Basics of Finite Element Analysis – Part II

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Lecture – 07
Errors in FEA Solution

Hello. Welcome to Basics of Finite Element Analysis Part II. This is the second week of this course and in the last week we had reviewed good number of topics which are covered in basics of FEA part 1, which was offered the in the first part of this year that is between January and March of 2016. So, what are going to do today is; we are going to look at errors, because every finite element solution is an in exact solution, the exact solution. So, there is difference between exact solution and then the in exact solution and the difference is error.

So, what we are going to do is, we are going to look at different types of errors and then we are also going to develop some matrix or measures as to how to quantify that error. So, that is another thing we will be looking at and the third thing we will be looking at there is conversions that as you keep on making you are finite element modal better and better; you are solution should approach the theoretical solution in a more close way, you should become closer to the to the exact solution.

So, that is another aspect we are going to look at that conversion. So, that is the scope of this week's lectures and specifically today we will be looking at errors what are the sources of errors and if we have time probably will also cover how we can measure errors in an objective way.

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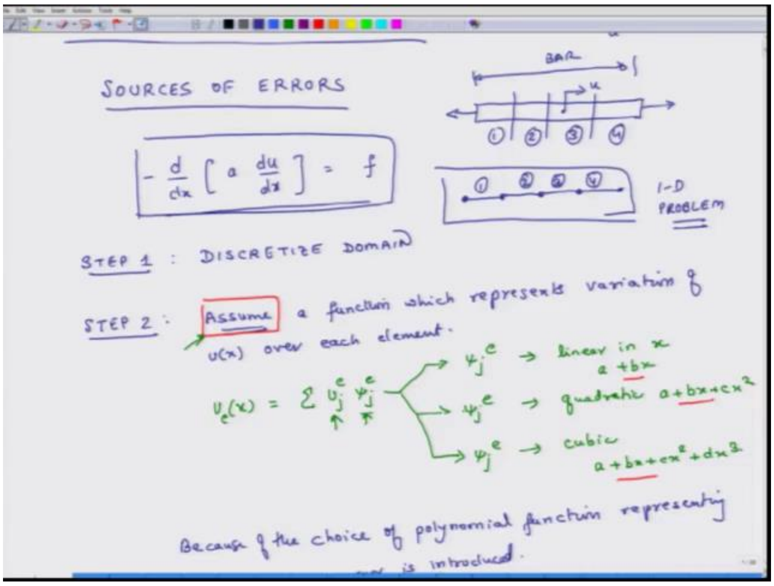

So, this is going to be our theme for today, now first we will look at different sources of errors, so suppose we have differential equations, second order differential equation and let us say this differential equation d by dx, a du over dx is equal to f.

So, that is why differential equation and what this deferential equation represents one of the things we should can represent is, that if you have a bar and you have pulling the bar like this and if I am interested in finding out the displacement of this point u, then and a here is equal to Young's modulus of the material and gross section of the material at a particular cross section.

So, this is the governing second order deferential equation and if you are able to solve this equation; we can get the solution for this bar problem and the first step we said was that, if we have to finite element analysis, we have to discretize this bar into several elements. So, let us say, this is element 1, this is element 2, this is element 3, and this is element 4, so in a simple way I can redraw this. So, this is element 1 2 3 and 4. So, these are the 4 elements.

So, this is step 1 is discretized the domain, so in this case we have discretized the entire bar, this is my bar into 4 different elements, element number 1, element number 2, element number 3, element number4, then the second step which we discussed in the last view was once we have discretized the domain, then the second step is that we assume a function which represents variation of u x over each element. So, this is where we can assume something; we have to assume a function, this is important thing, we have to assume a function.

So, one assumption could be that for the eth element u of x, for eth element could be sum of u j psi j and this is eth element. So, I am I mean superscript e, actually in past we have been doing like this. So, here this u j is unknowns, and this is the known function. Now I have can make several types of choices for psi j, psi j can be linear function, it can be linear in x. If it is linear in x, then it will have a form of something like a plus bx form, it can be psi j can also be quadratic, it can be cubic.

So, if it is quadratic it will be something like a plus bx plus cx square if it is cubic then thus function will be a plus bx plus cx square plus dx cube. So this which type of size away chooses depends on our assumption and based on our assumption, it is this nature of this assumption it introduces an error into the solution.

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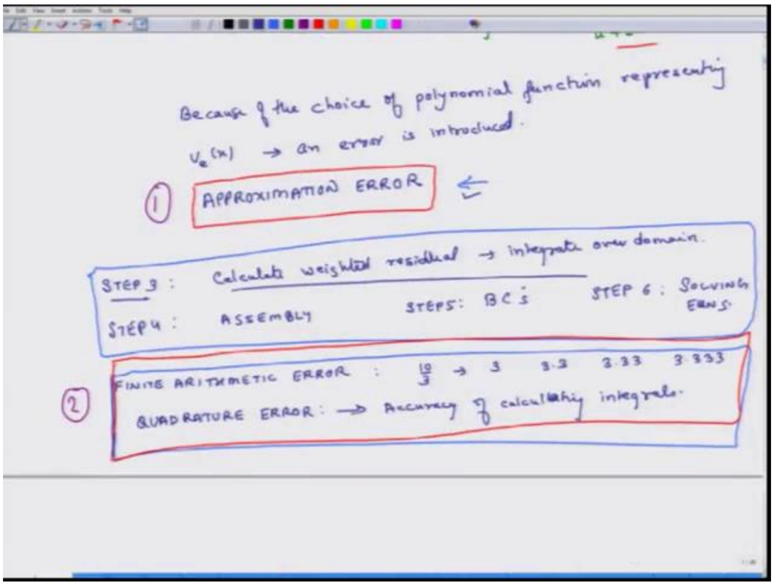

This is the first type of error gets introduced in to the solution. So, because of this assumption because of the choice of polynomial function representing; u on eth element and error is introduced and this type of error is called Approximation Error.

So, this is the first type of error which is their in our finite element solution, and the amount of this error is directly dependent on, what type of function we choose as the approximation or the extrapolation function. So, that is why because it depends on the type of an approximation function that is why it is called Approximation Error. It is important to understand this, if we go to higher order polynomial this approximation error becomes less; if we use lower order polynomial the approximation error is more can so on so forth.

Then in step 3, what do we do we calculate weighted residual, we take the error on the equation residual in the equation multiplied by weight function an integrated over the domain. So, we get element level equations and then step 4 is assembly, step 5 is what application of boundary conditions, and step 6 is solving the equations. So, in all these steps are step 4, 5 and 6 we are not making any assumptions here, we made an assumption in step 2, but here we are not making any assumptions. So, there is no approximation error which goes in to step 4, step 5, step 6 and. so on so forth, but different type of error is introduced in these step, so in step 3, step 4, step 5, step 6 all we are doing is we are doing lot of calculations.

So, the error how accurate are calculation are; how many number of decimal places we are calculating are results up to that is the related to the amount of error. So, in these steps, we introduced finite arithmetic error, this is one thing; finite arithmetical for instance, if we are dividing 10 by 3 in computer, it could be 3 or it could be 3 .3or it could by 3.33 or it could be 3.333 and so on so forth.

So, depending on how accurate we compute we do is computations are results will be proportionally accurate. Another source of error is when we do the integration, when we do the integration; this is something we will learned maybe next week, then we integrated using a method called Gaussian Quadrature method. So, depending on which type how many quadrature point repack are integrals because in step three we have to calculate integrals of weighted residues over the domain. So, how accurate we are in computing the integrals.

So, that error is quadrature error, and this depends on accuracy of calculating integrals, but both of these errors finite arithmetic error and quadrature error they are related to the numerical procedure, which we are going to fallow. So, this is the second source of error.

So, first source of error was approximation error, the second source of error is this thing computational error, which can be of 2 types, finite arithmetic error and quadrature error. So, this is the second source of error.

So, principally these are the 2 broad types of errors which are there in picture, when we handle one dimensional problem such as this. So, this is the 1 D problem. So, in one dimensional problems does not matter whether it is a second order equation or 4th order equation, in one dimensional problems we have primarily 2 types of errors, first one is approximation error and the other one is computational error, which can be of 2 types finite arithmetic and quadrature, but in 2 dimension and 3 dimensional problem we have a third type of error which comes into picture.

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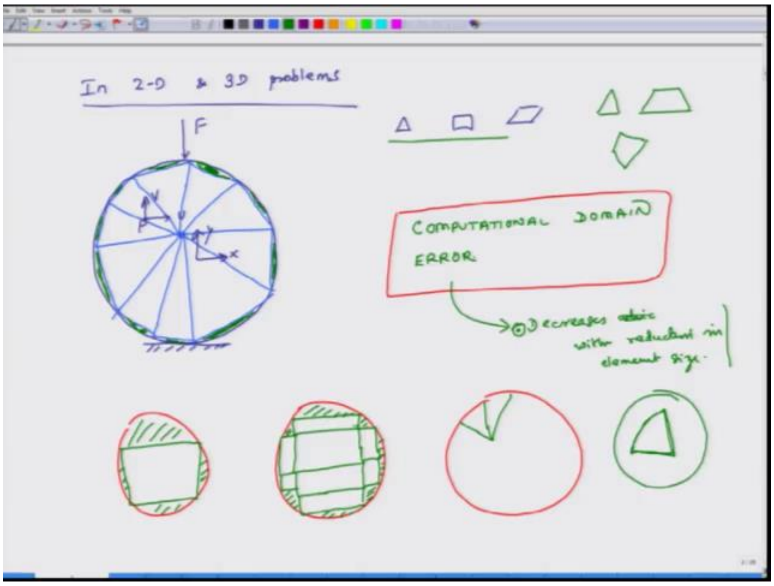

So, let us see this. So, in 2 dimensional and 3 dimensional problems we still have approximation error and we still have computational errors like finite arithmetic and quadrature errors. These 2 errors are there in 2 D and 3D problems also, but we have an additional error in 2 D and 3D problems.

So, in 2 D and 3D problems, we have an additional source of error and what is that source. So, will illustrate by example, so suppose I have a circle disk, let us say it is a disk an on this disk; I am fixing one end and I am putting a force on this end. So, this is the metallic disk and putting force on one end and at the other end I am fixing it and an I am interested in finding u and v u is the displacement of next direction and v is the. So, this is coordinate system x and y. So, add sum arbitrary point d, I am interested in finding the displacement in x direction, which as u and displacement in y direction which is v.

So, I can solve this using analytical approaches or by finite element method, but when I do finite element analysis, what is the first step, the1st step is that we discretize the domain does not change, whether it is 1 D or 2 D or 3D problem, the approach remain same, but in 1d problem there is no error introduced when we are discretizing the domain, when we discretize the domain in 2 D problem what do we do we essentially break this circle in to; either triangles or rectangles or some quadrilaterals something like that.

If I breaking it up into triangles. So, this is very course mash, then these are the boundaries of triangles, and what we have is this extra stuff; this green area. So, the green area is not covered by these triangles, these remains outside the triangles. So, in our finite element modal; we are not able to capture the geometry accurately, if I am using triangular elements.

So, this is third sourcing of error, which was not present, 1 D problem. In 1 D problem we have a straight line and that line is broken into smaller line. So, there is no problem of geometric approximation. So, here this is the third type of error and this is called "computational domain error", this is third type of problem, in the computational domain error; what it means is that, I am able to capture the domain computationally in an approximate way not in a not in an exact way. So, this is third source of error and this error how can we reduce it, if I reduce the element size right if I reduce the (Refer Time: 18:29) element size, suppose these are 3 things, suppose I have to represent this rectangle at the circle by rectangular elements.

So, here element size is 1, now another way I can do it is I can have smaller elements, so, this is one approximation So, here in the first case, all this was not included in the domain; here the domain error, computational domain error has become smaller this is the hazed area which is not included. And if I reduce the element size further; this domain are computational domain error were go down further. So, the computational domain error it decreases, with reduction in element size. The other way you can reduce this error is that, you do not only restrict; you expand the choice of element right.

Now, we have just talk that is elements are having straight adjust, so it could be a traposide or a quadrilateral things like that, but if am smart maybe I can also, device elements we are one edge is curved; So, then I have curved edges, then I can very easily reduce this error rapidly. So, if I have a 3 edge element, we are 2 edges are straight and the third edges curved; then I can fit this type of element like this, and it will be much more accurate. So, I can reduce computational domain error in 2 ways, one is by reducing the element size and second is by having elements which have curved edges, this completes the description of different types of errors which are there in our in finite element modal.

So, overall what we have seen is, that an one dimensional problems; we have 2 types of error which come into picture, the first error is approximation error, and the second error is computational error related to computational procedures mathematical error and those are could be related to quadrature error or finite element error, but in 2 dimensional and 3 dimensional problems. We have second error which comes into picture.

Third error which comes into picture, which is computational domain error and the nature of this error, lies in the fact that, straight edge elements cannot accurately capture curved geometry. So, either to address this error or reduce this error either I reduce the size of the elements or I introduced some fancier element were; at least one edge of the elements is curved in nature.

So, this is the overview of different type of errors which are there in FEA. And we will continue the discussion in the next class and specifically what we will cover in the next classes that how can we measure different types of errors. So, that is study much for today.

Thank you very much. Have a great day, bye.