Basics of Finite Element Analysis – Part II

Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Assembling element level equations

Hello, again welcome to basics of finite element analysis part II. Today we are going to continue our discussion which we having in the last class and last class we have developed element level equations for this bar problem which is shown here. This was the governing differential equation and for this equation, for the element level we had arrived at these equations.

So, now what will do in this class is that we will assemble these equations for the entire assembly .When we do this assembly we will enforce some continuity conditions and equilibrium conditions for the assembly process .Once we have done the assembly level equation then we will apply boundary conditions and finally solve the problem.

Start doing the actual assembly, i wanted to discuss the methodology for enforcing for doing this assembly level operation.

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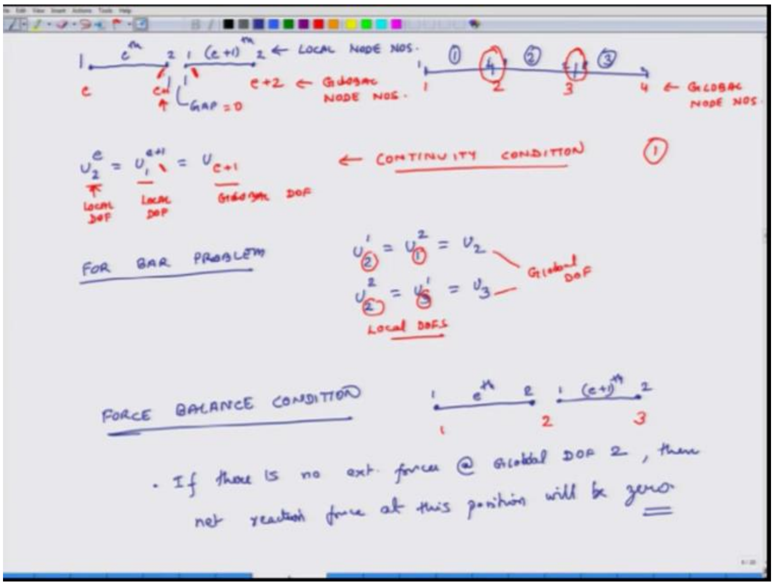

To understand this we have to understand, what happens when two elements meet. So, suppose this is eth element and this is e plus 1 th element .The first element the node number of eth element is 1 and 2. So, the first node is 1 and second node is 2. Likewise the first node does of eth e plus 1th element is node number 1 and this is node number 2. In this picture what i have shown is that there is a gap here, but in reality the gap is 0. The gap is 0 between these 2 elements because these 2 elements are physically touching each other. So, for instance if I have bar if I break it into 3 elements this element 1 this is element 2 this is element 3.

So, if I look at the node numbers for first element this is 1 and 2. For second element this is 1 and 2, this is for third element this is 1 and 2. The gap between this node and this node is 0. I can also assign global node numbers. These blues these are local node numbers. I can also assign global node numbers. So, what will the global node number? It will be 1, 2, 3 and 4. So, these are global node numbers. So, likewise for eth element my global node number for the first node will be e then for this one it will be e plus 1 and for this one it will be e plus 2. So, these are global node numbers.

Now, is the gap is 0 and the two nodes are fused locally .Then what does that mean? That the displacement U so, the displacement for the eth element at second node is same as displacement of e plus 1th element at the first node right? This means this is equal to U e plus 1. So, these are Local degrees of freedom and this is Global DOF.

So, once again the displacement of this node and the displacement of this node, the displacement of this node is signified by u 2 e. The displacement of this node is signified by u1e. Both of them are equal and the value is same as the displacement of this global degree of freedom which is e plus 1. So, this equality is known as continuity condition. That whenever there are 2 nodes which are merged if they are not merged if there is crack then they will not be same. But if they are fused together there is no crack, if there no breakage, then they will have the same deflection or displacement or same temperature.

So, this is the continuity condition. So, in context of our bar problem, for the bar problem which we are doing, we have 3 elements. For 3 elements we will have one continuity condition at this node and another continuity condition at this node. What are those continuity conditions? So, first one is U so, this is the first element 2 is equal to U 1 2 is equal to U2 .The second continuity condition is U 2 2 is equal to U 3 1 is equal to U3 .Once again this is global and these are local DOFs.

So, this is the first thing when we are doing assembly we have to make sure that when we assemble equations, these equalities are maintained. The second condition is more precise would be force balance condition. So, what does it mean? Again suppose I have 2 elements this is e th element this is global degree freedom 1 no local 1, local 2 this is e plus 1th element, local 1, local 2 and global degrees of freedom are 1, 2 and 3. Then what does it mean? It means that if there is no external force at global DOF 2 at global node 2, then net reaction force at this position will be 0. What does it mean?

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Suppose I have a bar and this is 1 2 this is e this is e plus 1 and this 1 and 2. The global degrees of freedom are 1, 2 and 3. When I break it at the element level when I break it what will happen? This is e plus 1th element; this is node 1 node 2 .Suppose I am pulling it like what will happen? They will be a reaction force which will when I cut it then I will see a force here.

Similarly I will just create a small gap for I will see a gap here force here. And on this element, it will be acting in the other direction. When I add these up, they should balance and they should vanish right? When I am doing this an assembly the internal forces should add up to 0. But this will happen only if there is no external force at point 2. Suppose at external point 2 I am putting sum at node 2 add this node global node 2 I am putting some external force F. Suppose I am applying some external force F then the sum of these two will be F. Because there is a no external force these twos will add up as 0.

So, this is important to understand. So, these are the two things we have to consider when we do the assembly process.

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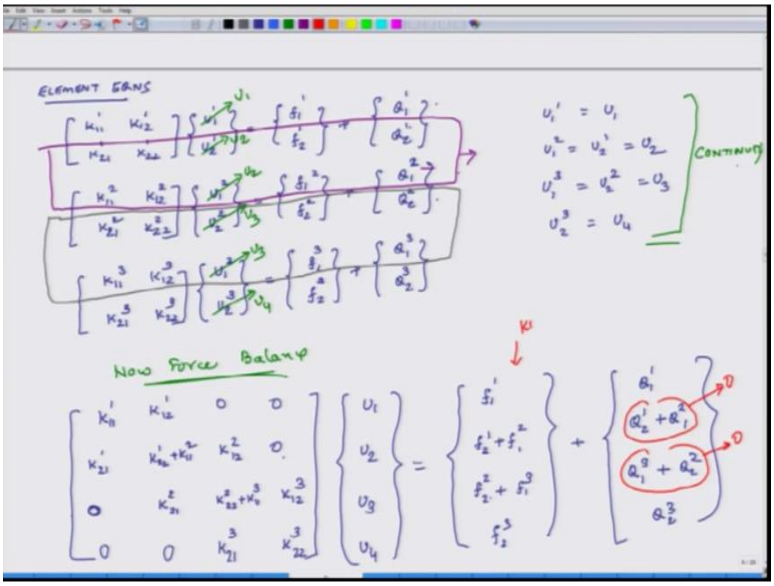

So, with this we will develop the assembly equations. First we will write the 3 element equations. It is K 1 1, K 1 2, K 2 1, K 2 2, U 1, U2 is equal to f1, f 2, plus Q 1, Q 2. This is for the first element. So, the superscript is 1 everywhere. Then I write the equation for the second element because our domain has been broken into 3 element right? So, we will have another set of equations for the second element. So, again it will be K 1 1, K 1 2, K 2 1, K 2 2.

But here the superscript is going to be 2, indicating that these equations correspond to the second element and then we have a third set of equations for the third element.

So, these are element level equations and we have to compress them into assembly level equations right? We have to assemble them together. So, when we are doing assembly level equations, the first thing we have to make sure is that these we have to transform these local degrees of freedom into global. So, we know that U 1 1 is equal to U1. U 1 2 is equal to U 2 1 is equal to U 2. U 3 1 is equal to U 2 2 is equal to U 3 right? And U 3 2 is equal to U 4. So, the first thing we do is we replace them U 1, U 2, U 2, U 3, U 3 and U 4. So, once we have done this we have enforced continuity.

The next step is that we have to do the force balance. So, we have enforce continuity now force balance. So, for force balance here we have to add these forces at the interface together right? This means I have to add these two equations and I have to add these two equations together. Why because when I add these two equations together Q 1 2 gets added with Q 2 1. If there is no external force then their sum will be, when i add these two equations this is the internal force. So, this is the internal force representing this thing and this is internal force representing this thing and if there is no external force at this point then, these two will add up to be 0. So, that is what we do. So, ultimately how many equations will end of with? See right now we have 1, 2, 3,4,5,6 equations. How many degrees of freedom we have right now? 1, 2, 3 and 4 we have 4 degrees of freedom. So, we have to reduce 2 equations these two are been reduce d by adding these two equations together.

So, when i do the assembly my assembly it looks something like this. So, this is the first equation K 1 1 times U 1, plus K 1 2 times U 2 and then in the first equation there is no U 3 and U 4. So, the coefficients on the K matrix are 0. Now let us look at the second equation. We have to add second and third equation together. When we add them this is what we get. So, this is the second equation K 2 1 for the first element times U 1 plus k 2 2, 1 plus K 1 1 2 times U 2 plus K 1 2 2 times U 3 and there is no U 4 in these 2 equations right? So, it is 0 here this is the second equation. Then the third equation is 0. In the third equation there is no U 1. So, the K matrix element is 0 there is no U 1 so, this element is 0. Then here we get K, K 2 1 2 and this is K 2 2 2 plus K 1 1 3 and this is K 1 2 3 right? So, this is third equation.

The forth equation is 0, 0, K 2 1 3, K 2 2 3. This is the left side of the equations. The right side is f 1 1, f 2 1 plus f 1 2 and then I have f 2 2 plus f 1 3 and then I have f 2 3. So, this is the matrix related to external forces. Then I have Q 1 1 and then Q 2 1 plus Q 2 1 and then this is Q 1 3 plus Q 2 2 and this is Q 2 3 right? And now I enforce the force balance thing. So, this is the sum of these two is 0 and the sum of these two is also 0. So, these two things are 0. They will not be 0 if there is an external force at that point which is being applied but if there is no external force then these two will be 0.

So, now this vector is known. Why? Because, we know q and h is also known right? So, this vector is known. All the elements in this matrix

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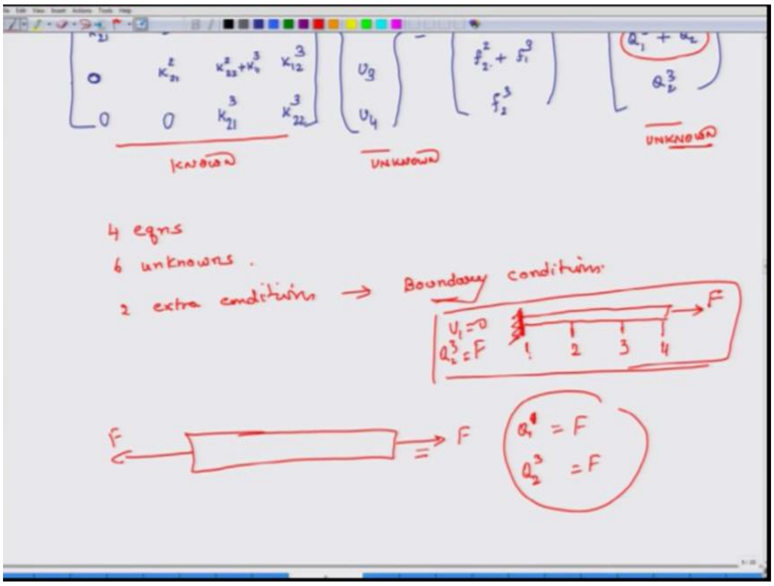

We have calculated. So, this is also known, this is not known. So, this is unknown .This is also unknown at this stage right? So, U 1, U 2,U3 and U 4 they are unknown and also this Q 1 1 and Q 2 3 are unknown . Everything is known, but we have 4 equations and 6 unknowns. So, to solve these equations we need 2 extra conditions. We need 2 extra conditions. These come from boundary conditions.

So, you can have a bar. Suppose this bar is like this and I am pulling it like this. So, this is node 1, this is node 2, node 3 and node 4. Then what are in this in case what are the two extra conditions that had node 1, U 1 is 0. And at node 4 is U 4 known .The first condition is that at node 1, U 1 is 0 .The second condition is that at node 4, we do not know U 4, but we know force. So, Q 2 3 is equal to F. This is one, but there could be other boundary. So, once we have found these two applied these two conditions then we will have 4 knowns and four unknowns and four equations.

Another situation could be you have a bar, and I am applying force here. In this case what is the case? U 1, U 2, U 3, U 4 is unknown, but force at node 1 and force at node 2 are known. So, Q 1 1 is equal to F. Q 2 3 is equal to F right? So, likewise we have to get 2 extra conditions .Those we get from boundary condition. Once we get those then I have an equal number of unknowns and equations and I can solve those and then I get the solution for the overall system. So, this is the very brief over view of whatever we did in the last course and going forward we will start looking at different dimensions of this course with newer topics. Thank you very much looks forward to seeing you in the next week bye.