Principles of Vibration Control
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Lecture-15
Springs for Vibration Isolation

Welcome to the course on principles of vibration control. Today we are going to focus on vibration isolation.

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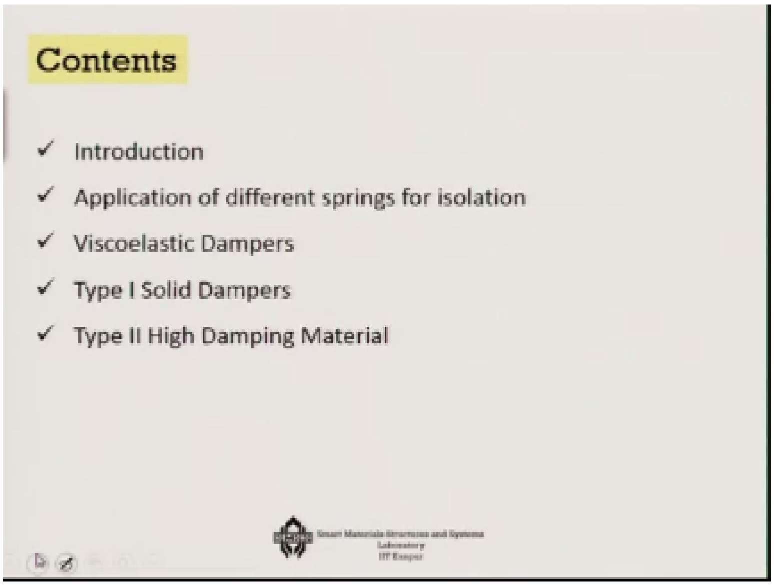

So vibration isolation we will look in to it from different elements point of view, so first of all we will talk about, will give an introduction on vibration isolation. Then we will talk about how difference springs can be used for vibration isolation and then we will talk about dampers for the same purpose and some of the sub divisions of the dampers which are solid dampers and high damping material. So essentially we are going to see how different damping elements are useful for isolating the vibration.

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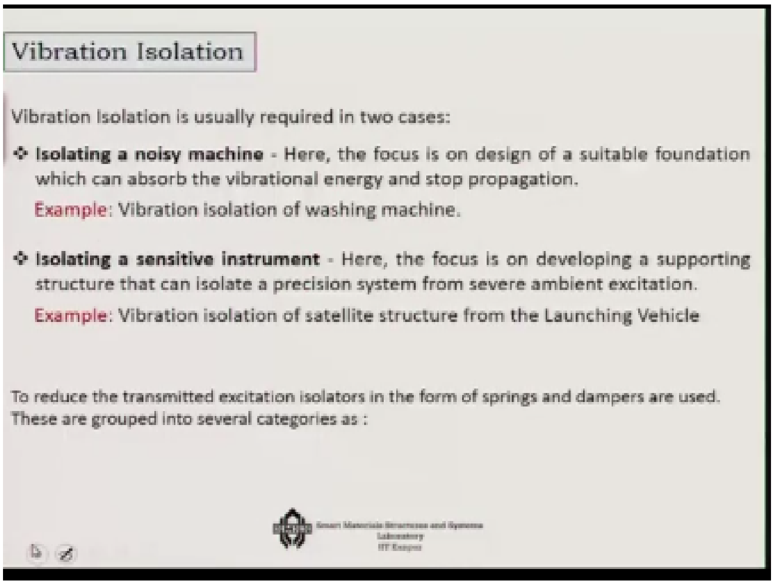

Now when we talk about vibration isolations they are required for 2 broadly 2 different types of cases. One case is that you have a noisy machine, let us say you have a washing machine which is generating too much of a sound. So you have a noisy machine and the one to observe the vibration sounds etc of the machine. So that is one type. The other type is that you want to isolate a sensitive equipment from the noisy environment.

So in one case you have a source of vibration and noise and you want to isolate that and in the other case there is noise and vibration in the ambience and you want to isolate a precision system from the effect of these noises. So these have the 2 different cases that in which vibration isolations are required and these are generally given in the form of springs and dampers. So let us see what are the different possibilities that are there for us.

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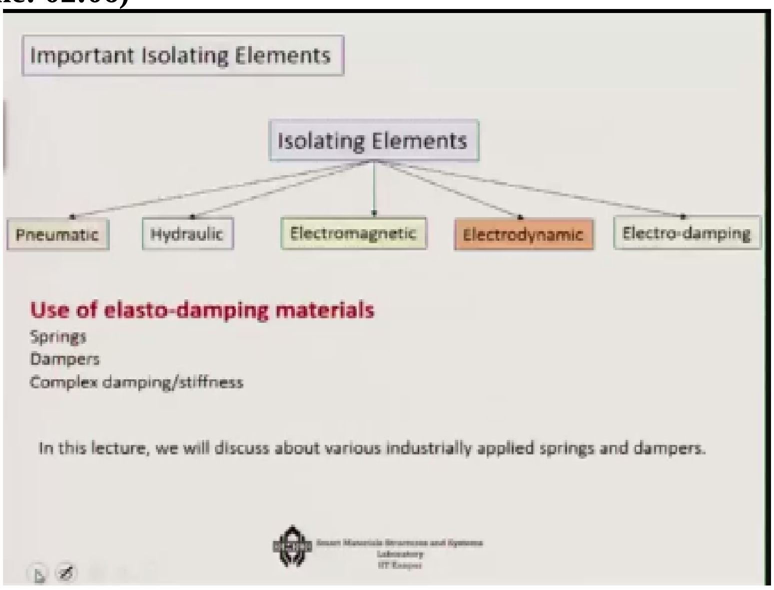

Well for isolating elements we have pneumatic, we have hydraulic, electromagnetic, electrodynamics, electro damping type of materials. And particularly in the electrodynamics damping category we have springs, we have dampers, and complex dynamics and damping and stiffness matrix that we generally use in the electro damping materials category.

There are other categories which after a point of time we will talk about active control i will touch if you want them. Now first we will you know focus on the springs and then we will go to the dampers in this category.

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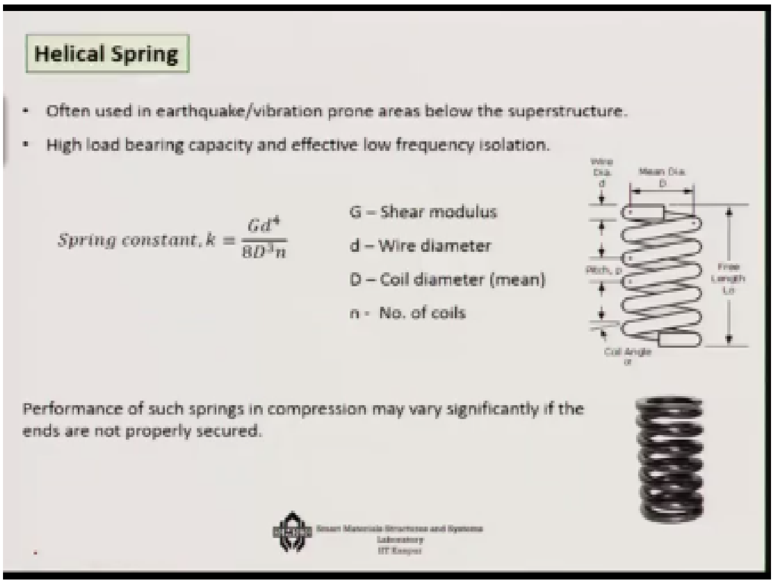

So let us first of all talk about the springs. Springs can be of many different types that most common spring is the helical spring which you will see in day to day you know applications. This are used for example in earthquake or vibration prone areas even below the superstructure and they generally have a very high load bearing capacity and a quite an effective low frequency vibration isolation.

Since earthquake happens you know it affect in the low frequency regime so this kind of helical springs are quite good for that and if you look at the spring constant K it depends on the share modules of the material, it depends on the wire diameter, very significantly d to the power 4 and it also inversely varies with the coil diameter, the mean coil diameter that is D here **as** has been showing here.

And also the number of coils that means once you know that what is the free length once you know the total number of coils and you will be knowing about the details of each, so using this information you should be able to find out that what is this spring stiffness K. So this is the helical spring and a this kind of springs in compression actually may vary significantly they are stiffness properties, if the ends are not properly secured.

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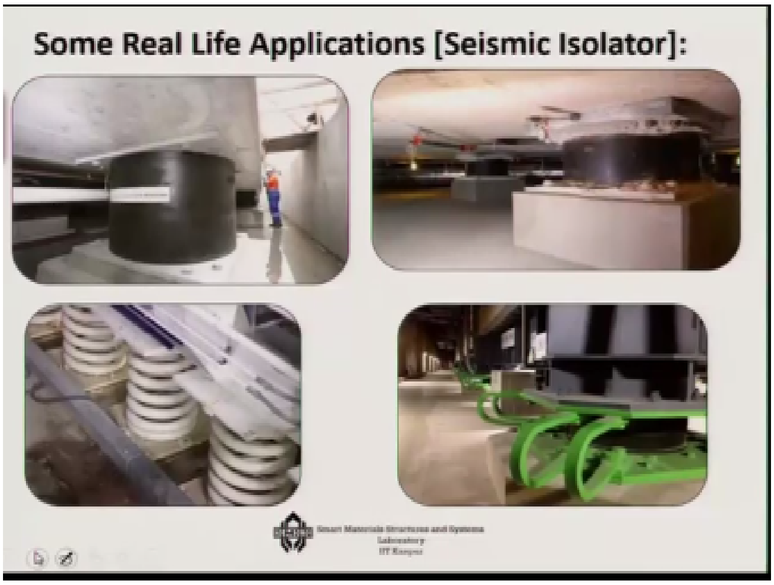

Now let us look into some of the applications of the helical springs as you can see here that is the huge supra structure and just below that inside this enclosures the springs are there. In some cases there will be springs like you can see this springs in this case here. In some cases this springs could be also with the dampers and here between the 2 images if you look at it in this case it is quizzed more due to the effect of an earthquake.

So you now its shows that this is very effective. In some cases actually there are quite an exotic additional springs like this U type spring as you can see here that is also added in some cases to further enhance these isolation capability. In all these cases helical springs used for the development of seismic isolator. So that earthquake resistant is possible in the system.

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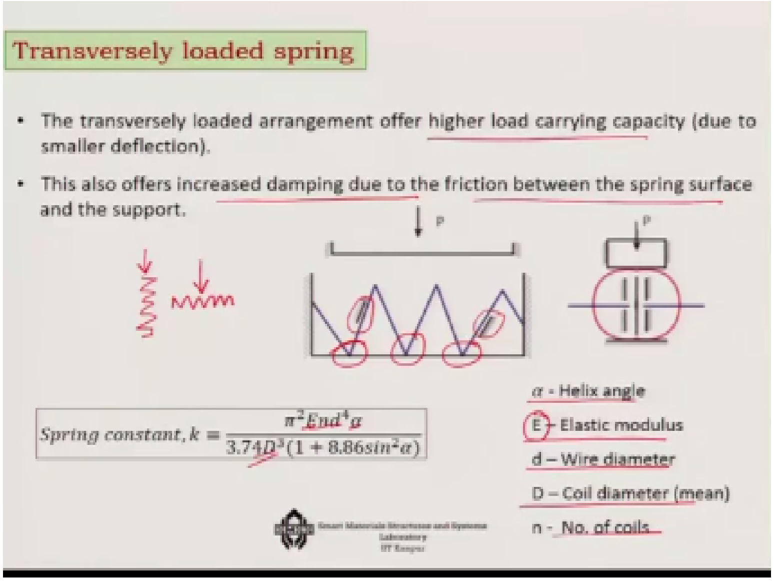

Let us look into some of the other springs like transversely loaded springs. Most of the times we think that the spring is actually loaded like from the top, so generally we consider a spring which is loaded form the top but in this case the figure is showing that we are not loading it this actual manner what transfer. So this transfers loading arrangement it is form that it offers actually higher load carrying capacity provided you actually need to have a proper these things.

So that there is no (()) (06:04) etc. that takes place in the systems. So you need to have proper guides in this spring reflection and this in this case also you will get an increased damping because there will be friction between this spring surface and the support, so that will enhance the damping, see there will be more frictions here in all these regions, so the damping will be enhanced.

So in this case the spring constant depends on the helix angle, the elastic modulus e, wire diameter gain is playing a crucial role, you can see all of them here, and then the coil diameter is coming here in the denominator inversely proportional and the number of coils also is coming into the picture. One thing you can see the difference between the last one is that instead of the share modular's here the spring stiffness depends on the elastic modules e.

And in most of the materials e is much higher than g and as a result you get larger bearing capacity and also good damping in transversely loaded spring elements.

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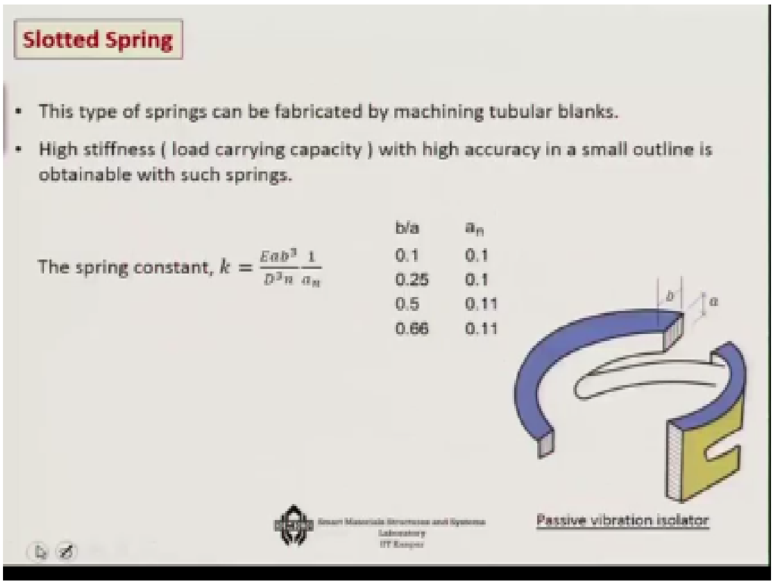

Now there is another variety of spring element, that is particularly used for small linkages and joints and this is known as a slotted spring element. This also extracts the advantage of getting stiffness from the modulus young modulus of elasticity rather than share modulus of elasticity, to do that the spring is actually slotted, so this is the typically slotted spring where these you know actually from the blanks it is directly made and the sizes are given here.

So with respect to the size ration you get a coefficient n and then you have this coefficient, you have the coiled diameter, all other things then you can get the spring constant and you can see here that it is directly proportional to the modulus of elasticity and it has a high accuracy in a small outline of a size you can get so that is all it is very popular particularly for the link developments etc.

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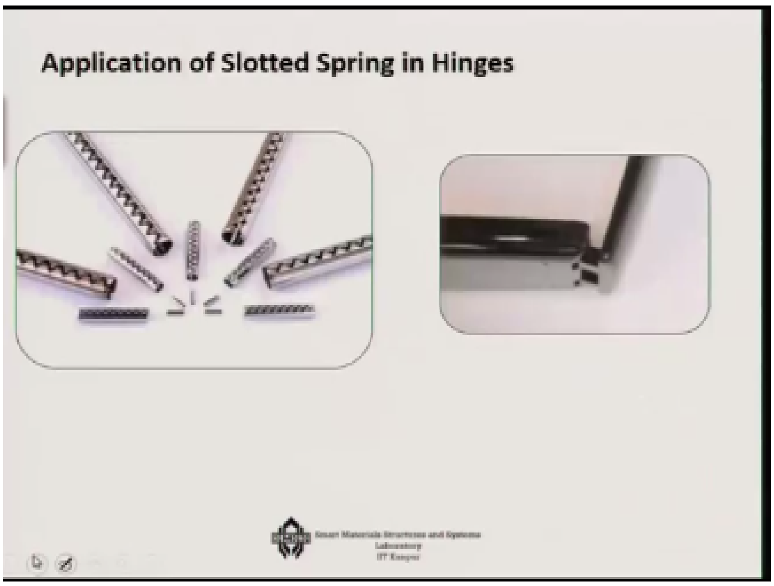

One application you can see here that this is the typical slotted spring system and it is used in this type of you knows link connections in hinges etc. So this is the second type of spring.

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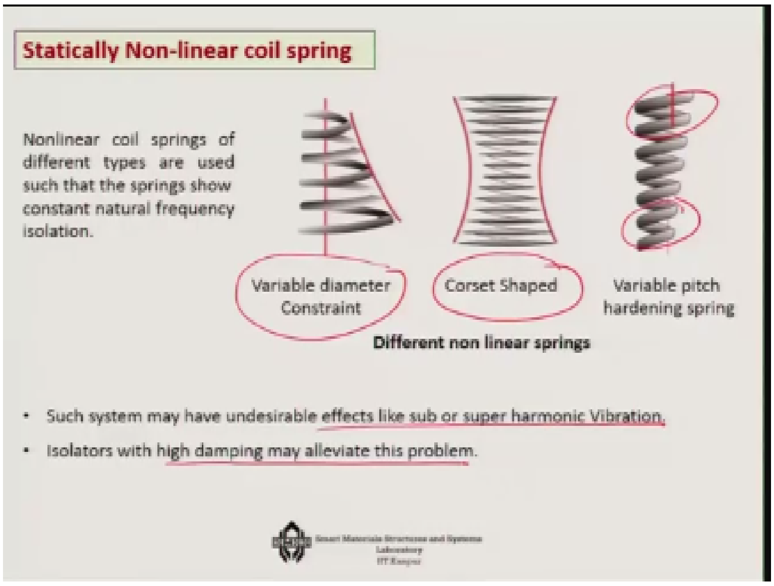

The third type of spring is that where we actually play with the coiled diameter. So in earlier 2 cases coil diameters are constant. But here the coil diameter is variable and it can be variable like in this manner or it can be variable like a corset shaped or it can be variable and the pitch also can vary in this case which is not varying but here the pitch itself is varying as you can see the pitch here and pitch here is not same.

So you can bring non linear it in many ways, you can by varying the coil diameter also by varying the variable pitch. Now such systems are usually done in such a manner that they actually with respect to different frequency level of a excitation they work well and however, sometimes they are actually assessable to sub or super hormonic vibrations and isolators with high damping are generally used to alleviate this type of a problem. So this is a statically non linear coil spring system.

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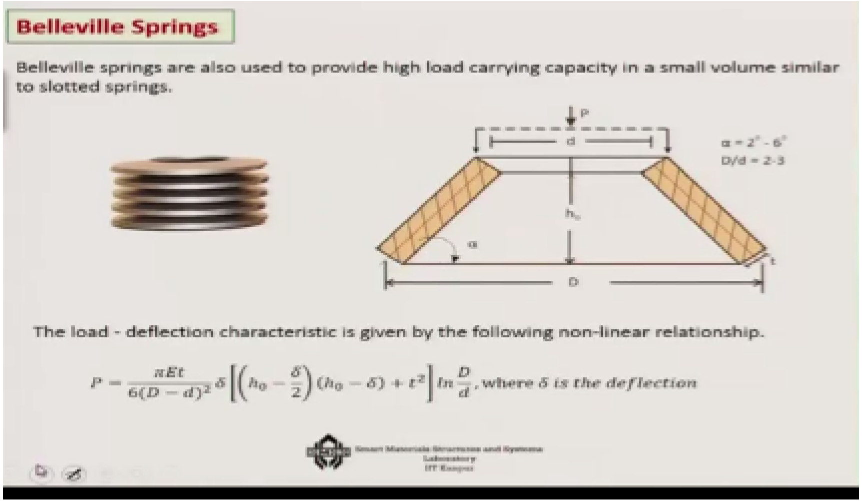

The other very useful spring system is known as Belleville spring system. Now you cn see that there are actually 1, 2, 3, 4, 5 Belleville springs like washer type of an arrangement that they are and in the Belleville spring there are many parameters you can control, one is of course the base diameter, and then this angle and which is generally between 2 degree to 6 degree and then this d by d ration the which is 2 to 3 and then of course the height.

So you can play with so many parameters, in fact one can find out the load deflection relationship for each system and if you look at it here also you are getting the modulus of elasticity for stiffness, but here additionally what you are getting is a complex expression with respect to the reflection of the load deformation relationship and that if I try to actually seat in certain regimes.

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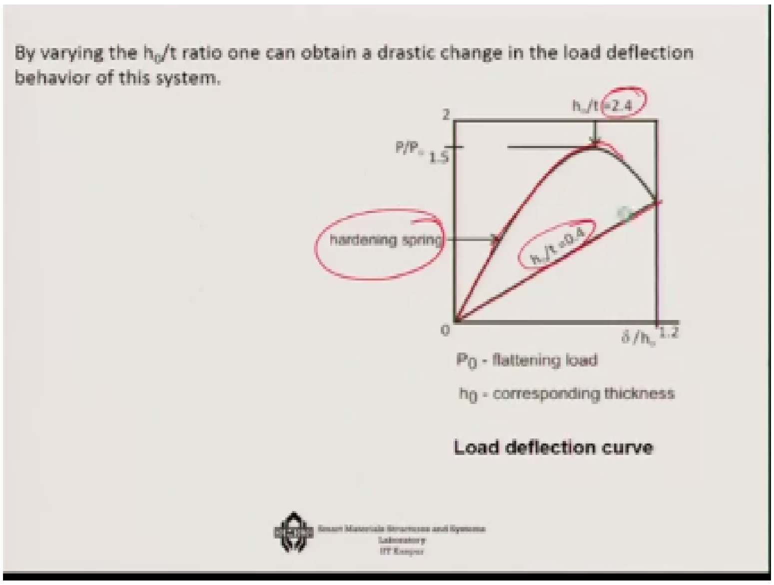

For example when the height to thickness ration is 0.4 you would see that this almost it is like a liner system. But you change the height to thickness ration to 2.4 you will see that this is behaving in a completely non linear manner and behave in more like a hardening spring. Again if you vary the h by t ratio below a certain level you may also get it you know some of the other type of deformations here. So thus by varying these h by t ration you can get different types of load deformation relationship in such a system.

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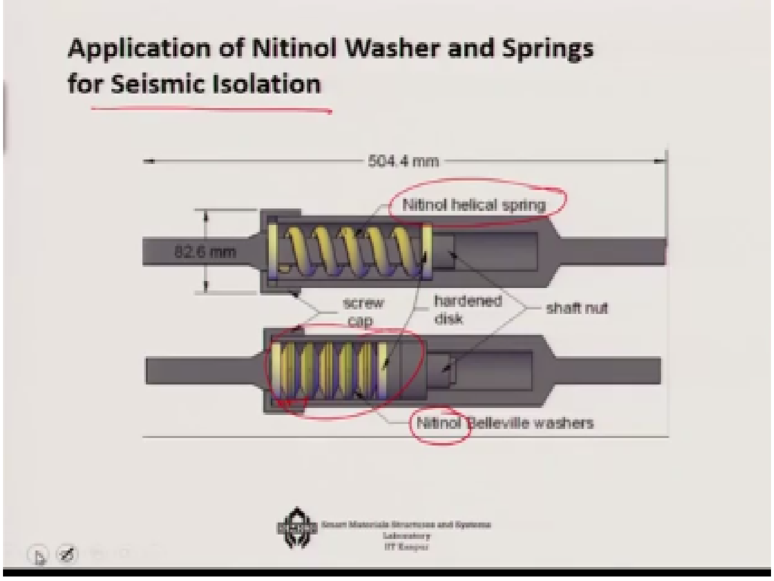

Now keeping this point in mind **ah** it is actually used in nitinol washer and springs as you can see here that nitinol helical spring and this is a valuable nitinol washers. In this cases it is done intentionally because what you can do is that the h by t ration being important in terms of showing the linear relationship or the harden relationship and you can vary the h by t ration by passing a small current in each one of them.

If you pass a current then there will be phase change in the nitinol because it is (()) (12:33) material and then because of this phase change you would observe that the h will be changing the height will be changing and as the height will be changing the geometric parameters will change and the same system will behave like a hardening spring etc. so you can actually effectively using for various types of seismic isolation systems.

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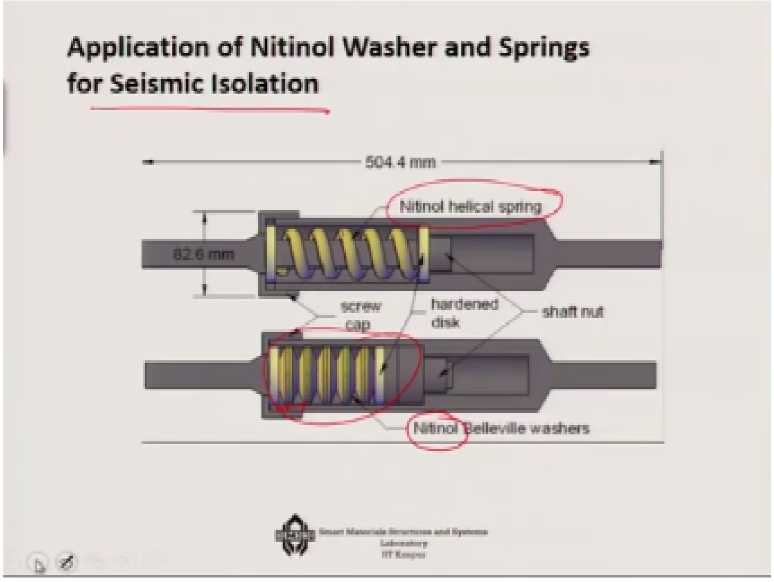

So that is valuable spring, now another type of a material which is also used board like even though it is used as an isolator but its stiffness and damping both are equally important in this type of cases because there are interlayer damping that takes place in it at this are known as wire mesh material. In such a system actually there are 2 stiffness's the dynamic stiffness and the static stiffness.

So the static stiffness is you can simply obtain through aquatic static load deformation relationship but then if you also know the material parameter and ratio of vibration amplitude to the height you can find out the dynamics stiffness and this is a highly non linear material but it is very effective for vibration isolation in machine foundations particularly for applications like you know hospital floors or for appellations like some marine machines are to be isolated noisy machines wire mesh material are very good used for such cases.

A very similar material like wire mesh is actually the failed material but in this case it is made generally of nylon or rubber fibre. And in this case the dynamics stiffness is given by this relationship where you have again 3 material parameters N, M, and A1. Now how does the wire mesh material look like.

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This is a typical wire mesh type of a thing and you can see that this used as an isolator element or machine foundation, so that means if there is some vibration here it will not be transmitted to the base. So base will remain vibration feel that is the idea of using the wire mesh damper for machine foundation.

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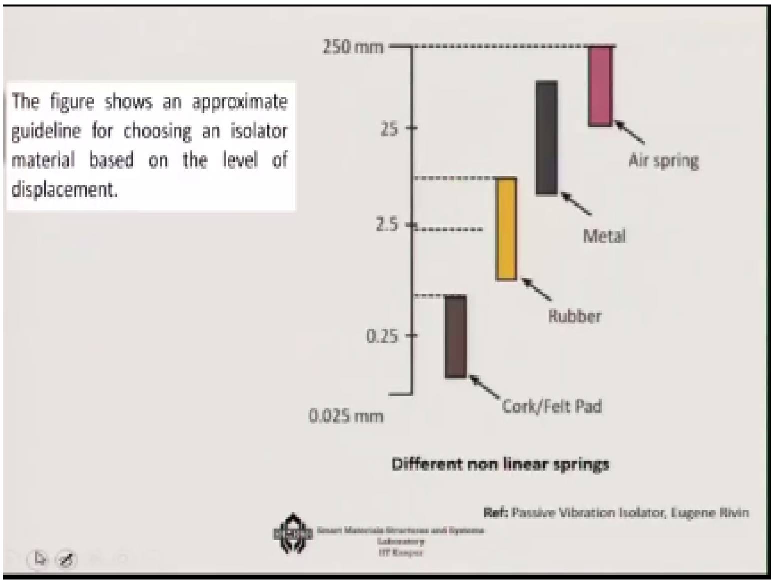

Now we have seen so many verities right we have seen these kind of felt pads or coil bases then we have seen metallic variations, there are also some variations like air spring, and rubbers have not discussed them but these do exist as a good you know type of spring for vibration isolation and each one of them has a typical area in which they work and this is generally define with respect to the level of displacement.

If the displacement amplitude is quite small then it is cork or felt pad, if it is more then rubber even more metal, it is the maximum then the air springs are used, something like 250 mm kind of a displacement. So depending on the level of amplitude of displacement the isolators are actually chosen.

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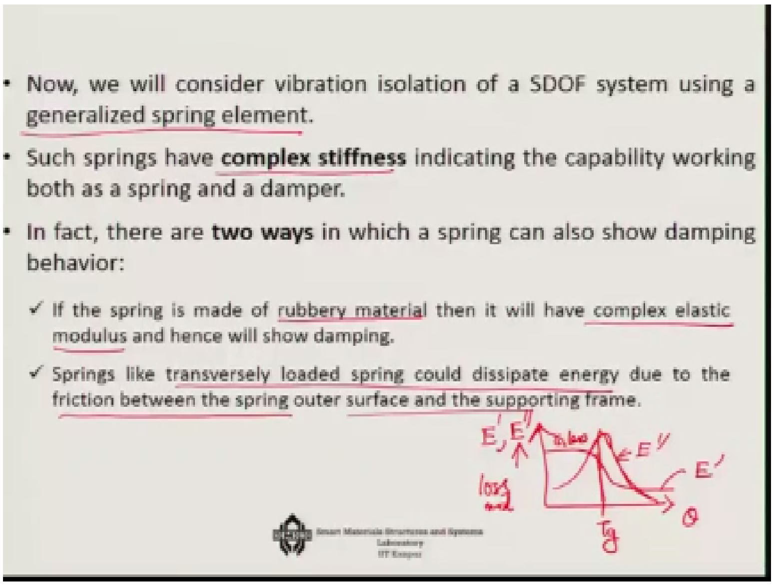

Now we will consider the vibration isolation of a single degree of freedom system and this is subjected to an isolator which is a generalized spring element that means it is not a pure spring but it is a spring + damper. Such kind of a system as a stiffness which is complex stiffness. So that means it will you know observe bear the load by getting displaced but as well as it will work partially like a damper.

Now there are 2 ways in which a spring can also show this kind of a damping behaviour, if the spring is made of rubbery material then it will have a complex elastic modules and hence that complex elastic modulus the imaginary part of it actually shows the damping. Because if you remember that we plotted basically with respect to temperature the 2 modulus of elasticity E prime and D double prime, okay.

So E prime is the glass modulus and we have seen that from the glassy region it goes down like this, so this is the glassy region. E double prime which is actually the loss modulus that varies on the hand in this manner, so the point where these transitions is happening is roughly it is the same point where you will see that the maximum peak is of the E double prime, so this is the E double prime. And this is the E prime.

So the last modulus also maximizes along the temperature which is known as the glass transition temperature. So this actually contributes to the last part of the damping. Now springs like transversely loaded spring could dissipate energy due to the friction between the spring and outer surface and the supporting frame. Whereas in this case it is not friction induced loss.

But in this case the loss is happening because of the rearrangement or the motion of the chains you know atomic chains which are there inside a polymeric material.

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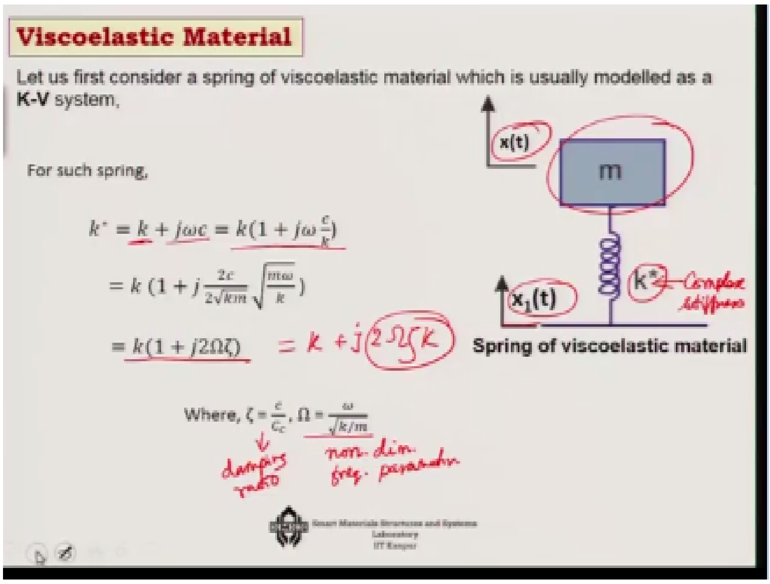

Now let us consider a simple problem where we have a mass here and we have a spring but this spring is having a stiffness which is K *. The * denotes that this is a complex spring, that means the spring with complex stiffness. So that is why we are using the * symbol to it. Now suppose this is subjected to a, you now a kind of a harmonic excitation okay, so at the base you have x1 t and here it is xt.

Now what is our K*. K* has a real part let us say it is K and just as an imaginary part and we are writing it with an imaginary J along with omega c and in that case I can actually the K out and it gets into this particular form 1+J omega c by k and this can be further written as 1+j you know this 2 c by 2 root k n and root over omega by k which means it will become k times 1+j 2 omega zeta where omega is the non dimensional frequency parameter.

So it is non dimensional frequency parameter. And this is the damping ratio zeta. So with respect to this the K* can be obtain as K which is the real part of it, so this is K+j 2 omega zeta k, this is you know imaginary party of the stiffness which contributes to the loss of the system. Now let us say this is a generic definition because here we have considered both the stiffness of the spring as well as the loss due to the viscoelastic part of it.

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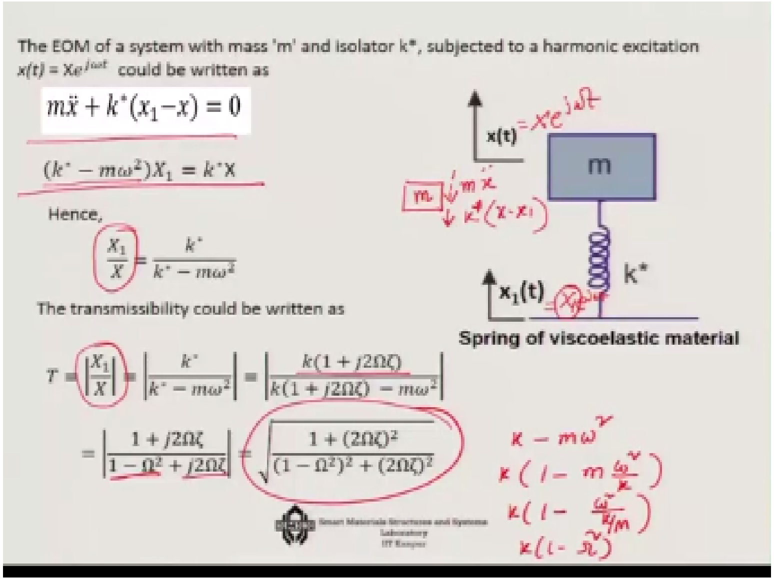

Now if I try to form the equation of motion, so again we will look into the equation of motion near in this case you do not have any other excitation, this is the base excitation problem. So we have mass m here and it is moving upward so which means it has a enough here force m x double dot and also it is getting a spring force which is resisting the motion so that is a real force, that is K * the difference between x and x1.

So if I add them together we get this relationship where it is giving us as Mx double dot+K* x-x1, so it is mx double dot + k* x-k1 and then if I you know apply x equals to or x1 equals to x1 e rest to the power g omega t and we apply here also as xe rest to the power g omega t, then this equation could become further simplified in the frequency domain.

And then it would become k*-m omega square x1 capital x1 where capital x1 is the amplitude here at the base and that equals to K* x where x is the amplitude of the mass m, the displacement amplitude of the mass m. So from this equation we can actually write what is x1 over x and x1 over x is K* over k*- m omega square. So then I can write down the transmissibility which is the amplitude of the 2 ration.

So it is k* over k*-m omega square and what is k*, we already say that k* is k into 1+j to omega zeta so you can write it here and we can cancel the K form numerator and denominator so it is a ration of 1+j 2 zeta omega over 1-omega square okay k we are cancelling. So this part how it is coming you need to just keep in your mind that denominator we have k-m omega square.

We are taking the k out so we are getting it as K-m omega square by k and that is k times 1-omega square by k by m and that can be written as k times 1-omega square. Okay because this is the natural frequency omega m, so that is how this 1- omega square term comes and of course the imaginary part is j2 omega zeta both in the top in the bottom.

Now I can find out that what the amplitude is in the top case it is simply 1+2 zeta omega square and in the bottom which is 1-omega square square+2 zeta omega square. So thus we can get the transmissibility of the vibration that means what is the ration of t you know displacement that I am giving here and the response that I am finding for the mass m. So I can find that out.

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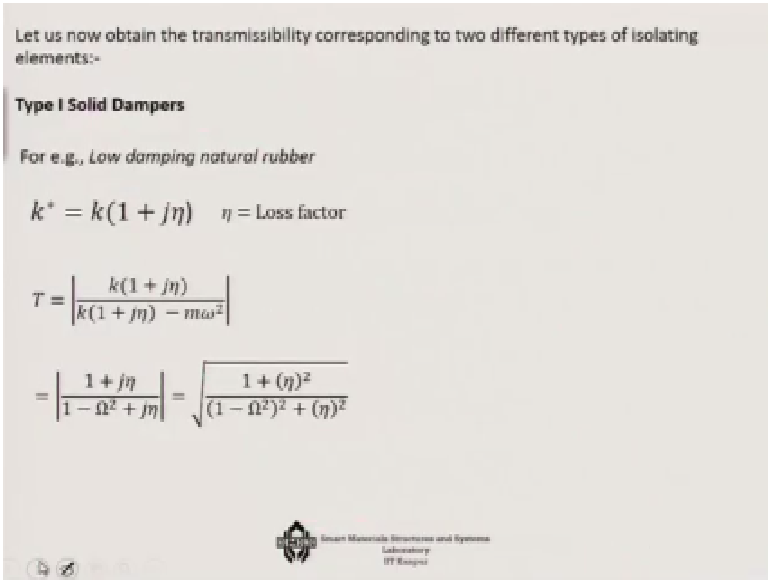

Now this transmissibility can be obtained for different types of cases. For example if I consider type 1 solid damper. Type 1 solid dampers are like low damping natural rubbers. Now here the k* is simply K+K into 1+j zeta where zeta is the last factor. So this is not a function of the frequency, the complex stiffness and it is simply it has a real part and it has an imaginary par.

So you can find out the transmissibility now by applying this expression of k* that it is k into 1+j zeta in a numerator and denominator k into 1+j zeta-m omega square and then you divide both numerator and denominator by k you are going to get 1+j zeta here you are going to get 1-omega square just like last time and then j zeta separately.

And that would mean that if I have to take the amplitude it will be amplitude of the each of the individual part which is square root of 1+zeta square divided by 1-omega square square + zeta square. So that is the transmissibility corresponding to no damping natural rubber or solid dampers. Now let us look at another case.

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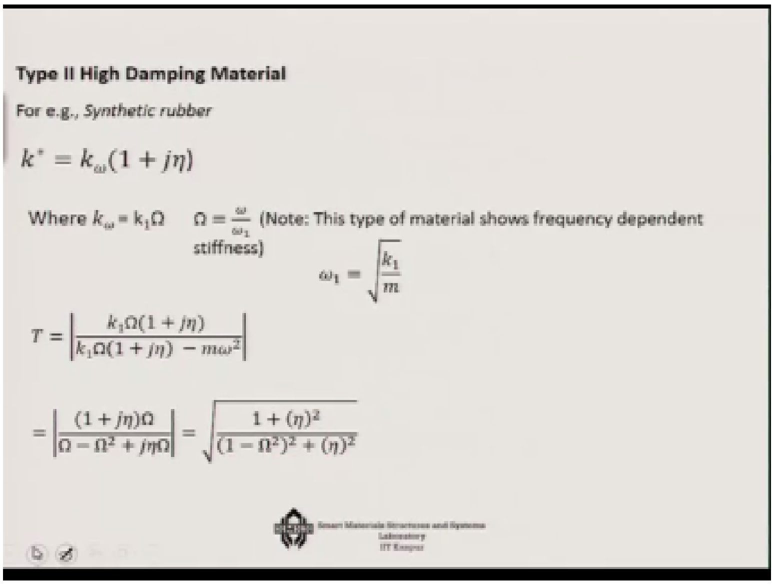

If it is high damping material, in that case the K real is not constant, it is a function of frequency and k omega is k1 times omega where omega is known to us already. So here it slightly changes and in the final relationship it becomes very neat it is 1 over square root of 1 + zeta square divided by 1-omega square+zeta square. So high damping material you can see that in this particular case, it is the omega and the zeta which is directly controlling the transmissibility of the system.

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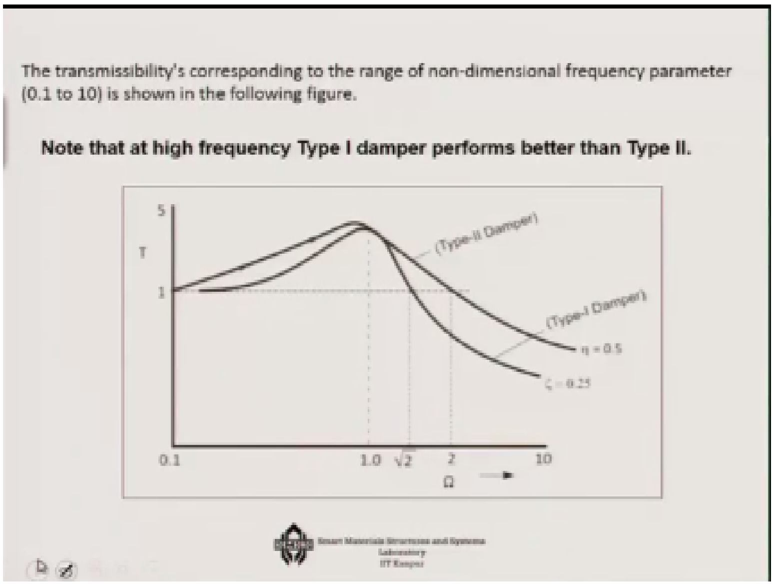

Now if we try to actually plot this so we have discussed damping plot, we have type 1 damper, we have type 2 damper. So if we plot this for a frequency ration value let say from 0.1 to 10 if we plot this 0.1 to 10 and then we will see that for all the cases when omega equals to 0 it always in variably from unity. You can even check it. If you check it in a last systems if you check it you just you now you can just put the omega ration omega to be 0.

And in that case you would see that this omega will be 0, so this will be square root of 1+zeta square over 1+zeta square meaning there by that the transmissibility will be unity. So all transmissibility is actually start from unity. So and the same thing happens for the type 1 damper also. From there at omega equals to 1 then excitation frequency equals the first fundamental natural frequency.

You are going to variably see the peak. After that the changes important because this is what is showing the decay ration how it is decay, now here we can see that a type 1 damper is actually decaying much faster than a type 2 damper. So some configurations are good for high frequency damping particularly in this case type 1 is better than the type 2 damper for isolated design.

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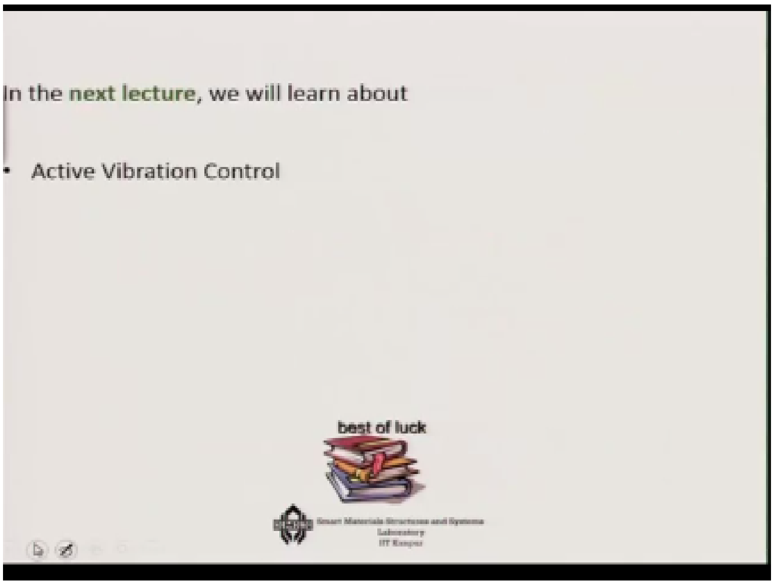

So this is where we will put an end and in the next lecture will learn about active vibration control. Thank you.