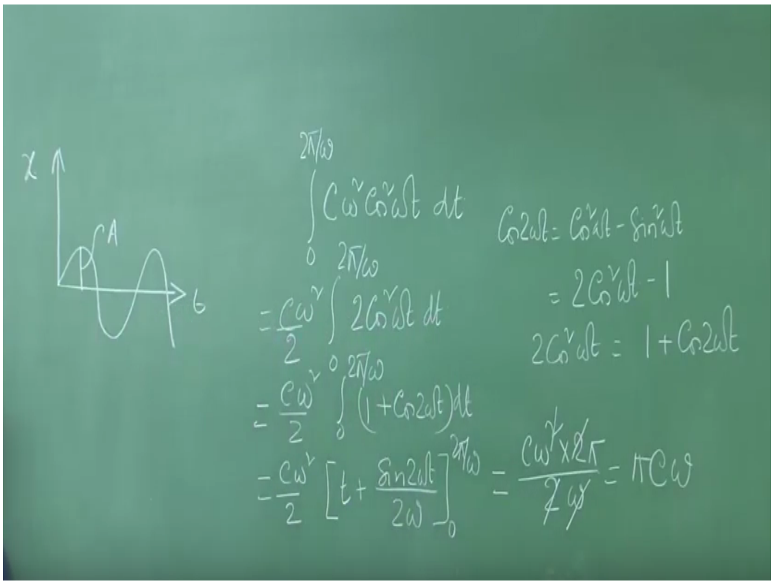

So we will do that and the total energy that is dissipated in the system can be retained in terms of fd that is the driving force and that times x dot dt. In other words, this will become integration of 0 to 2 pi by omega and this will become c a square omega square cos square omega t. So that is the energy that is dissipated per cycle in the system.

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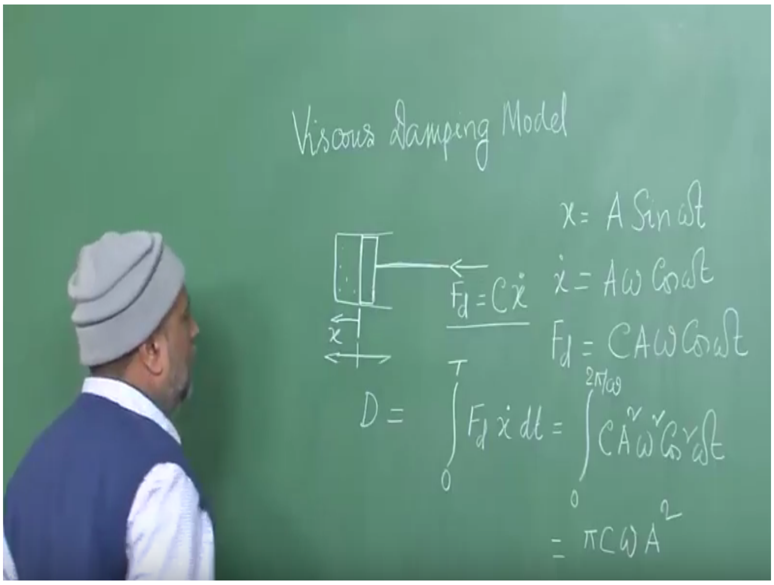

Now let us try to evaluate this. It is a very simple integration let us try to do that. That means what we have to find out is what is this 0 to 2 pi over omega and c omega square cos square omega t where c is our damping coefficient, a is of course amplitude that is the constant. So we can write in terms of c and let us say we divide it by 2, so we get 0 to 2 pi by omega, omega square also you can take it out. Ok.

So there what we are getting is 2 cos square omega t, dt. Now you already know that cos 2 omega t is cos square omega t - sin square omega t which means it will be 2 cos square omega t, this 1 - cos square - 1. In other words, our 2 cos omega square t can be simplified as 1 cos 2 omega t, so we can write this here as c omega square by 2 and integration 0 to 2 pi by omega 1 plus cos 2 omega t dt which means this will be c omega square by 2 and then t plus sin 2 omega t divided by 2 omega 0 to 2 pi by omega.

If we further go ahead then we will see that this will become only the sin terms are going to get cancelled because sin 4 pi will be 0, so and sin 0 is also 0. So this will become c omega square times 2 pi by 2 omega. This 2 are cancelled so we are going to have only pi c omega with us. So if that with the thing, we can borrow this part here so that we can now write that this is essentially the amplitude of the system and pi c omega a square. Ok.

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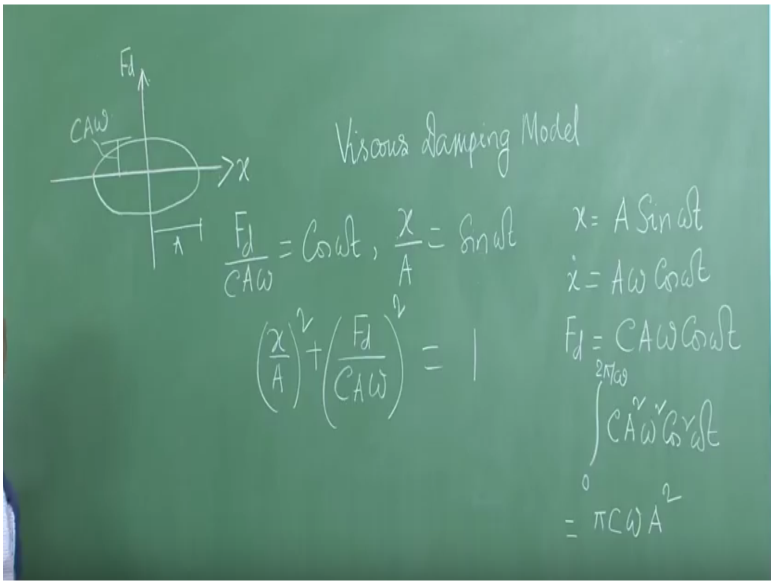

So that is the energy dissipated per cycle. So we can see from this expression that d energy dissipated per cycle that is with square of the amplitude. So if the amplitude is very high, the energy dissipation will be very high. It also varies with the damping coefficient c so for larger damping coefficient energy dissipation will be mare and it also varies with omega. So that is what his expression is going to give us.

You know that how the energy is dissipated from a viscous damping model. Now we can also visualise these in terms of a kind of geometric interpretation of the system. In order to do that we will simply need to plot this viscoelastic damping force. So if we do that then this will become, we have to consider that how this you know force versus deflection relationship a will come into picture.

So we already know that if d over c a omega ca omega equals to cos omega t. That is 1 relationship we know,. And also we know that the normalised displacement x over a is sin omega t. So this essentially tells us that x over a square plus fd over c a omega square equals to unity. This gives us the clue that we can actually represent you knows this whole viscous energy damping in terms of the work d1 in the system.

So that means if we plot fd over x, this will take a form of an ellipse. This will be like an ellipse. Such that the semi major of the ellipse is the amplitude a, and the other direction this is ca omega and the area under the curve is essentially the energy that is dissipated per cycle in a viscoelastic material.

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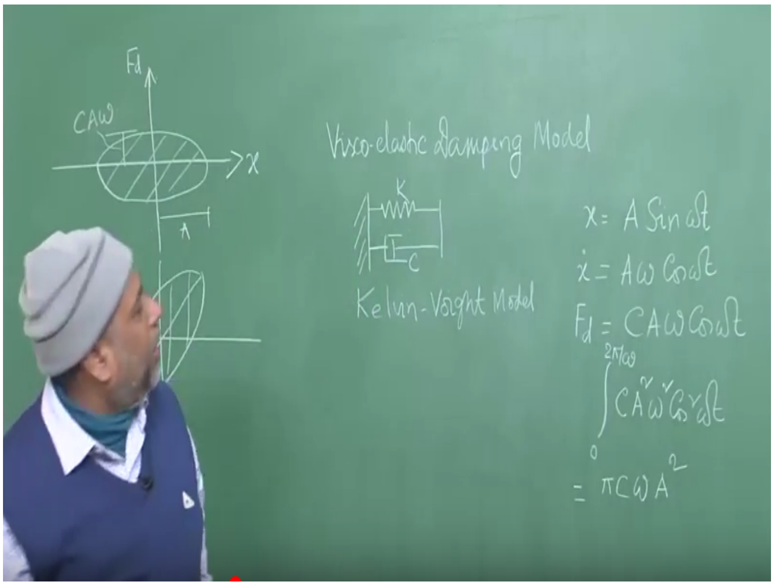

In a viscous damping material, now if it is a viscoelastic damping system then this is going to change slightly. What will happen is that in that case there is something more in case of viscoelastic damping. So if in state of viscous damping it is viscoelastic damping, in that case what you have is not just a simple damper but a combination of a spring and a damper there can be various combinations possible.

This is a combination in which the spring and the damper are in parallel and this is combination is known as Kelvin Voight model. Now if I use this combination then this diagram will slightly change the energy dissipation will now take care of these dissipation as well as the spring factor in a way that the orientation the ellipse will depend on the spring's stiffness and instead of you know the ft versus x relationship this manner it will become an inclined ellipse system.

So that is what the viscoelastic damping model is. Now what we Are going to find out tis actually that how in terms of this model now we know that what is the energy dissipation and what is the you know a kind of a geometric interpretation of the system but going further we would like to integrate this system with the a single degree of freedom system. Let us start first with the single degree of freedom system and let us try to prove what are the concepts that.

We have already discussed that for a single degree of freedom system what are the parameters that affect the vibration control at various points of time.

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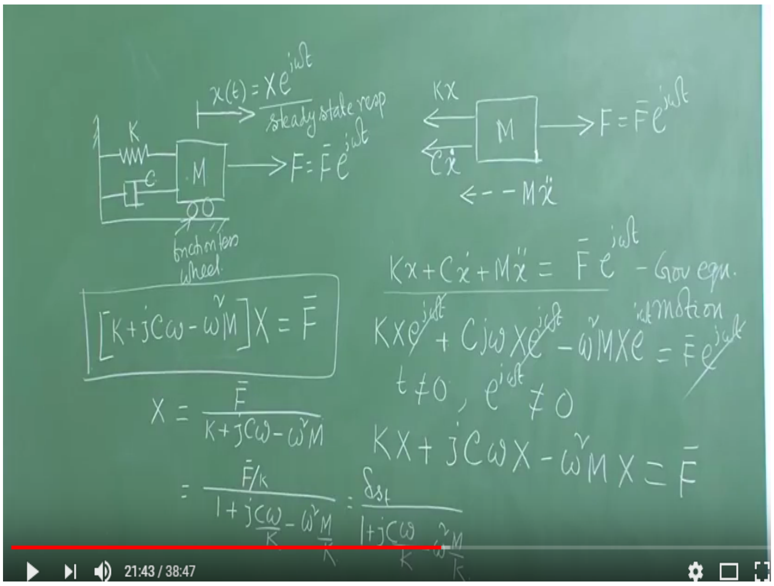

To do that let us first draw a single degree of freedom system. So let us say this is our boundary and we have a single degree of freedom system which has a kelvin Voight model type of a damping and the spring and the damper is attached to a mass in this case and the mass is resting on frictionless wheel. So these wheels are frictionless.

And let us say that we have applied an excitation force f which is harmonic in nature where we will can represent it as f bar e raised to the power g omega t and let us say we are tracking with the help of a sensor that is the displacement f this mass m when we are subjecting the system to this harmonic excitation and let us say that this displacement is xt

Now presumably if i give this harmonic excitation to the system xt the response will also be harmonic and that we can write in terms of x e raised to the power omega j omega t. Our intention now will be first of all to get the governing equation of motion of such a system. And then to show that how we can predict about the nature of the response and various parameters will be affecting the nature of response.

So first of all let us to ge the governing equation which we can do very easily by applying Newton's law. So if we draw the free (( ))(15:43) diagram of the mass m then 1 of the forces is working is f which is f bar es to power g omega t. And we have this harmonic force is getting resisted by the spring and the spring force is proportional to the displacement. So it is kx and also by the damping the damping force is proportional to the viscous damping here to the velocity of the system that is what we have just now discussed.

Now in addition to that in dynamic equilibrium there has to be an inertia force in the system. So since the displacement is in the right side so this inertia force we will be showing it as a pseudo force, (())(16:42) force and that we will be writing as mx double dot. So these are all the forces that ar working on the system. Now I can sum up all the forces that means all the forces in the left hand side together that is kx plus cx dot plus mx double dot equals to all the forces in the right hand side.

That is the force in the right hand that is the force acting on the system. That is f bar e to power g omega t. So with the help of the force equilibrium along the direction x it is unidirectional motion we are going to get the governing equation of the motion of the system. In this course many a times we have to derive such governing equations of the motion.

Now we can try to find out that what will be solution of this system not in the transient case but in the steady state. Ok. That means after some kind of a finite time what will be the nature of solution of the system. So in the steady state, the response is also harmonic in nature. This is the steady state response of the system. And if I substitute this steady state then what I am going to get is kx e raised to power g omega t plus c j omega x e raised tot the power g omega t - omega square in x e raised to the power g omega t equals to f bar e raised to power g omega t.

Now since t not equal to 0, so e raised to the power g omega t not equals to 0 any steady state time you know after t equals to 0 you are considering t not equals to 0. E g raised to the power omega t not equals to 0 as the result you can this from both the sides and you can get these as the steady state response of the system as kx plus jc omega x - omega square mx equals to f bar.

If I slightly reorganise this I am going to get the steady state response of the system as k plus j c omega - omega square m times x times equals f bar. That is the steady state response of the system. Now we can from that try to seem this steady state response, you know in terms of as we change the excitation frequency so lets us try to do that. So now what ew can say is that x equals to f bar over k plus jc omega - omega square m.

And If i work on this I can also write this as If I divide the numerator and denominator by k then it would become 1 plus jc omega by k - omega square m by k. I have a purpose in terms of dividing the numerator and the denominator by k. The purpose is that f bar by k is nothing but the static deflection of the system. So we can also write it as delta static, static deflection of the system and the rest of the system this we can write it as 1 plus jc omega by k - omega square m by k.

We can now try to further simplify this whole thing. In order to do that, we have to keep few basics in our mind. So let us try to work these few basics here. And then we will attempt to write this expression in a manner by which we can actually get the nature of the response easier for us to understand. In order to do that we have to keep first of all in our mind that there are 2 things that is there.

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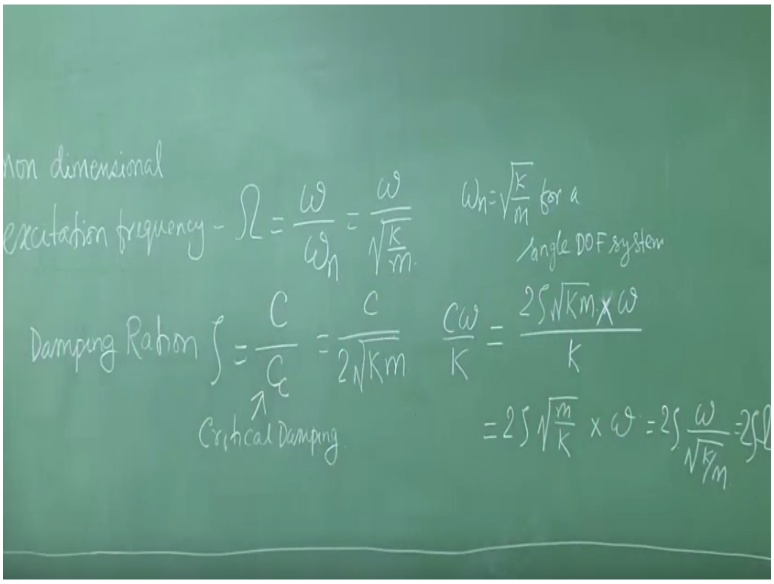

1 is called a non dimensional excitation frequency we will us which is also a kind of a normalised excitation frequency and that we will define as capital omega which will be excitation frequency divide by the natural frequency of the system omega m and that it means it will be m for a single degree of freedom system. We know the omega n equals to square root of k by m for a single degree of freedom system.

Also we have to think of the damping coefficient and there we would like to introduce a new term called damping ratio. Zeta which is once the damping normalised as c divided by cc where cc is known as the critical damping. Now I will not going to the basics of it you can check the details of it from any basic text book on mechanical vibration which is perquisite of this course.

Thompson's book let s say and you can see this derivation. So cc is actually derived as squared 2 times square root of km. That is what is the definition of the cc. So we need to find out that what will be the c omega by k term. So we can try to find it out and also the other term is then already there with us. So if we try to find that out from based on this expression that we write that c omega by is what we are planning to find out.

And this c mo omega by k we can write in terms of the damping ratio of the system in to get that , that means we can express the c as 2 zeta root km and that is what is our the expression of c as we can see here that it is 2 zeta root km. So this times omega divide by the k. In other words this can be written as 2 zeta root m by k times omega. Ok. This is a multiplication simply.

Please do not confuse it with you know the steady state amplitude of the system. So it is 2 zeta root m by k omega. In fact we can write it further in terms of the non dimensional frequency then that it is 2 zeta omega divided by square root of k by m. That means it is nothing but 2 zeta capital omega. So this is the relationship that we will like to borrow here and then we are going to get a better simplified normalised relationship.

So let us try to write that steady state normalised relationship which will be useful for us in order to predict the response of the system and now we can write that, that this steady state response x is nothing but delta static over 1 plus jc omega by k. We have already derived the c omega by k is 2 zeta omega. Ok.

That is what we have already derived. So it is j 2 zeta omega - this part of course you can very easily do the same way that omega square m by k is nothing but omega square by k by m. That means omega square by omega m square. That means it is capital omega square, so - omega square. So that means I can writhe that x equals to delta static over 1 - omega square. Tats 1 part plus j 2 zeta omega.

Now that is the response that we are getting from the system. Now let us say that we want to find out the amplitude of the system. So if I am interested because this is having a complex as you we can see the response ids complex. So if we want to find out the amplitude of it then it will become the amplitude of delta static divided by 1 - omega square whole square plus four zeta square omega square root of the whole thing.

That see you know because it is the ratio of the numerator amplitude of the numerator and the denominator and the denominator is a complex quantity so we have to find out the amplitude of it which we have do not in this manner.

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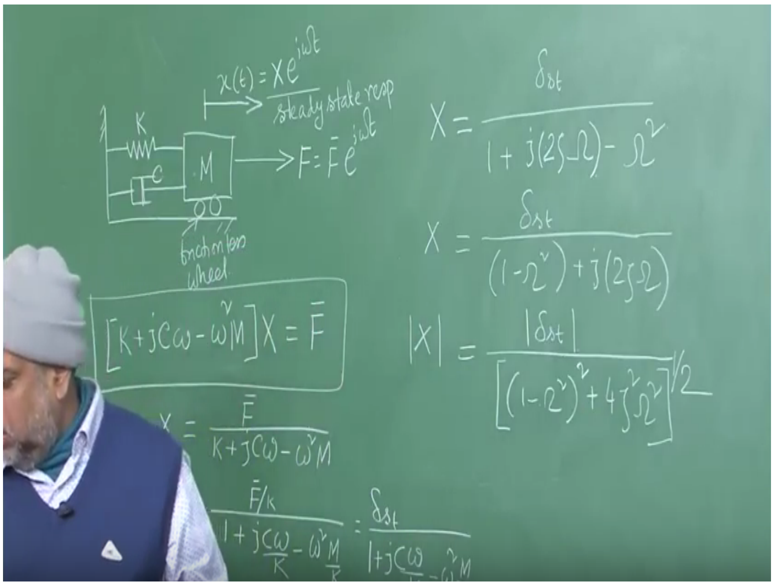

Now we have everything that we require in front of us. All we have to do now is that for certain conditions we have to find out what are these conditions we are talking about. The conditions we are talking are just 3 conditions right now. 1 is when omega the excitation frequency ratio is very much less than unity which means this omega is actually very small in comparison to omega natural frequency of the system.

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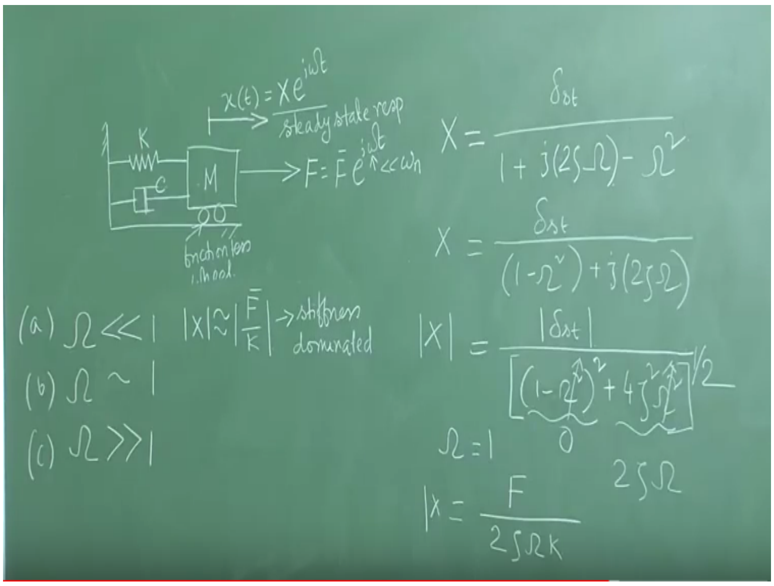

So this is your capital omega to be very less than the unity and then omega is approximately unity and then omega is much greater than unity. Now let us try to see as an engineer what will be approximately the response of the system under these 3 conditions. Ok this is our case A. This is case B and this is case c. So if you look at case A in that case x will be approximately you can see that omega is very small so that means I can neglect this it will be very small.

Omega to power four also will be very small also omega square will be very small. So x will be approximately what it will be the amplitude of f bar over k. That is what is our f bar over k. That is what f bar over k, is our first case. So in this case we can write that x is approximately the amplitude of f bar over k which means my response is stiffness dominated when the omega is very small. Second case, omega equals to 1.

If I substitute then what will happen is the this star will become 0 and this star will become after square 2 zeta omega. So in the second case, x is f over2 zeta omega k. Now let us try to find out what's this zeta omega k. We already know all the definitions here. So we can try to apply all these things here and try to find out that what it is 2 zeta omega k.

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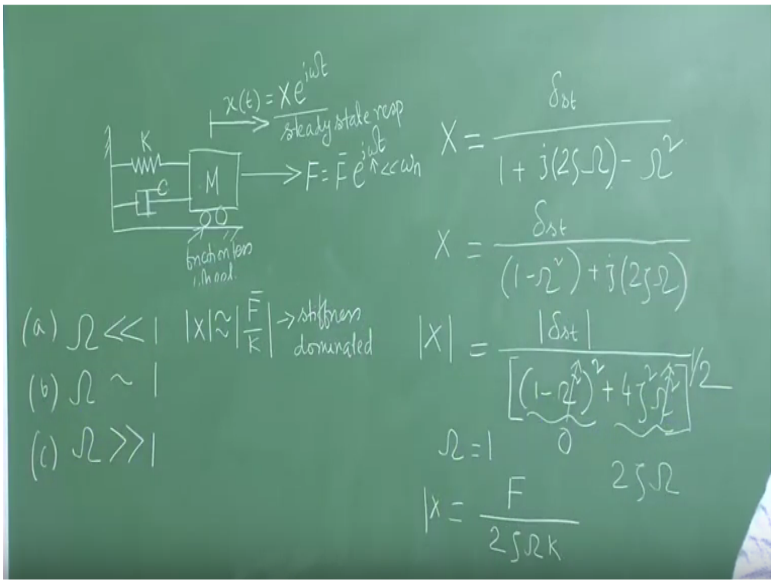

What does it means for us in terms of the damping constants and all the other properties of the system, so 2 zeta omega k would mean 2 c over 2 root km that is what our cc is. then omega over omega n that is square root of k by m and then multiplied by k. So this 2 and 2 will cancel and the square root of k , the square root of k cancel k and the square root of m, the square root of m will get cancel and we will get only omega c.

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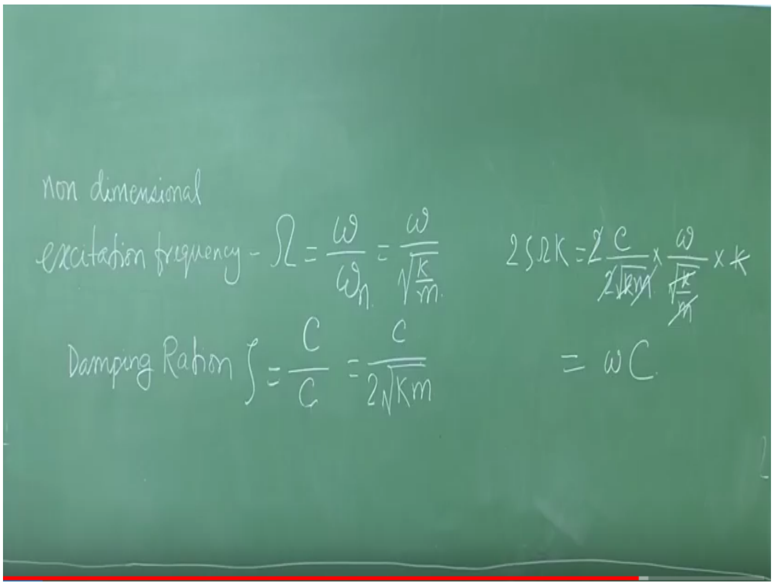

So what it means is that this expression is actually nothing but f bar over omega cor c omega. So that means here, x approximately equals to f bar over c omega. So that means at this region when omega is approximately equal to unity this region the vibration response is actually damping dominated. Let us now go to the last part that means omega is much greater than what are we going to find in such a case.

When omega is omega is much greater than unity let us try to see what's going to happen to expressions. So it will erase all the other things. We have the basic expressions with us. Ok. And here also we have it has four zeta square omega square that was our basic expression and here now omega is much greater than unity that would mean that in this denominator the highest term is actually omega square square that is the only term that will be much larger in comprising to any other term.

So what it will mean is that for such a case x will be approximately over k times our omega square itself. And that means it will be approximately f over k times omega is our as you can see here, that this is the expression of omega. So you can write from here that omega square is nothing but omega's m, omega square or k. We can derive it by looking at this expression itself. So this can be written as k times m omega square over k this k and this k cancels so the approximate expression is f over omega square m.

That is the approximately expression so in this case x is approximately lets put it as f bar region missing so f bar . So it is f bar over omega square m. This also interesting, so that means in a particular region vibration control has nothing to do with damping but do it with the mass of the system when the omega is much greater than omega n. This omega when it is much greater than omega m in that case it is all the mass that matters. So this is mass dominated.

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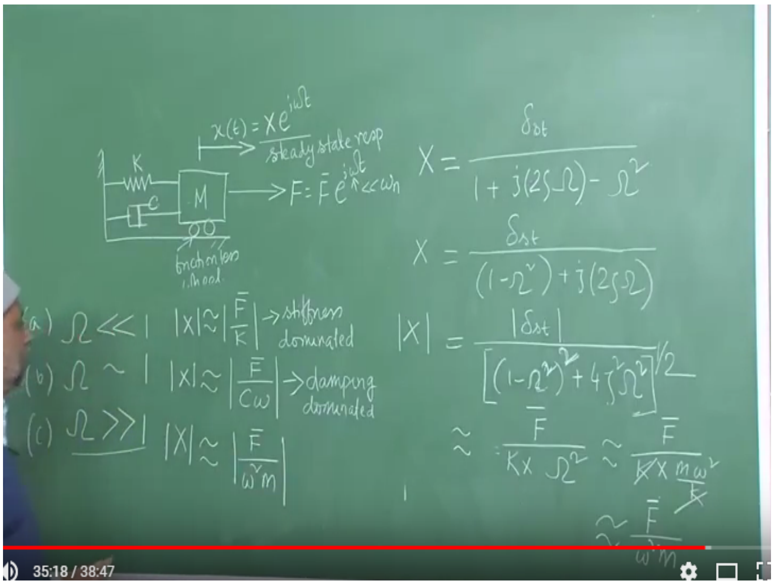

So thus if I try to sum up this total result in terms of the transfer function representation of the system and then what we are going to see is that this clearly tells us that there are 3 regions. You can plot this omega with respect to the transfer function of the system. We are going to see that these representations will take up the prom like this. In which there is a part up to which this excitation frequency omega can be considered to much less than unity this is the peak point, where omega is unity so this part is actually stiffness dominated.

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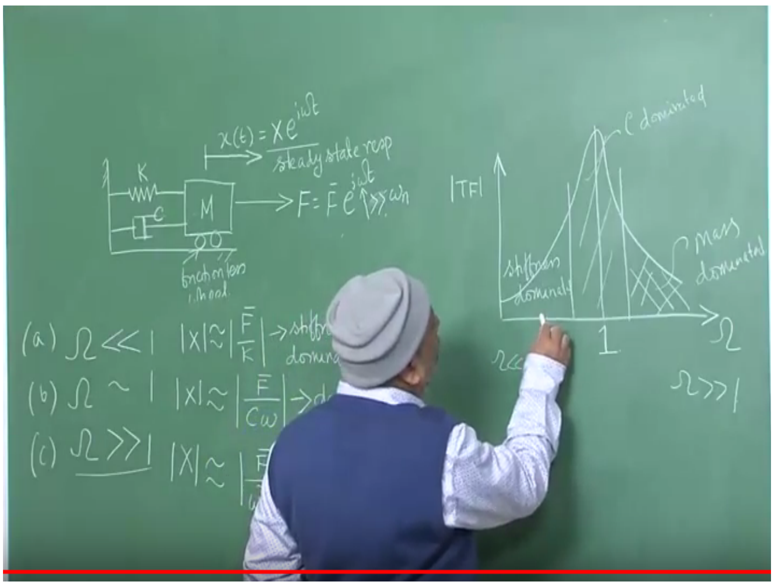

Then in between there is a part which is actually c dominated, damping dominated and omega is much greater than unity then you are going to get the part which is actually mass dominated. So the important take home message for us is that, that is why the characterisation of the system is important if the excitation frequency of the system is low in comparison to the you know the resonance or the first natural frequency of the system we should not look into c or mass as the parameter.

C or mass would not be important. Only k will be important. Because it is stiffness dominated omega is much greater than. If I come here, Close to the resonance we should not look into the stiffness or mass it is only the damping constant which will be important.

If we come to the right hand side where it is the excitation frequency is much greater than the unity then the entire response of the system will be dominated by the mass only.
Nothing else will have the impact on the system. So that, we can prove it with the help of all these basic definitions of the system. Thank you.