Principles of Vibration Control
Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology-Kanpur

Lecture-14
Proof Mass Actuator

Welcome to the course on principles of vibration control, and today we are going to talk about a very special type of vibration controller.

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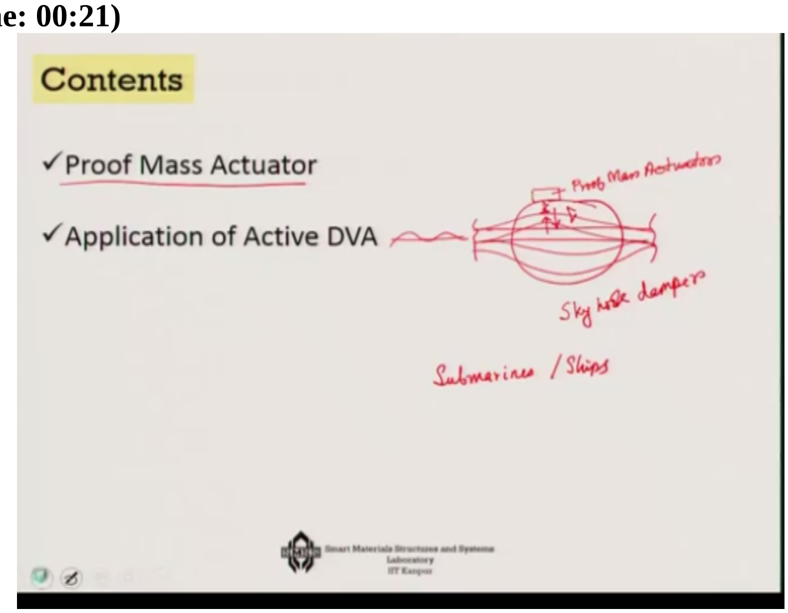

Which is known as proof mass actuator, see that how this kind of proof mass actuators can be applied in terms of active dynamic vibration absorber, now the basic principle behind the proof mass actuator is very simple suppose you consider that there is a you know a body like a submarine hove or some such system or an aircraft platform which is subjected to so, this is the body of it and this is subjected to some kind of harmonic excitation.

Now, wave is propagating through it, now, in order to resist the you know response of the system so, under certain conditions it may go up or down like this, but in order to resist this kinds of vibrations of the hall what it is generally done is that there is a magnetic proof mass system which I will show you the details of it. In a minute which is attached to the system, it maybe with or without damper, for this is what is known as a proof mass actuator.

And, the role of the proof mass actuator, is to apply once again a harmonic way back to the system, ok, so, when for example, this is going up at that time you apply a force which is going down dynamically and back and forth like that, so, that you can supplies the dynamic response of the system, so, that is why it is also sometimes known as skyhook damper. It is a very effective technology for particularly submarines and ships or ships.

Because, both for submarines and ships there is the problems that you know the power source of it particularly for defence applications, the power source are very much acceptable to this kind of vibrations and an if depth charge or if an enemy shock wave created by an enemy you know kind of a bomb or something if that creates a problem in these particular generating system.

Then the entire system will be down, so, as a result people generally apply these kinds of proof mass actuators on such defence equipment so that this kind of vibration is not allowed in to the system.

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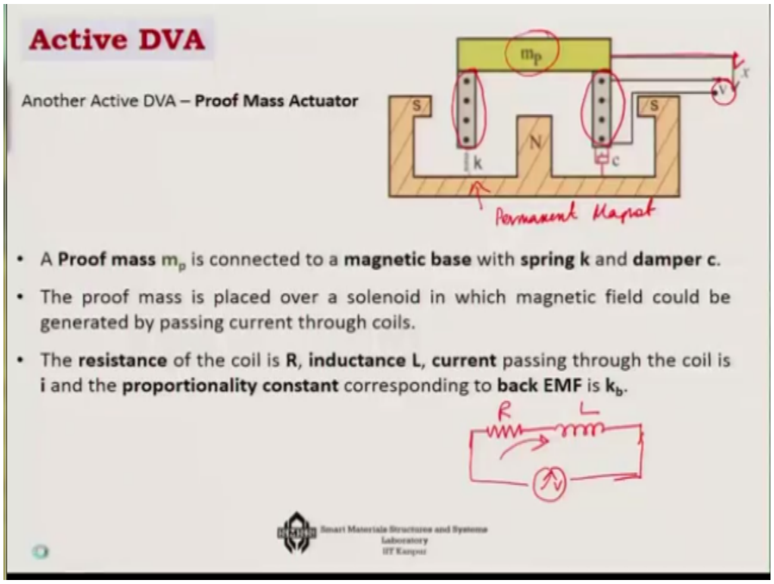

Now, let us look in to how, such a system would be actually you know what is it inside this proof mass actuator, so, if you look at it that this is the proof mass or so, to say that the mass part inertia part of the proof mass actuator, and then it has basically permanent magnets, so, this is actually this is the permanent magnet part of it, and if you look at it, that this proof mass is sitting on electromagnetic coils. That is the electromagnetic coil system.

And a kind of a spring an damper is a symbolic representation but basically attached with the where it is attached to the permanent magnet, may be consider through a spring and damper, so, that is what is the total kind of the construction of proof mass actuator. Now, this particular coil that it has you may consider that this coil as can be represented as something which as a resistance or and an inductance L.

So, the coil can be represented in terms of resistance or under inductant sale, and there is the voltage that we are actually applying to that system ok, and through a some kind of a source we are applying this voltage ok, power source and the current is passing through this whole thing and it what happens when I pass the current through the solenoid this becomes a magnet, and, you can design this magnet in such a manner that you can generate the poles in it.

And this poles will essentially control the motion of these particular proof mass actuator system, because, if there is a north creation then there will be a repulsion and the southern will be in a south-south part will be actually pulling it so, you can actually generate a kind of a motion in the proof mass actuator by controlling the current in the system. Now if we actually try to find out the governing equations of these.

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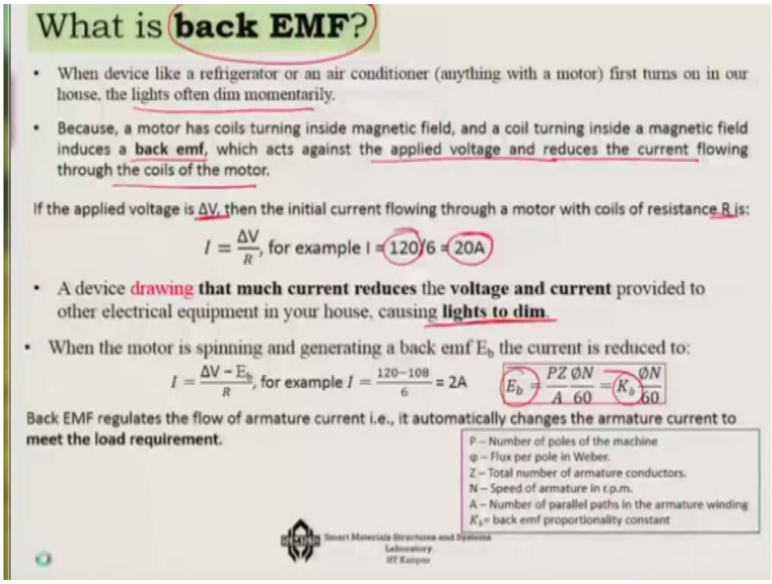

Will find that one important thing that will come into the motor model is called the back EMF ok, in motor actually you need two things one is called the armatia constant which will be knowing it typically depends on these you know coils, the coil density, coil materials etc., but the back EMF also is an important factor and you might have noticed it many times in your day today experience, suppose whenever a refrigerator or the air conditioner.

You know the anything which has a compressor in the motor, when it first starts you see that the light of the dims (()) (06:21), now the reason is that because the motor has coils and you know the inside the magnetic field and the coil is turning inside the magnetic field will actually induce a back EMF, in that coil itself which will act against the applied voltage something that is producing the magnetic field, so, thus, it will reduce the current flowing through the coils of the motor.

You can do a simple calculation like if the applied voltage is delta v for example and you know the coil resistance are then you can find out that what is the current that will come up in the system, and for that you know delta v applied voltage of suppose 120 and resistance of 6 you will get 20 ampere of current. So, that means this much of current will be actually drawing from the system quit to in order to actual counter this magnetic field.

And since, this current is getting drawn from the system so the lights dim because that current is getting drawn back to the system. So, thus you know you can actually see this kind manifestations of the back EMF, now in the motor we can actually find out that what is the you know EMF and what is the back EMF constant depending on the various factors of the motor and the load requirement.

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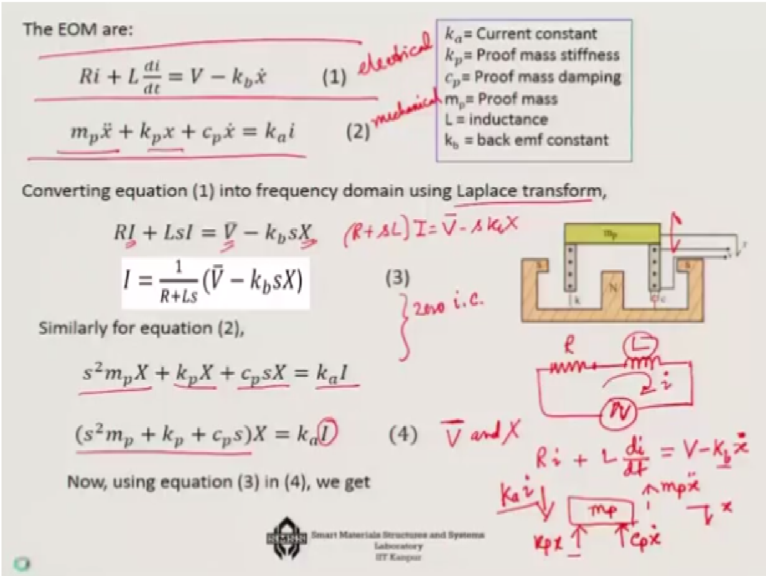

So, if we look in to the governing equation of the system, so, what we have is that we have two systems with us, one is an electrical part, so, let us first discuss the electrical part in the electrical part as we have seen that we have a you know resistance or and along with that in a series we have an inductant scale and we have a voltage source here from the motor okay and there is a direction of the current which is flowing through it right.

So that is let us say it is high so that means the drop is here are i there is the first part + the second part is actually l di dt and that equals to the applied voltage v- of course the back EMF and that is heavy times the velocity of the proof mass actuator because as it will be moving up and down proportionately this back EMF will be generated. So, heavy extra the faster it will be moving the more will be the back EMF.

And the slow of the lays extra so, that is what is my first equation and the second equation if i look at the that is simply once again, you have to draw the free body diagram because these, this is electrical equation okay from the electrical part of the system and this is from the mechanical part of the system. So today that is becomes a electro mechanical system, So for that mechanical part of the system.

We have the Mp as the proof mass with us and it is moving down which means an inertia force of Mp X double dot is working on it also this spring resistance Kp X corresponding to the displacement here and the damper if you have that is Cp x dot and all this thing is happening against working force which is working on the system and that is Ka that is a armatia constant and that is proportional to the current that is passed into the system.

So if I pass more current I get more Ki and i get this power training but the my Kb also once it transferring if, my Kb also will increase. So this is the mechanical part if I consider now this equilibrium mechanical part will be clear to you. What you can for that who is that this are this are two odes right this is the first one is a first order and second one is a second order ode. So you can convert both of them in tra the frequency domain.

So, then you get actually I as one of the you know frequency domain variable capital I, capital V bar and capital X. So, I get these 3 frequency domain variables and I get a new set of equation the first equation is Ri + LsI which equals to v bar – KbsX that is opta carrying in the laplace transform, so, from this you can get a relationship because you know that R+Sl times i equals to v bar – s kex, so, you can find out what is I, which is one over R+Ls times – v bar –skbx or kbsx.

Similarly, for equation 2 also you will get, now in both the cases I have considered of course 0 initial condition, now in the second case what you are going to get is s square mpX that is for the first term, then kpX from the second term, and then scpX third term equals to KaI. So, here also I can take all of this things and I can take the X out, so, I get this particular this particular equation, and combining this 3 and 4, because you already have this relationship.

Of what is I, you can get a relationship between v bar that is the applied voltage in frequency domain, and X. Now let us see the what is the relationship will get.

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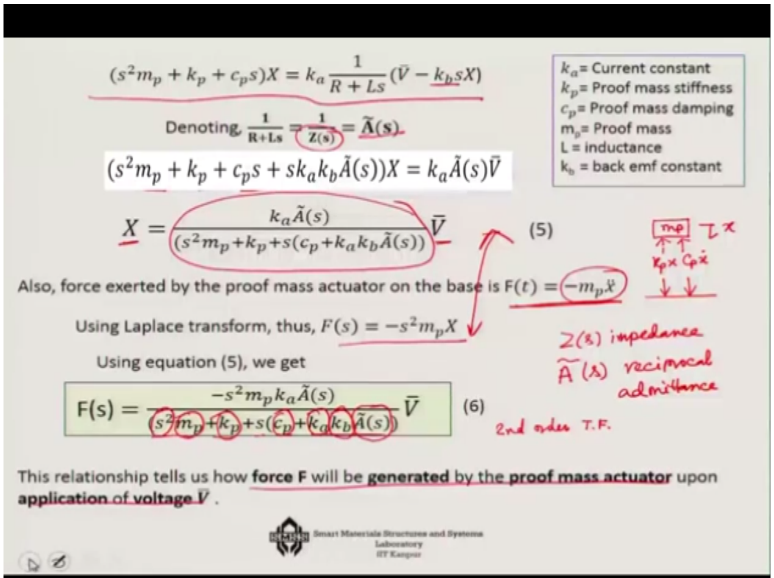

So, if you look at it, that if I combine the two equations and going to get this particular relationship and a little bit advanced if, I put one over R + Ls, now the R+Ls this LS in R+Ls can be written as the impedance. So Zs is actually Zs is actually the impedance of the system and A tilda is actually reciprocal of the impedance. So this is the reciprocal and it is also known as admittance.

So, if I denote 1 over R+Ls as 1 by Zs and then that equals to A tilled s which is the admittance then I get a bit of simpler relationship where I can cloak all the X terms together that means I get S square Mp + Kp + CpS + SKaKbA tilled S okay. So that is from these part of it. This whole thing times X equals to right hand side A tiled s times V bar. In other words my relationship between the output and the input is actually governed by this particular part that is Ka A tilled s v bar in the denominator S square Mp + Kp + SCp + Ka Kb A tiled s.

So that is what is my representation and hence I can write out also another thing. If you want to express this whole thing, in terms of the force then you can check that the force that is transmitted to the base is –mpx double dot. How can we see that that is what is happening well because if you consider that, this is what is your mp and only consider the mechanical of part of it, so, mpx, so, you can see that here because it is in dynamic equilibrium, so, it has this two balancing forces one is kpx and another is cpx dot. So, on the support also it is the same thing that is going to what is the system.

That is the combination of kpx + cpx dot just in the reverse direction, so, that means if you look at the earlier governing equation that we have if we just away look at it.

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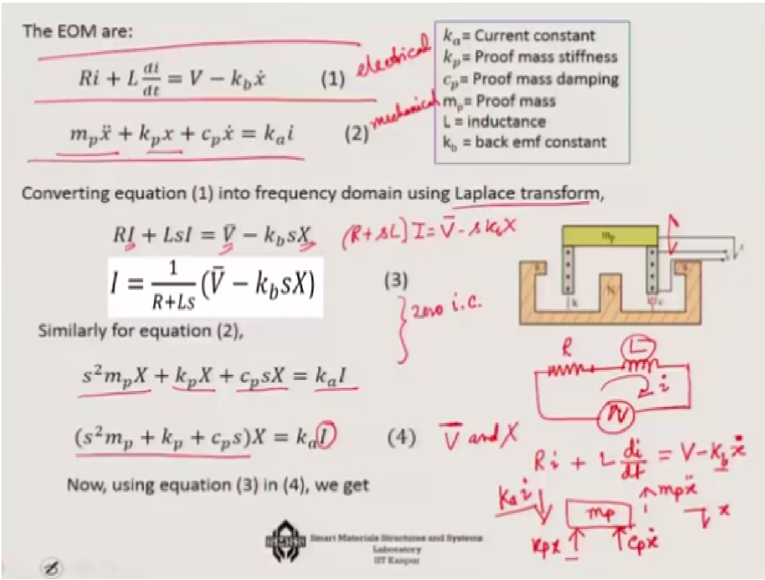

You will see that we had these relationships here, where we have this mpx double dot + kpx +cpx dot equals to kI and since we are considering this effect itself, so, this will be equivalent to.

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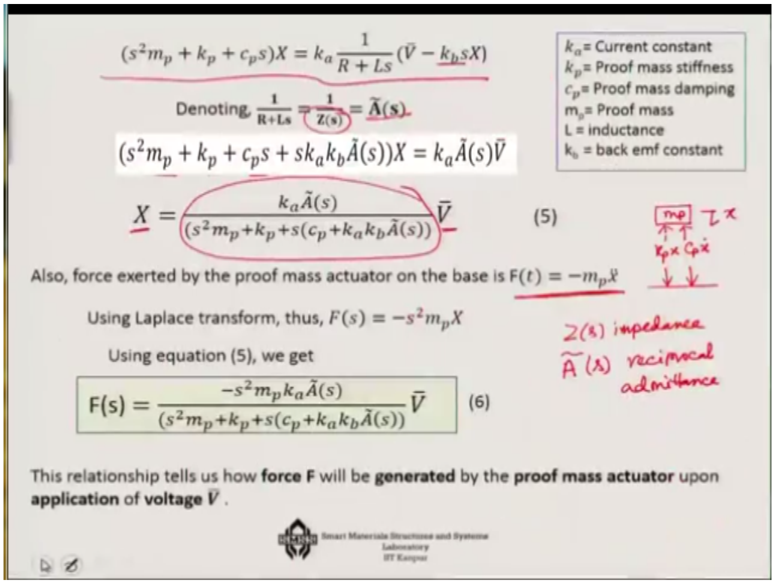

As you can see in this case, that this kpx + cpx dot the total of them is equivalent to what is the called the mpx double dot part of the system. So, the force that has to generated, has to be the negative of it, because it is in the opposite direction. So, then Ft is –mpx double dot, now if you use laplace transformation then it will become –s square mpX and now you can combine this 2 equations together.

And if , I do that then I am going to get these relationship between Fs and V bar and where we can see that we have in the numerator –s square mp k8 tilled s and the denominator we have s square mp + kp + s times cp + ka kbA tilled s. Now, we have to keep in our mind that the first up all the order of this transfer function is that this is a second order transfer function. When we will discuss close loop system will talk more about it.

But, actually the order of the polynomial of the denominator is talking is telling us about what is the order of the transfer function and because of the presence of the S square. We can say that S it is as second order transfer function secondary the response of this second order transfer function can be actually controlled by controlling the mp the put mass itself, the kp that is the you now the spring stiffness.

The cp the damping of the spring the armature constant ka and the back here may kb generally k and kb are same. And finally the impedance of the system which is one over zs, so, while varying all this parameters one can actually the response of the second order transfer function. Now, we will see the that ok, if, this is the force that F generated by the proof mass actuator with an application voltage v bar, can we put it back in to a system where we need to control the vibration, similar to what I have discussed in the very beginning.

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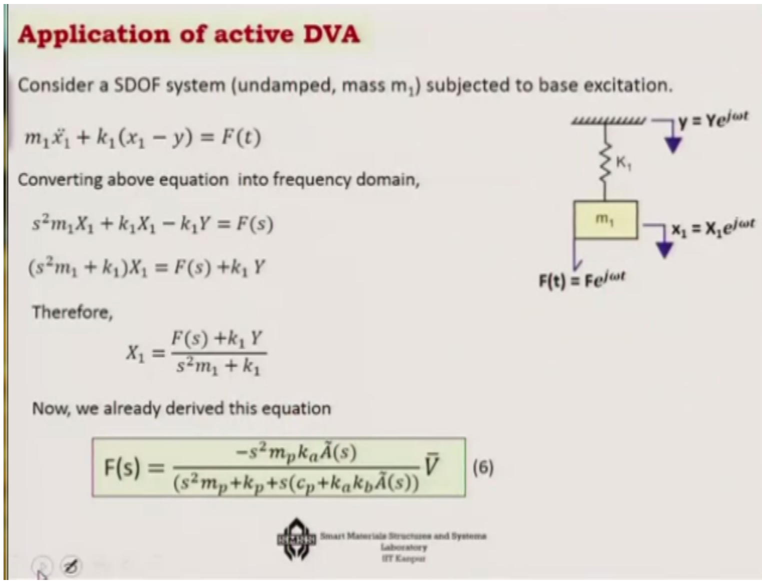

Ok, so, now we will be now talking about how we can apply this proof mass actuator in assistant which is undamped or under damped. Now let us say we disregard the damping part of that system, and let us say that the mass of the system for whose vibration I want to control using proof mass actuator is M1 stiffness is K1 and it is subjected to a dynamic force which is coming the source is the proof mass actuator.

So, that is what we have to keep in our mind because we have to plug in this proof mass actuator equation into the whole system. First thing is the equation of motion once again that is pretty simple for us. Because we have the mass M1 here and this mass is of course subjected to what you call the base excitation. So, corresponding to its displacement X1 here it is getting an inertia force which is M1 X1 double dot and so, M1 X1 double dot.

And it is also having a spring force which is K1 and X1-Y. Okay so because it is subjected to a base excitation that base excitation is the vibration way that I told you is coming because of the shock that the structure has been subjected to. Now, so, this are the two forces and now if suppose if I didn't have the proof mass actuator. Then it will happen M1 X1 double dot and +K1 X1-Y equals to zero, that would happen in the case of a base excitation system.

And you can see that this since there is no damping presenting in the system. So you would have seen with respect to the frequency domain with respect to the transfer function. You might have seen that it is going to have an unbounded resonance of the system, but, now if you introduced this particular force in function part. So that means I have an additional force in function part which is coming due to this proof mass actuator and that is Ft.

So, there my equation is non homogeneous and I am going to get this equation with us. Now this equation let us quickly convert it using in the frequency domain using laplace transformation So I am going to get it as S square M1 X1 that is from the X1 double dot and then +K1X1-K1Y equals to Fs. I can take all these X1 related terms in one sides, so, S square M1+K1X1 equals to Fs+K1Y. In other words X1 equals to Fs+K1Y by S square M1+K1.

So, this is what we are going to see in terms of a relationship between X1 and the Y as the, you know the base excitation input. What we have to Fs with us already we know the relationship of Fs which we have already seen that there is a second order transfer function between Fs and that applied V bar. So let us plug in this relationship here, so let us put this part Fs into this relationship.

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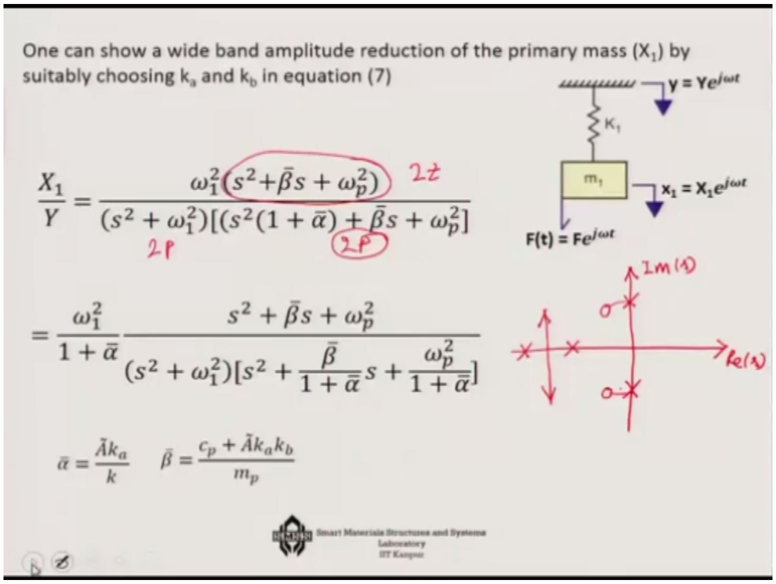

So, what we are going to get now is X1 as a function of k1Y that is the input and the voltage that have applying to the system. So I have a mechanical excitation this is the mechanical excitation part, it is just a post with two things right and this is the electrical excitation part. And, I am trying to surprise the mechanical excitation with the application of the electrical excitation. Now when A tilled s let us say the a very special case where you know the admittance is constant.

Then let us denote omega 1 square as k1 by omega 1 and omega p square as Kp by Mp and applied voltage V bar is K X1 is proportional, suppose I put a control law this is what is my control law that my control law is that to apply a voltage which is proportional to the amplitude of the displacement here. So, I do that then am going to get an relationship where the V1 is called now and you have relationship between X1Y and the you know you have this a voltage which is proportional to X1 itself.

Now let us for the in this particular relationship of course where is the control part, you have to keep that in your mind okay so, alpha bar here is having a bar ka over k, which means that is comparing the control bar, because, you have the control law there, which is embedded inside the system, we also have another parameter beta bar, which is cp+a tilled kakb over mp, now since you have already choosing cp, ka, kb, mp, and you have consider the a tilled s to be constant.

So, you are not going to get any change here, but this alpha bar you can actually tuned, so, now you can rewrite the relationship as if you take both X1 terms in once, then it will become 1+ alpha bar x square bar s square + beta bar s + omega p square, times X1, and that equals to omega 1 square over s square + omega 1 square Y. So, that is the very special case kind of a thing, where the relationship between X1 and Y is of this nature.

Now in this particular case you have to see, that in the absence of these alpha bar part, so, suppose the game case 0, so, then this alpha bar power do not part in there. So, then what would have happen, your X 1 then would have been simply omega thing the case when alpha bar 0 omega 1 square over S square+omega 1 square Y. Now that is the classical case of an undamped system.

In fact this system has two roots right if I find out the two roots S1 and S2 then this resistant will be having your roots as square root of S1S2 as square root of omega 1 square which will give you two solutions that is we will be having S1as either omega 1 or you can actually plot them in the you know – omega 1 also as another solution. So you can plot them in the imaginary axis itself.

Both + and – as the two solution of it. So because the imaginary so you are getting + - so you can say that + - j omega 1. Both of them will give you the solution because it is S square equals to – omega 1square right. So you are going to get a + - j omega 1 and as it an in the root locus if you plot it that means that root locus of real over S and imaginary of the root S.

Then this is going to give me two of the poles here this numerator is going to give as the two poles and both the poles are on the imaginary axis and that is the till tell sign of an undamped system. So that is what it would happen if my alpha bar would not happen there. If, I introduced the alpha bar what may happen to the system. So, in that case we can actually write this whole equation together.

And we are going to see that we have a denominator in which we have a numerator in which we have two zeroes and we have a denominator in which we have two poles here which have undamped and we have two poles here, and we have two zeroes here. So, essentially this will be a case of if I just try to plot it here once again. Because of the introduction of this alpha bar term, then we are going to see that we have two under damped undamped poles here.

And, along with that we have two additional poles there and this 2 additional poles you can + anywhere depending on wherever you wish to do it, so, essentially we can place this poles on the real axis or somewhere as complex conjugate you can put it and you have two zeros, in the system, so, these two zeros are once again complex conjugate and you can put them anywhere in the system.

So, if they are in the left up line, then of course it is fine but, they go to the right up line then you will have problem, so, you will see that in certain cases it can happen so, that they can go to the right up line or and in other cases when they are in the left up line, then it get attracted towards, these poles may get attracted towards, these zeroes and these poles may actually diverge, so, you can get various solutions with respective the change of the games.

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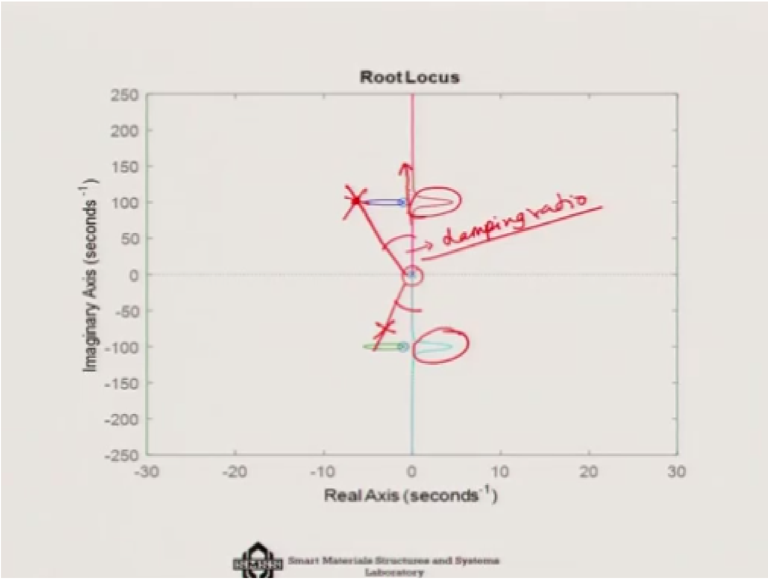

And, we can see one typical solution corresponding to a unit scale system, where you can see here that you have these are the two poles here as you can see and also as the third pair is somewhere here which is also very close to 0, and corresponding to a positive game you can see that a part of it, can actually go to the right up line. So, a proof mass actuator if it is not control properly, then, it is possible that it can add actually energy to the system.

And, it can generate instability, otherwise if these thus not happen then it can happen that will follow approximately this course and provided I put up proper poles in replacement and which I will talk about it when I talk about root locus, and you get these two systems as the two damp systems, with whatever damping that you want to apply to the system. So, this is the damping ratio.

So, by designing the alpha you can make the system very stable as well as very well done suppose this are the positions of the close group poles that could be some of your goals, and the two undamped modes just do not you know make sure that it do not come to the righter plane to create an instability. So, this various possibilities will come up with the proof mass actuator, and that is a what is a good control design problem, and will talk about such a problem in our discussions on active control.

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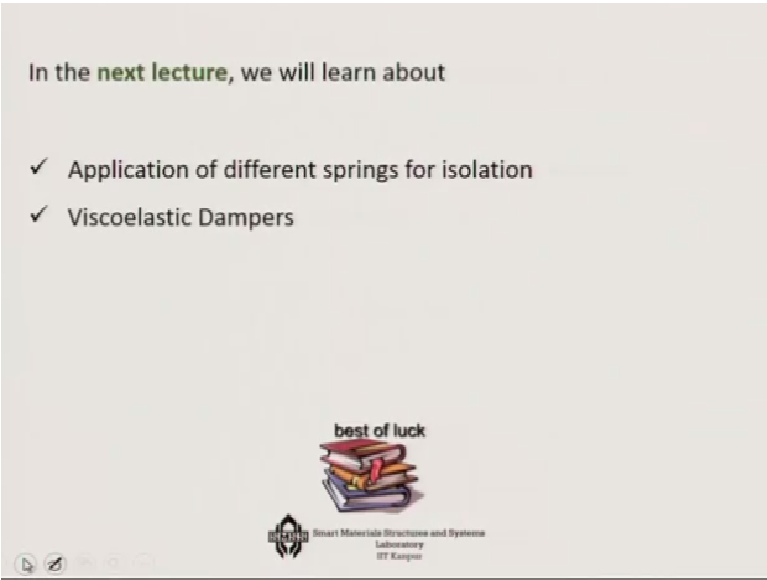

So, this is where we are going to finish today, and we will also talk about in the next lecture on different springs for isolation and, viscoelastic dampers. Thank you.