Principles of Vibration Control
Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology-Kanpur

Lecture-18
Basics of State Space Control

Welcome to the course on principles of vibration control. And in the last class I have told you that how to design a classical control system with the help of various types of control block diagrams, but there are cases where you may not get a single input, single input control. Let us say that we are considering a system which is lumped mass parameter system something like this type of a system where there is more than one degree of freedom.

So let us try to draw a system which has at least 2 degrees of freedom input, so we have this type of system here which has a mass first mass which is M1 and then we have a second mass which is M2 and the stiffness is K and this is subjected to some kind of a force Ft. Let say this damping is coefficients is C and let us say that this is X1 that represent of mass M1 and here it is x2, x1 as a function of t, xt as a function of xt.

So this is not a single degree of freedom system any more. This is a 2 degree of freedom system and hence we cannot apply the same way in which we have done it for the last case of the classical control system. So first of all let us try to find out that what the given equation of such a system is. So if I try to draw the free body diagram of the system then we have this mass M2 the force working on it is Ft just put it as Ft.

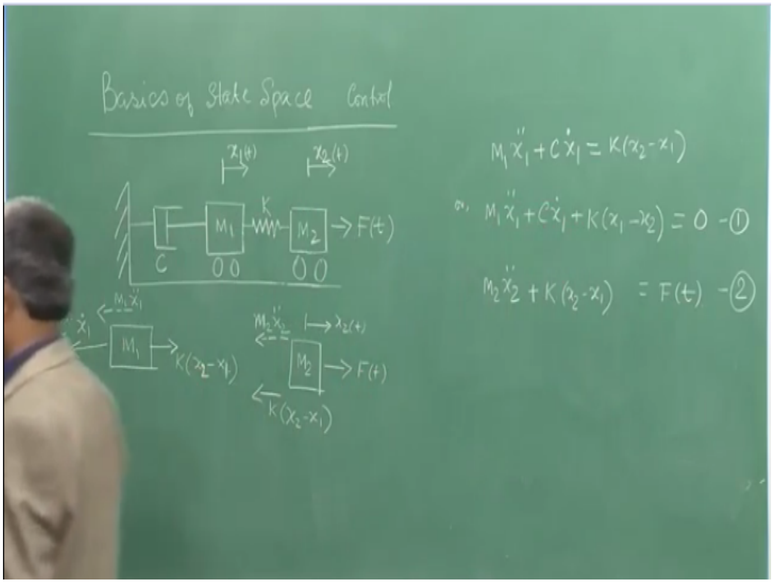
And then you have response as x2t which means the (()) (02:26) force will be in the opposite direction which is M2x2 double dot and also you have a spring here K and that spring is having a relative this must be x2 and x1 which is x2-x1. That is the first free body diagram, then we will be having the second free body diagram for the mass M1. Now in this case there is no direct force for this spring is working so it is K times it just the reverse direction X1-x2.

And then there is a damper that is resisting the motion, so that is C x1 dot and also of course we have the enough force which is in the reverse direction of the this + makes it so it is M1 x1 double dot. So see this is what is a free body diagram that is I can write down the equations of the system in the first case which is M1 x1 double dot and then we have these of course this is K x2-x1 in the reverse direction.

So Kx2-x1 and hence if we want to put it all in the same directions I can do it in 2 steps c x1 dot equals to K x2-x1 in other words we can write it as M1 x1 double dot to the form which we know more Cx1 dot+Kx2x1-x2, that means we keep it as a homogeneous form and the second one let us try to write it that will be M2 x2 double dot and + there is no damping here K x2-x1.

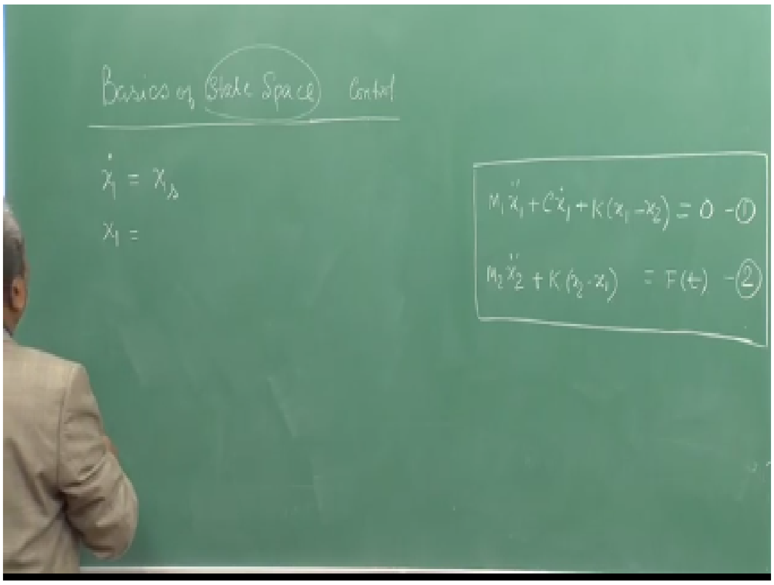
And that equals to ft. So this is my first equation and this is my second equation. Okay so I can erase all our things now because this are what my given equations are so I want only keep my given equations for the system with me. Once we understand the systems we need is everything else to focus on the given equation itself.

(Refer Slide Time: 00:36)



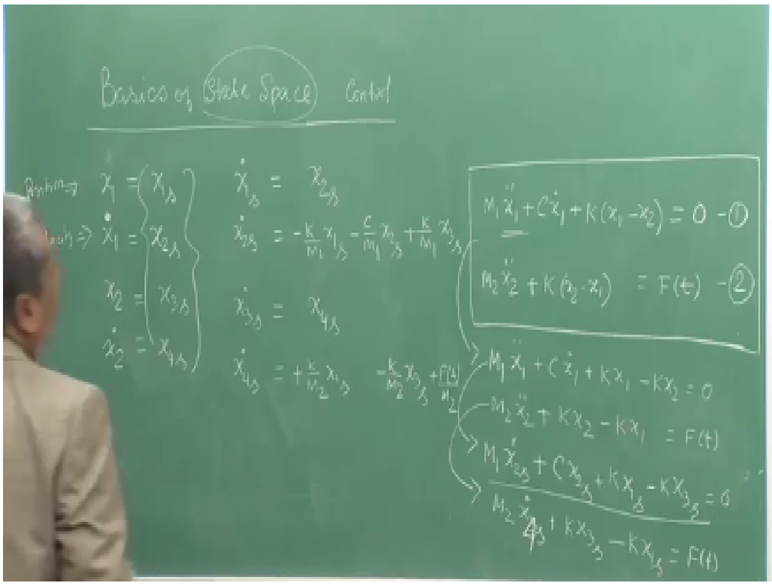
So this are the given equations of this particular system which is my first step. In order to be that state space model of the system. So as you can see that these given equation consists of 2 second order ordinary differential equation. In order to decompose it into the form of a state space form we have to convert each all of these second order OD to 2 first order OD systems. How do we do it well to do that let us assume now that a new set of vector switch elements which will call them to be the study state elements of the system.

(Refer Slide Time: 06:26)



So we can try to find that out that means it will be x1 dot as one state so let us call this as x1 state and then x1 itself as another state so let us call that also as a second state of the system in order to just confirm to the usual model we may also do the other manner that means we can first put the x1.

(Refer Slide Time: 06:59)



Then we can put the x1 dot and equal that as x2 study state, then we can put the x2 as the x3 study state and we can put x2 dot and we can write it as x4 study state. So what have we done here we have simply considered for each one of these degrees of freedom, we have considered 2 states, okay. One for the position, so this is the position and another for the velocity.

Now we have to keeping our mind here that states space representation is not a unique representations in the sense you could have considered any other liner combinations of the systems or you could have consider velocity and acceleration as the 2 states or position and acceleration etc. etc. But in this case the most common one is to go for position and velocity that is what we have done for the 2 degrees of freedom of system.

Now keeping this point in mind let us try to rewrite this equation in a manner that it will become easier for us to convert it into 2 states space form, so it will be M1 x1 double dot + Cx1 dot+kx1-kx2 equals to 0 as the first equation and second equation will be M2X2 double dot+we can write it as Kx2-kx1 equals to Ft. So if we keep these points in our mind let us try to see that how this representation is going to help us in terms of the states space representation of the system.

First of all by looking at this itself we are going to get 2 you know identities state forward, so the first identity is telling us that x1 s dot is actually x2s, because x1s dot means x1 dot. So I can write x1 is dot equals to x2s. That is the first thing that I can get from here. And then I can also write similarly here that x3s dot equals to x4s, so this 2 identities we can derive from this relationships that we have created.

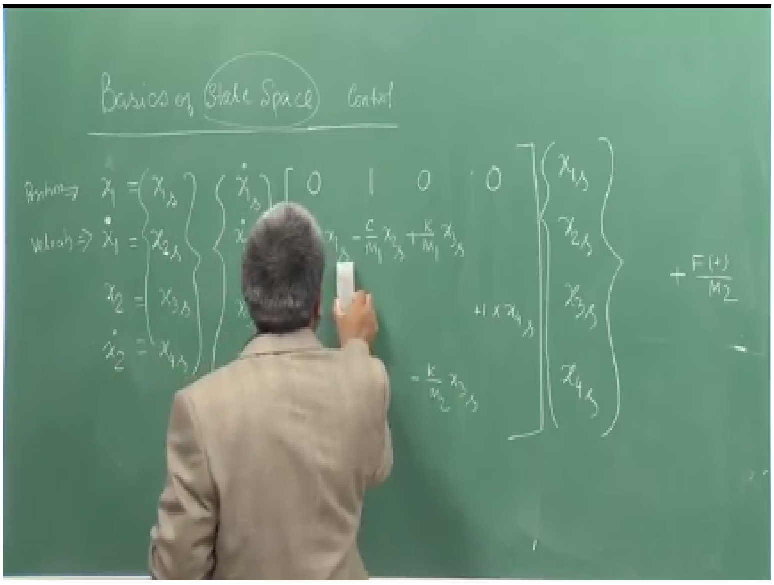
Now let us look into this equation. In this equation now we have x1 double dot so that means it is x2s dot. So that means I can write this equation so let us write this equation now as M1 X2s dot+Cx2s+Kx1s-Kx3s equals to 0. That is what is my first equation, rewriting it and the second equation also we can attempt to rewrite the same manner that it will be M2x3s dot+Kx3s now-Kx1s equals to Ft.

So we have this 2 equations also represent it in terms of this new variables fast and then I can try to write them down, so my next equation which is this equation. Let us try to keep x2s dot in one side, so that means this equation if we work one more stage on this particular equation, how it look like if I keep x2s dot in one side it will look like x2s dot equals to that directly go to the other side now equals to-k over M1 x1s and then - C over M1 x2s.

And then+K over M1 x3s. That is what will be this equation. And the last equation let us look at it the same manner we can write here as X2s dot in this case we should have written x4s dot because it is x3s double dot okay, so x4s dot so that this last equation will be for us x4s dot equals to let us put first x1 term - K over M2 x1s.

And then there is no terminated to x2 then there will be - so this - K over since we are going to the other side it will become +so-K over M2, it is this one now-K what into x3s+Ft divide it by M2 because M2 here diving everywhere. So +K over M2 x1s and-K over M2 x3s+Ft over M2.

(Refer Slide Time: 13:49)



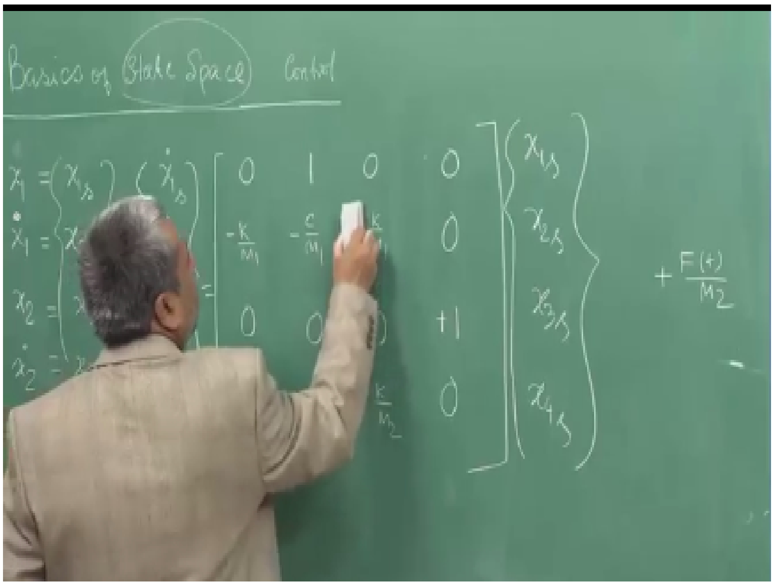
So that is all we are going to get all the 4 equations based on which now we can develop the state space representation of a system, so let us try to erase everything else and focus on this 4 equations either slightly more organised form. So what we can do is that it will do that we slightly shift the position we keep here x2s okay so+1 times x2s that is what is one equation and we keep here the last equation +1 times x4s.

And we shift this further away so we keep this as +Ft over M2. This will help us in terms of visualizing the system and slightly shift this also -KyM2 x3 which is here -K by M2 x3s and here it is x4s okay. So we are nearly true with this arrangement, all we now need to do is that to write the system in a vectorial representation form like this. So this is the complete matrix equation will get left hand side we have x1 dots all x1 dots together.

And we now put them all into a single matrix so we bring these things together, we bring these things, we are building up the matrix now, we are going to write all these vectors here, so let us try to shift this + Ft by into further away + Ft over M2 will talk about it but let us just shift everything now for the time being so we have first one as x1s and then for x1s first entry is 0, second entry is there x2s corresponding to x2s then x3s.

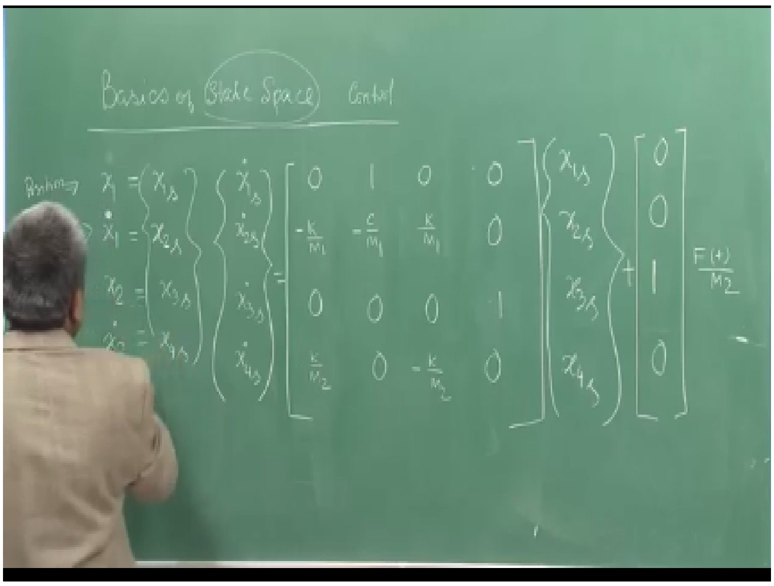
And then x4s. So we have 0 then we have + 1 and then we have 0, will have 0. That is what is our first equation right. So we can write it as 0, 1, 0, 0 x1s, x2s, x3s, x4s for this case.

(Refer Slide Time: 16:20)



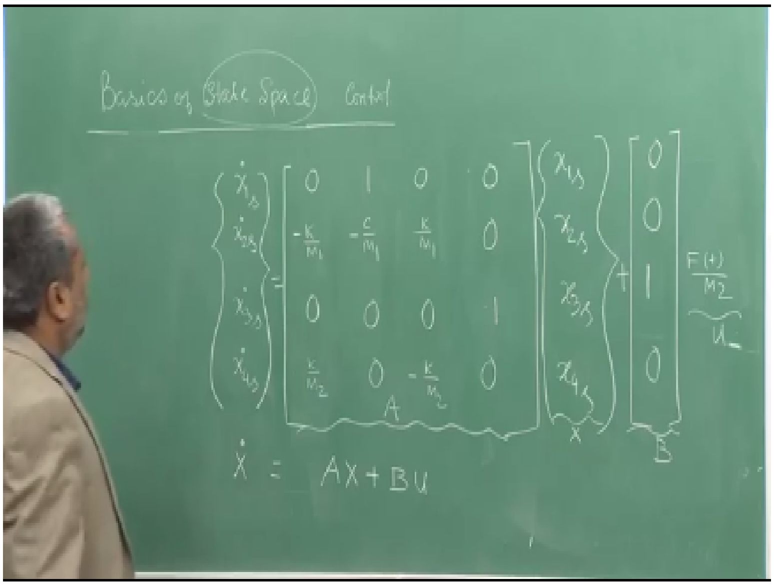
Then we have the second one x1s we are already taking care of it x2s, we are already taking care of it, x3s we are already taking care of it, there is no relationship with x4s, so that is 0. Then we have the next one this is 0, this is 0, this is 0 and this is +1. This is +1, and then we have the last one which is + K by M2 and then we have 0 nothing there. Then we have -K by M2 and then we have 0.

(Refer Slide Time: 17:03)



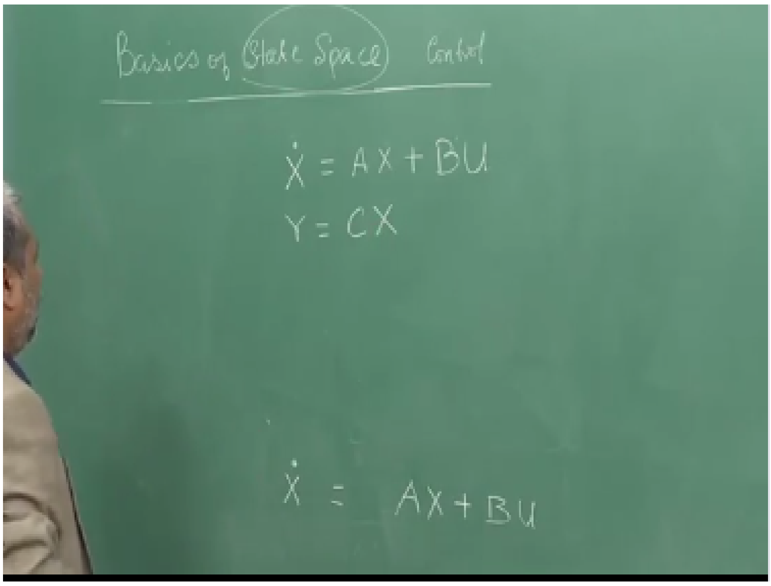
Actually now that we have all the things there so we can even erase the first part because it is understood now. So we have this is what is the first of it. We have to still work on this guy so let us try to put this as an additional thing what we will put it in such manner that we have it in terms of something like a 0 and 1 keeping in mind that this is not affecting here, so it is 0 here, the force the not affecting the second one. It is 0 there, the force is affecting here, so it is unity here and the force is not affecting here. So it is 0 here. And this is a constant term.

(Refer Slide Time: 18:00)



Now in terms of state space representation this whole representation has a very formal way of writing this part we will then call as x dot where x dot will have all these variables. This one is known as A matrix A then that +x+B, so this is the A part, this is what is our A and this is what is our X, this is what is our B and this what is the control input using which we are going to control the vibration.

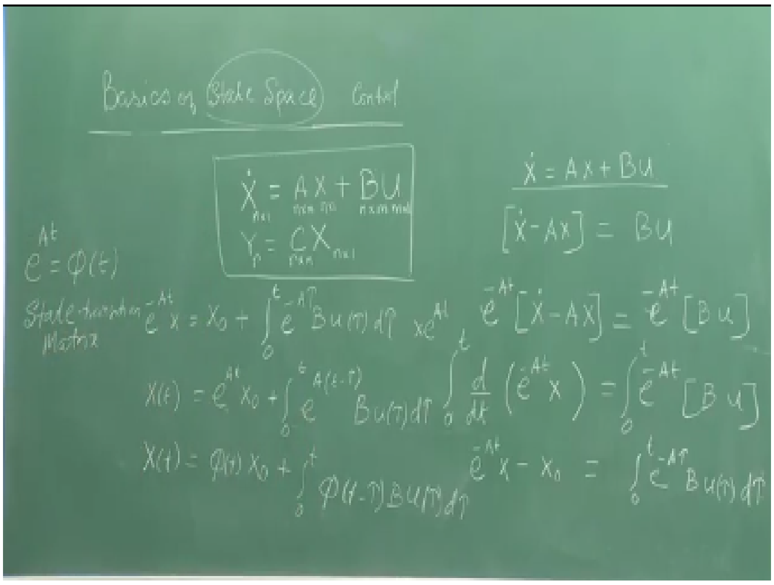
(Refer Slide Time: 18:46)



So x dot equals to Ax+Bu and we are going to erase even this now as in front of us we have a very formal state space representation of the system. So we have seen that starting from the equilibrium given equation how you can gradually get the state space representation of the system which we can write here as x dot equals to Ax+Bu.

Now depending on what we are going to sense the output we can also develop a similar equation for the output, in it is there is no you now dot on there but simply Y as Y equals to CX that means some of these positions and velocities with some of the you know sensor gains we are going to observe. So this is what is a kind of a you know representation of a state space model of a system.

(Refer Slide Time: 19:51)



In this model x is generally n by 1 vector so x is n by 1, u is the actuation force which is generally less than n, it is n by 1, so that means this is n by n is the structure of the in our last case it was 1 so that is why it was just a single vector and A is n by n now this one once again depends on you know how many outputs suppose if it is r then it is r by n. So this is how we generally you now define the sizes of the vectors.

So this is what is my steps space model. The next thing is that how do I solve this state space model, we can talk a little bit about that. So that will take the following form that you have x dot equals to Ax + Bu and what you can do is that you can take all the X where you see once side, so you have x dot - AX equals to Bu. No let us multiply both the sides by e rest to the power - At.

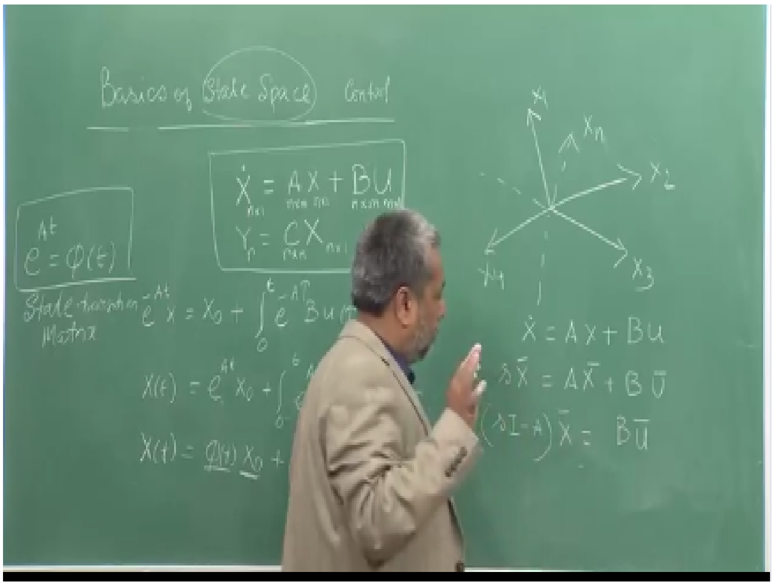
So it is e rest to the power - At x dot - Ax equals to e rest to the power - At times Bu. Now once we do that we know that we can write this part as ddt of the product of e rest to the power - At times x, so this is known to us and this equals to e rest to the power - At times Bu. Now in order to get the solution if I integrate this from 0 to t, integrate this from 0 to t.

Then we are going to see that we have this first part as e rest to the power minus. At the differentiation and integration will cancel for the first part -X0 and that equals to integration from 0 to t, e to the power - A let us give because this 2 are same, so let us give it a different variable just e rest to the power - tow Bu tow d tow. So now we can actually write down the solution of the system as e rest to the power - At X equals to X0 + integration 0 to t e rest to the power - e tow Bu tow B tow.

And so that would mean that I can actually find out Xt as e to the power At X0, so pre multiply again both the sides by e to the power At, we can do that + integration of 0 to t e rest to the power A, now this will go inside t-tow Bu tow d tow. So that is what is the solution of this particular state space equation. So can simply use these method in order to reach the solution and if we denote e to the power At as phy t which is also know as a state transition matrix.

Please keep in mind A being a matrix e to the power At is the matrix, so state transition matrix. Then our Xt can be written more formally as phy t x0+integration 0 to t phy t-tow Bu tow d tow.

(Refer Slide Time: 24:44)



Now if you look at it that there is a philosophical kind of the state when that we can make actually in this system that the beauty of a state space system is that if you know what are the X0s which is the initial states of the system. Then if the phy t is available to you which is the state transition matrix which is nothing but e rest to the power At. Then you should be able to find out the states for any time t in future.

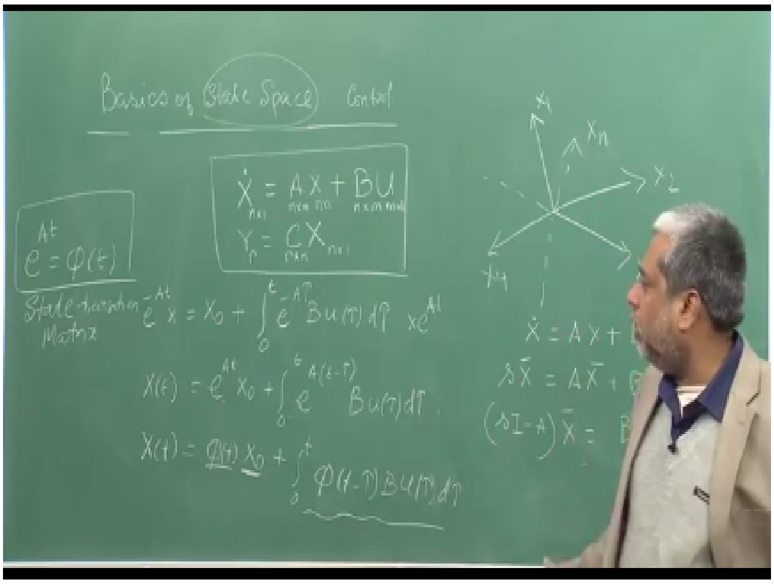
That was the neutonial model of a system of the entire universe even that knowing the initial condition you can find out what will the condition at any point of time t. So just you need 2 knowledge's of course X0 and phy t and if it is a system you see have some force part then you have to add this forcing part in it. If there is no for force person only xt will be phy t X0. You initially disperse it and then you see how the whole system is working.

So you can start to visualize that this X0 is actually in this particular case it is a 4 dimensional system right, so you have a 4 dimensional system that you have here X1, X2, X3, and X4. This 4 dimensional system is giving me the state space of the system and in this particular case in this 4 dimensional system I am finding out how the entire states are evolving. Now you can have a generalized case where you can have many more states like this and have to have something like Xn.

So you can have n dimensional system in which you can solve the same problem with the help of the state space formulation of the system. The other important point that we want to make here is that unlike the natural frequencies the count of part here is actually is the A itself, so you can in fact once again find it in this manner that X dot equals to Ax + Bu.

So if you use laplase transformation then it will be sx equals to Ax+B let us call it as X bar and Bu bar. So that means s identity - A times X bar equals to Bu bar. Now for a homogeneous case where I am not applying the force to the system when this is 0.

(Refer Slide Time: 27:53)



So that means I will be having Si - A times X bar is equals to a now matrix, so for the nontribal solution of these is determinate of si - A must equal to 0 and this is going to give me, so this is what is my new characteristic equation. Earlier we have talked about the characteristic equation. And this is the new characteristic equation of the system. So you are going to get let say A is a n buy n size.,

So you are going to get a polynomial of s which is you know Sn+ some constant let us call it alpha 1 sn - 1 + dot, dot, dot, dot, +alpha 0 equals to 0. So instead of getting actually a second order characteristic equation, now you have nth order polynomial of new characteristic equation which you can get from this determinate. So you will be getting actually not 2 roots part s1 to sn, number of roots you are going to get from the system.

These roots are also known as Eigen values of the system. So instead of having just a pair of Eigen values in the case of the state space system you are going to get you know n number of Eigen values and of course if they are of complex type then they must be complex conjugate because otherwise my equation here is having all real terms, so you will not get it.

So that is why you will get it either as you know s1 to sn real or s1 to sn as a combination of real and (()) (30:10) I means of just complex conjugate roots. So that will be a complete representation of the state space system. With this representation in the next talk we are going to talk about that how we can use a full state feedback control for another kind of multiple input, multiple output system. Thank you.