Principles of Vibration Control
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Lecture-20
Basics of Classical Control System

Welcome to the course of principles of vibration control. We have reached the last lecture now in terms of active vibration control and today with the help of all our earlier knowledge's of state space system we will try to develop controller for the state space model and which will be in the form of full state feedback control.

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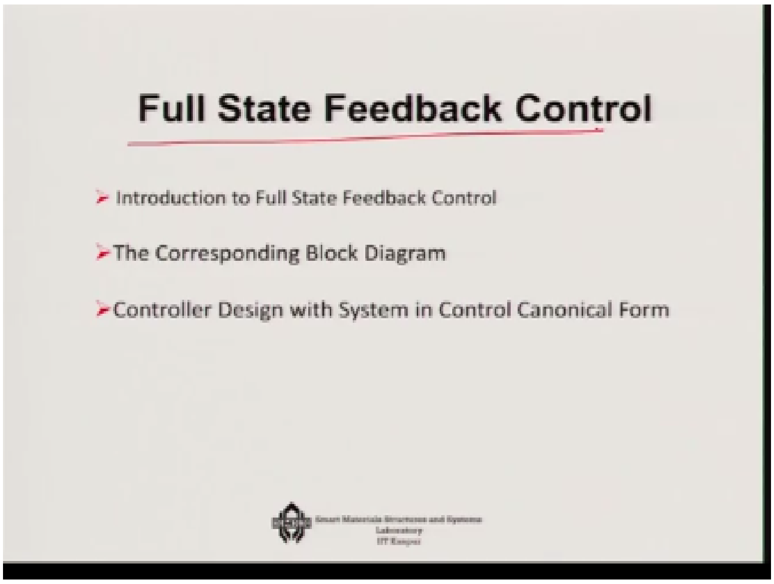

We will talk about the full state feedback control. We will start with the introduction to full state feedback control corresponding block diagram and then we will talk about the controller design system in control canonical form.

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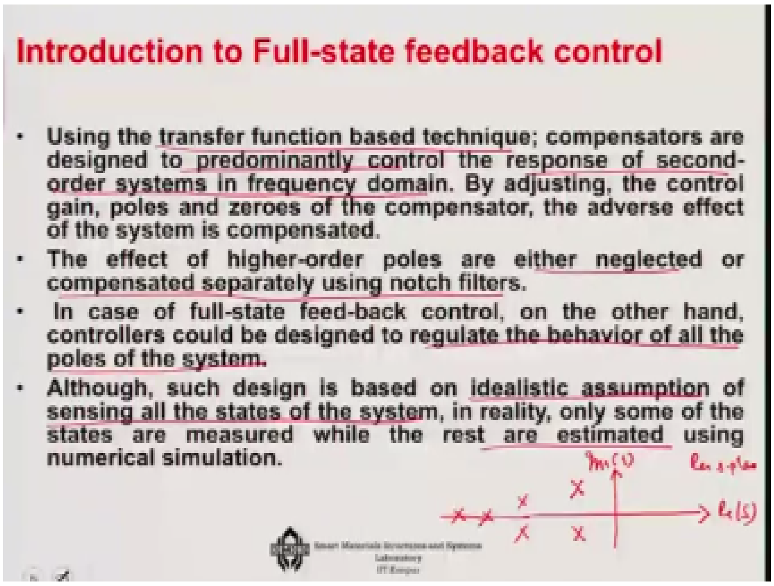

So the first point then we will talk about the feedback control is that, if you compare with the transfer function based technique which we have earlier worked on then the compensators for the transfer function based techniques are design to predominantly control the response of second order systems in frequency domain. So by adjusting the presence of gain, poles and zeroes of the compensator, the adverse affect of the system is compensated.

But if there are higher order poles that means as you can see that for a second order system if I just draw it very quickly that for a classical control you know that is the S plain and this is the real of S and this is the imaginary of S and the poles if you remember there only 2 poles right and that was the second order system. Now if you have a multi-body system multiple degrees of freedom, you may have more poles, may be the poles will be here, may be it will be here.

How this higher order poles are maybe there will be far away from the imaginary on the imaginary axis you know much far away. So if you have this type of higher order poles you will either neglect it or you will compensate it separately using notch filters. However, on the full state feedback control we actually regulate the behaviour of all poles of the system. We really do not neglect anybody.

So that is the good part of it. So although such design is based on an idealistic assumption that you can sense all the steps of the system in reality of course you cannot sense the states then some states are estimated. And then those estimates states you have design or observe or separately to make sure that estimation is good enough so that we can is virtual estimated states can be passed by as just like the regular states and our full state feedback control model remains intact. So that is what you know is the kind of a discussion between that two.

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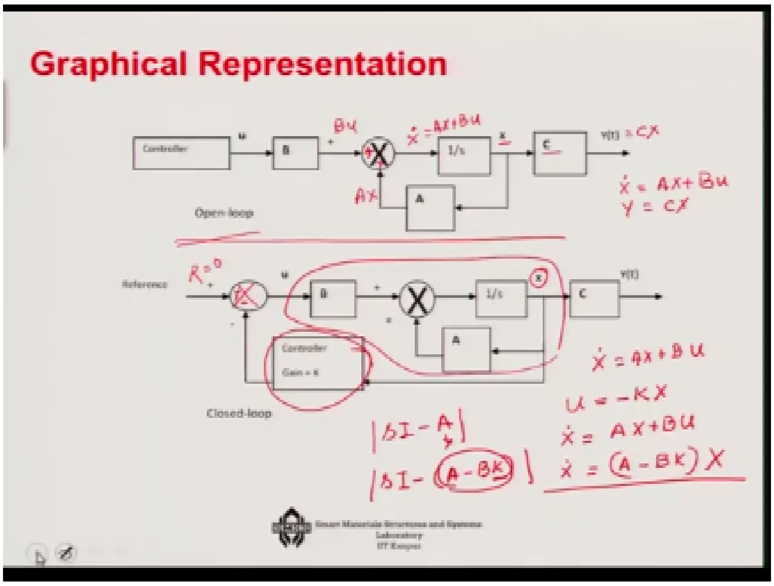

Now this the top part is the representation of X dot is equal to AX + Bu. If you just give valid of time you will be able to see that suppose this is Bu here and you are adding this here and then this is you consider it to be X dot you are getting. So one over is integrator will make it X and then here it is this X is coming out here with the pre multiplication with A becomes AX.

So this is + and this is + which means we are going to get X dot equals to Ax + Bu. So that is the you know transfer function open loop form and also y equal to CX we can see that this is X, this is C you are getting Y equals to CX. So this is the block diagram graphical representation of the system. Now in this system if we try to put the games extra how does that come into the picture?.

So as you can see here that this part we have already discuss this is actually X dot equals to this is actually or X dot equals to AX+Bu. But the question is what will be these control effort you. If you are sensing this steps X let us say this is where you now have your control gain, so your controller gain is in the feedback loop and you are taking it here. This is your reference signal and this is - and this is +.

So that means you suppose for a very simple case are equals to zero then no reference signal have to simply bring down the vibration then by U will be simply -KX. So that means what is by system?, it is X dot equals to AX+Bu, this is what my active control system is. Let macro environment substitute the value of u there, so it will become A -BKx. This would become my new system.

This A-BK is clearly telling you at least something that earlier system when you did not have this feedback the characteristic poles are the Eigen values are determinant of Si - A. But now you have the new system whose determinate is determinate of SI-A-BK. So this A got changes so that means Eigen values will get changed by suitably choosing this gain K I can actually place all the Eigen values to my required position that is what we will be learning today.

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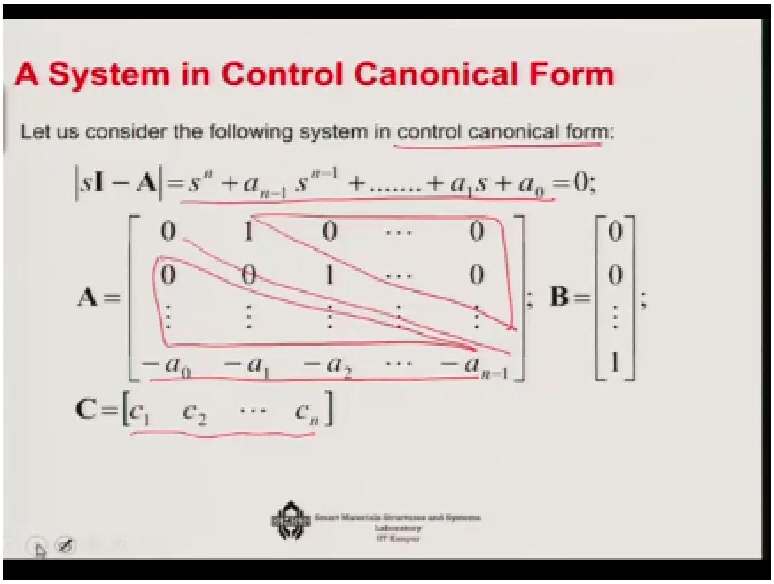

So whatever we have discussed let us bring it in the context of fast control canonical form. So the determinant of Si - A is Sn + an - 1 sn - 1, a1 s + a0 equals to 0 where if he is in a control canonical form so I can very easily contrast it from the characteristic equation at only this last row we will all this coefficient, then this are the zeros and then the super diagonals are unity.

The rest are all zero, that is the control canonical form, if you remember and the terms below are also all zero except the last row where we all the coefficients. B also has this very special form 000 to unity and C is of course having all the you know fully populated with the coefficient it need. But we will be bothered those with A and B and we will be deciding the controller in this system. C will take care of when we have to do the observe what is I.

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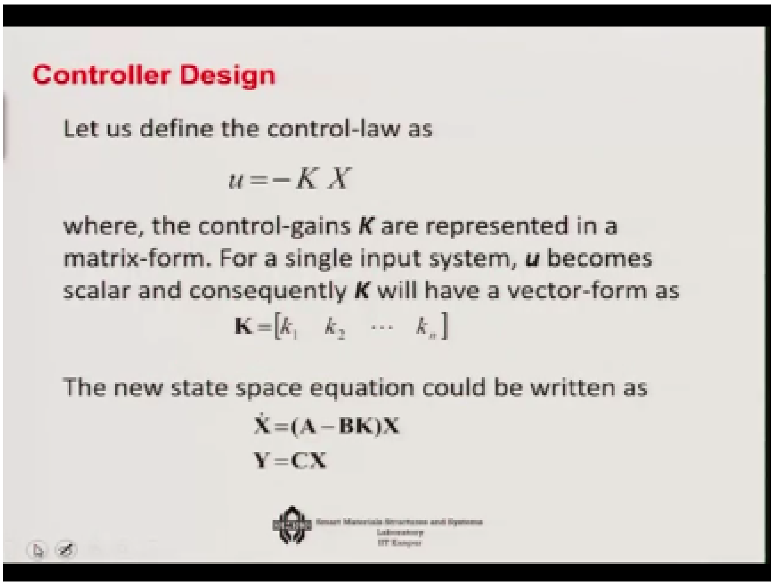

Now let us define the controller and as we have shown you in the earlier block diagram. Let us see u have negative state feedback control which is u equals to - KX. So let us say my structure of K is K1 to Kn. All these instead of single gain now you have a gain vector, okay so these are all the gains. So together they form a gain vector and by deciding the controller means I have to get what are going to be this gain vectors.

So by new state space equation is X dot equals to A - BKX and Y means as it is as CX. So it is only the fast state equation which gets effective by the introduction of the control law.

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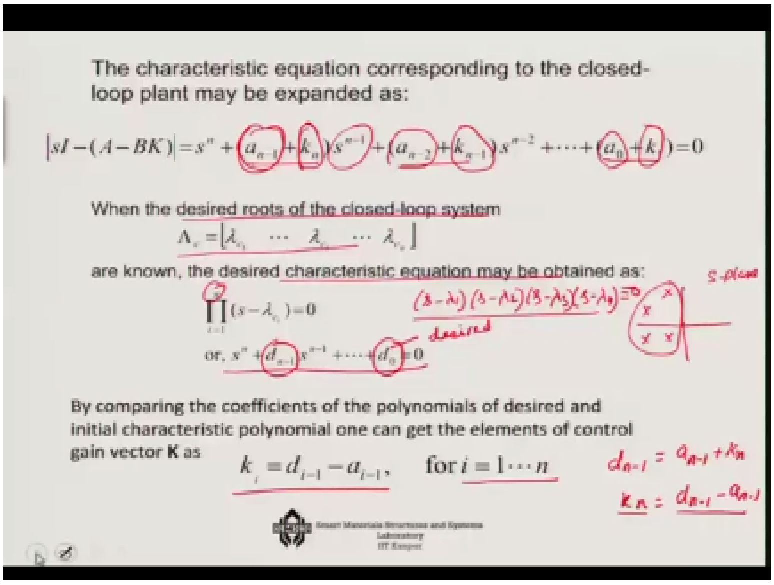

Now what is the characteristic equation of Si - A - BX. When the good part is that A is in a control canonical form and B also in a canonical form, so if you just work it out you will see simply there are additions in the coefficients. So your earlier open loop system was having coefficients like An - 1, An - 2 to A0 and the controller is simply these control vector is getting added with these coefficients, okay that is what is happening to the system.

Now let us say our client has did not ask what are the desired roots of the close loop system that means for any dimensional system let us say you know for these type of a system they are telling me that in the S plane where these lamdas should be there, the desired once are let us they given to us, that this is the way you arrange it. So I know beforehand what are my desired roots.

Now since I know the roots I can find out the characteristic equation of the system. If you consider it for this kind of a 4 pole system and you can write it as S-lamda1 into S - lamda2 into S - lamda 3 into S - lamda 4 equals to 0. If it is up n then you can write it like that I means the long hand once you do it, you are going to get the characteristic equation.

This characteristic equation if you see that its coefficients are with D Dn - 1 to D0. The D is telling us that this is the desired coefficient. So in one hand I have the desired coefficient and in the other hand I have the plant coefficient and some unknown gains. Now it is very simple because for each and every polynomial you know order like S to the power n - 1. I have to get it as Dn - 1.

So what will be my gain, well the gain will be such that the Dn - 1 will become An - 1 + Kn. In other words Kn will be Dn - 1 - A n - 1. So that means for Ki will be Di - 1 - Ki - 1 and this principle go from i equals to 1 to n. So in some sense it is actually a distance between the you know characteristic polynomial that comes from the desired Eigen values.

And the corresponding coefficients which comes from the open loop plant system, that t stands it is for that means I need more Eigen if it is very close that means I need less gain to have a full state feedback control.

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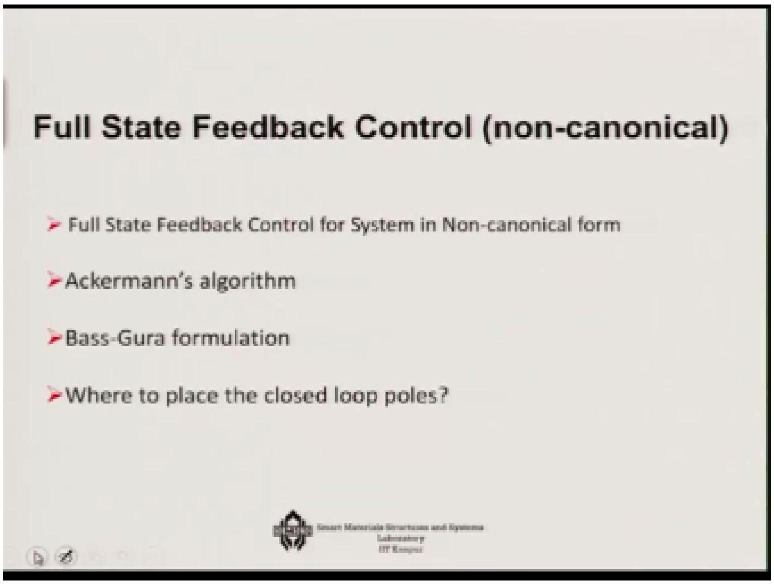

So that is what is my full state feedback control in the you know canonical form. Now how did look like in a non canonical form. Well if it is in non canonical form I have to transfer it into canonical form. There in fact certain algorithms there Ackermann's and Bass Gura formulations. I will come to it later. First of all let us see how we can do it in a state forward manner.

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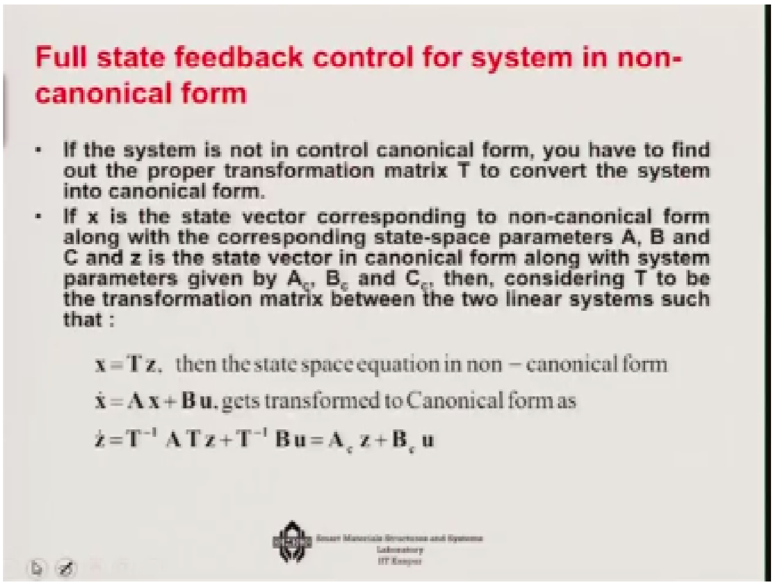

Okay if it is in a non canonical form let us say that these transformation matrix exceeds. So that means this X dot equals to Ax+Bu will now become Z dot T inverse A to Z+T inverse Bu which is A control canonical Z+BcU. That is the first part, we have to convey the system into the canonical form.

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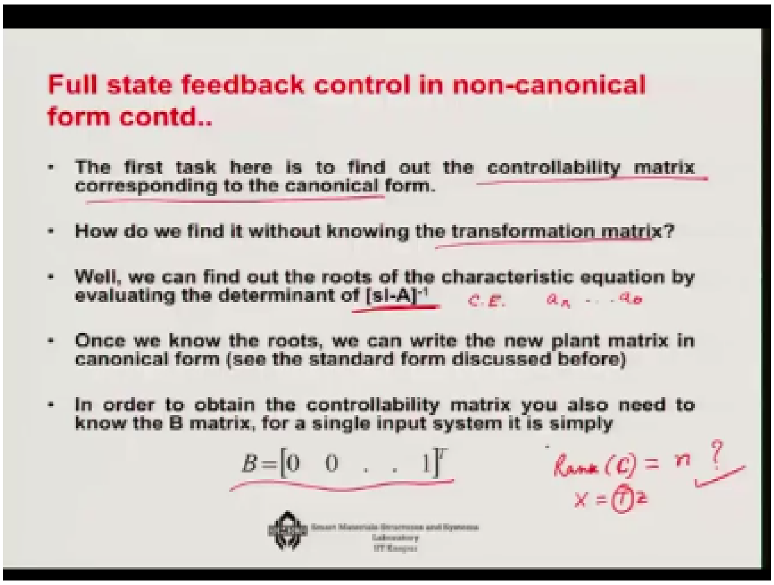

Now in this case before you do anything you please first check what is the controllability matrix and whether the system controllable at all or not. That you can very easily do if you remember, if you check the rank of that controllability matrix and say see whether it is equal to the state you know all number of states of A, okay. If it is satisfy then you proceed. Otherwise, the controller design is not possible.

How do you find it. Now the next is that in order to do all these things so I need to know what is X equals to Tz, what is the transformation matrix. How do I find out this transformation matrix to begin with, well that is simple. First of all you can find out the roots of the characteristic equation that is Si-A inverse so the moment you get these equate it to 0 you get the characteristic equation.

The moment you get the characteristic equation you can develop few control canonical form because once I know this characteristic equation then I know all these coefficients An to A0 and also I know B has to be in this form.

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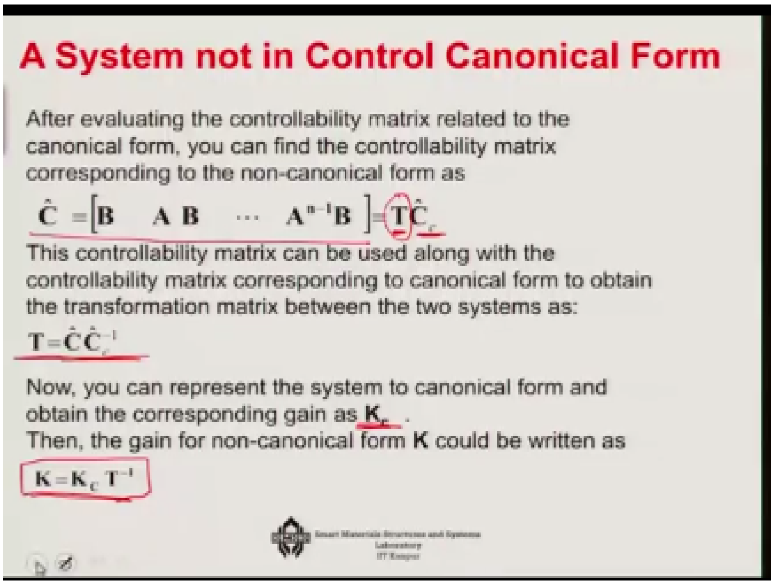

So why you know combining these 2 knowledge I can very easily find out that what is my controllability matrix because you know I already know and this I know and also I know what is the transformation matrix. Now since I know the transformation matrix and this equals to T up Cc hat let us see for the canonical form, so then the T is the transformation matrix easiest way to find out is that it will be C hat Cc hat -.

So, I should be able to find out these. Now let us actually imaging that the system is in canonical form, so I can very easily now get the gain Kc and once I get the gain Kc I can find out the gain in the non canonical form which is Kc times T inverse. So I can go back from non canonical form and this from canonical form to non canonical form and finally give what will be the gain of my system.

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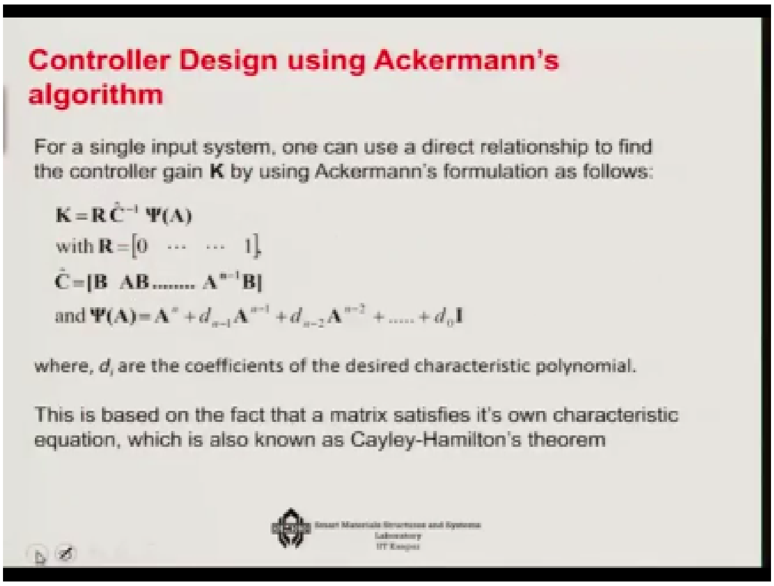

Let us try to actually discuss these with the respect of some kind of a formulation. So one easy way to do it is by using a Ackermann's algorithm were K is considered to be RC hat inverse iA where R is of 0 to 1 and C hat is actually the controllability matrix and shy A is having all the coefficients of the desired characteristic polynomial which you know. So you can find out the gain K very easily.

This is actually based on a principle that a matrix always satisfied its own characteristic equation which is also known as Cayley Hamilton's theorem.

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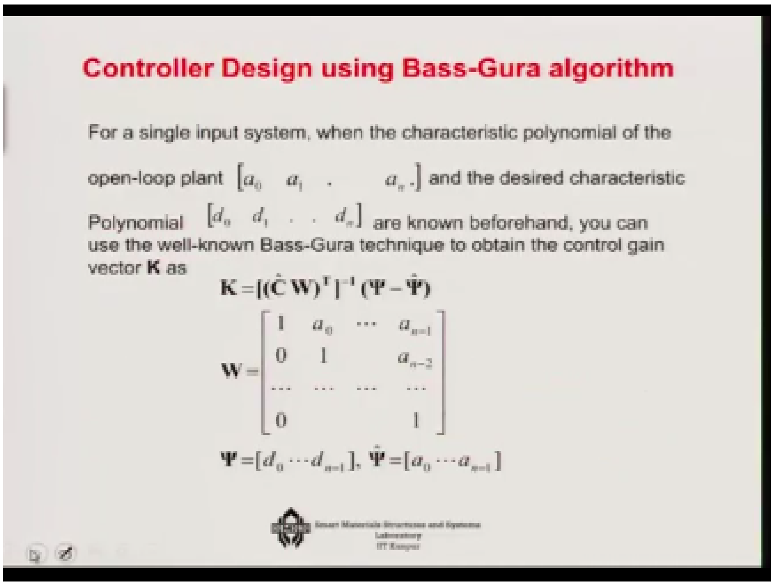

So that is you know if you want to do it directly and there is also another formulation that is used that time which is known as Bass Gura algorithm. Here you have the open loop plants a0 to an and you have the desired characteristic one. You can get the gain very easily K provided you know C hat you know (()) (15:25) , you know shy and shy hat where shy has a desired one and shy hat is the existing one.

So what is the (()) (15:31) is a matrix which has all the diagonals as unity and then the a0 to an - 1 this is the special formation and that is how if you can develop that you can get the gain K of the system. So you can use either of this formulations, this 3 formulations to design the gain K.

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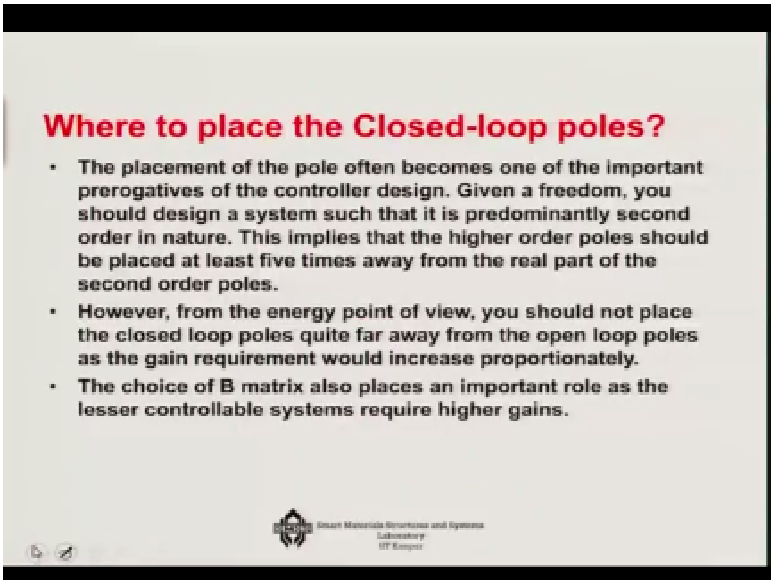

The question is now where to place this closed loop poles? because you can actually place it anywhere that is the freedom that you are getting in full state feedback control. Now given up freedom of course you should design the system such that it is predominantly second order in nature. So that means higher order poles should be placed at least 5 times away from the real part of the second order poles.

So that way if you do then you can keep a system predominantly second order, which may be good form. However, if you want to do it from the energy point of view you should not place the close loop poles quite far away from the open loop poles because then the gain requirement would increase proportionately.

So one way you can play is with the choice of the B matrix which place an important role as the lesser controllable systems generally require higher gains. So B matrix choice is proper then the gain will be requirement will be smaller and hence I will be easily able to place the close loop poles to the desired location.

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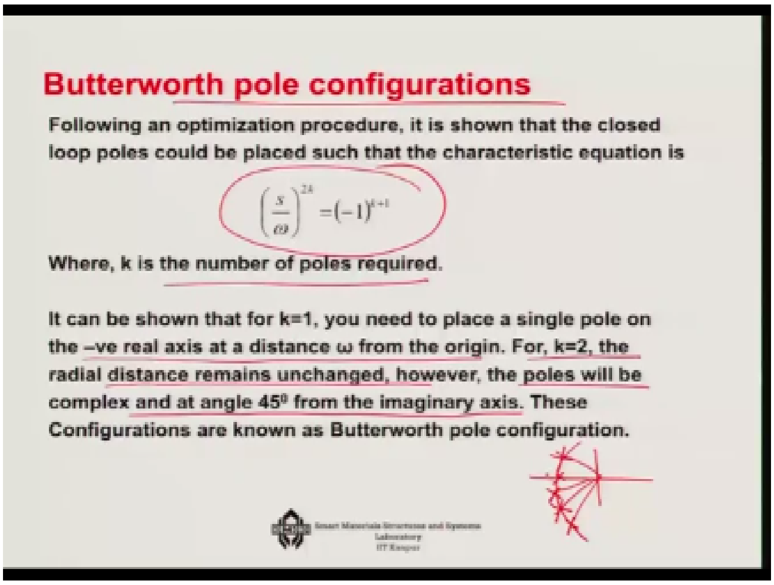

The one way to do it is actually to use something which is known as Butterworth pole configurations and this actually tells us that desired locations can be find out if you can solve this characteristic equation where K is the number of poles that you would need and to place a single pole on the negative real axis at a distance omega from the origin. K equals to 1 and similarly for K equals to 2 the radial distance remains unchanged.

However the poles will be complex and at an angle of 45 degree from the imaginary axis. So basically what it does is that it actually tells you that if K equals to 1 then you put it on this axis itself, if K equals to 2 then you put it in that same circle you take a circular arc and you take the 2 desired poles in this way that is what the Butterworth pole configuration rule.

If it is 3 then actually you divide these angles into 3 parts and you divide each one of them into 3 of it. So like that you increase, so that is all the Butterworth pole configuration tells us about the pole placement of the system. So this is where we are going to put an end. Thank you.