Basic of Finite Element Analysis - Part II
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Lecture - 47
Plane Elasticity Problems: Element level equations

Hello, welcome to Basics of Finite Element Analysis Part II. This is the last week of this course and today we will continue our discussion for plane elasticity problem. What we did in the last problem was that we developed in the weak form for two equations of motion.

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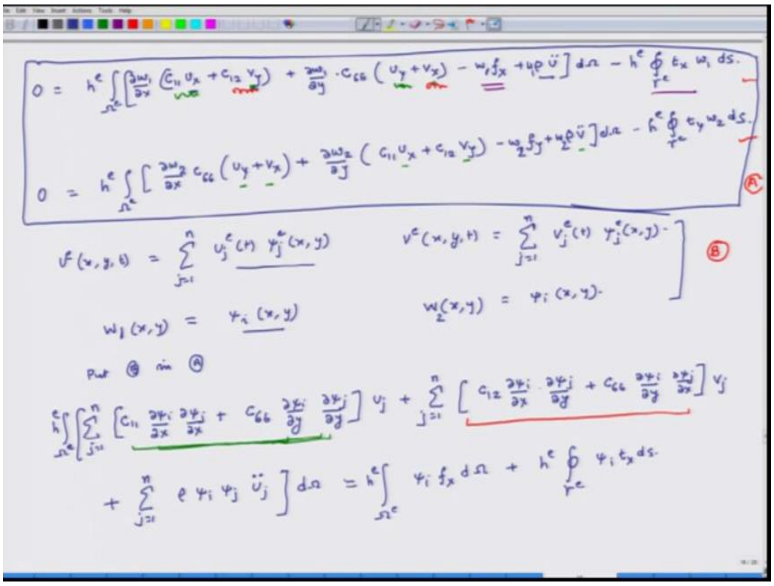

Associated with the problem and these are the two equations which we have developed. The next step in this sequence is that we have to approximate these Us and Vs right. So, we have to. So, approximate them in terms of approximation functions.

So, now we know that U for any element this is the dynamic problem. So, it depends on x it depends on y and it also depends on time. So, the way we do this approximation is let we say that the approximation in the space views are approximation functions and the values of use at nodes they themselves are functions of time. So, in this way we separate the variables. The nodal values are functions on time and the special variation does not depend on time, this is what we do and this is exactly the same thought process which we adopted for 1 dimensional dynamic problem, which we did in the earlier course.

So, the approximation for this is J equals 1 to n U J and U J is the function of time, but it does not vary, in a space it is specific to a node and then Psi J x y. Similarly, V for the eth element which depends on x y and t equals summation over 1 to n V J which is a nodal degree of freedom for the e-th element times Psi e which is the function of x and y. So, we can use these relations in all these terms U x V y U y V x U double dot all these terms. The other thing is we have to make a choice for weight functions. So, I say that W 1 is a function of x and y and it is same as Psi i x and y and the second weight function is also Psi i x and y.

So, once we do this. So, let us call these equations A and let us call this B. This is W 2. So, we put B in A and what we get is so we will do this for the first equation. So, the first equation looks like this. So, I am integrating over the element h e, that we have plugged. We are going to plug equation B into A and we are also clubbing equation with same coefficients. So, for U the coefficient is U J and for V the coefficient is V J. So, we are clubbing this term, this term and this term together. Similarly we will club V y term and V x term together. So, that is how you will be able to club them together. So, C 1 1 U x corresponds to C 1 1 U x is Del Psi i over Del Psi j over Del x and that is multiplied by W 1 which is Del Psi i over Del x plus the other term is coming from C 6 6. So, C 6 6 and this is multiplied by Del W 1 over Del y. So, I get Del Psi i over Del y and I am getting U y. So, I get Del Psi j over Del y and this entire things, is multiplied by U J and U J is the function of time, U J is the I am not putting it here, but it is a function of time.

So, this takes of care of green terms. So, this term the second derivative I will keep it separate, I will keep this term separate you will see it why then. So, I have collected all the terms which are underlined in green next I will collect all the terms under lined in red. So, I get plus summation J is equal to 1 to n. So, first is Del W 1 Del times C 1 2 times V y. So, Del W 1 Del x it gives me C 1 2 Del Psi i over Del x times V y and V is V J times Psi j. So, I get Del Psi J over Del y because V is been differentiated with respect to y plus I get a term from C 6 from C 6 6 here, I have Del W 1 Del y. So, I get Del Psi i Del y and I have V x. So, I get Del Psi J del x. So, this is the second block of terms and then I get a third term related to this inertial term. So, here I get oh there is, this has to be I am sorry I should have multiplied this by see in this earlier equation.

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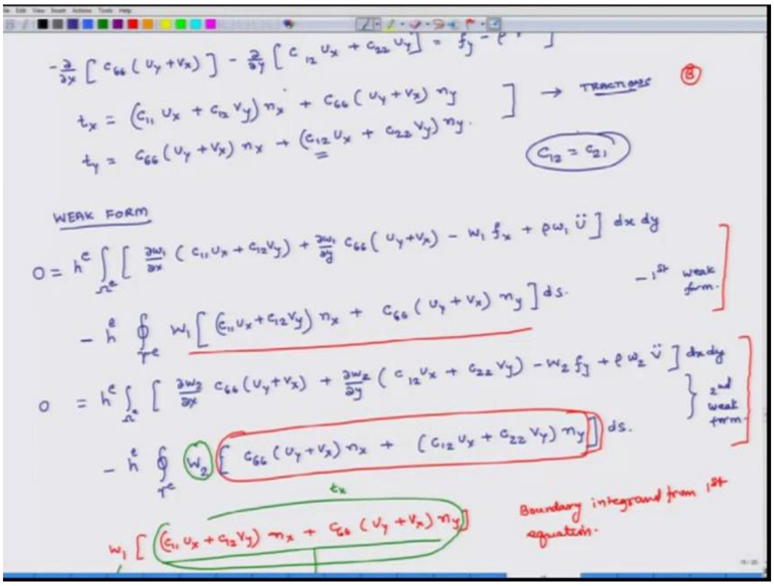

The entire expression is being multiplied by W 1.

So, we missed writhing W 1 here and we also missed writhing W 1 here and the second equation W 2. So, there was an error there. So, the inertial term I get it as actually it is summation J equal 1 to n rho from W 1 I get Psi I from u double dot I get Psi J, but it is being differentiated with respect to time twice. So, which term will get differentiated C J. This V J no sorry, U J will get differentiated twice right. So, it will be U J double dot and this entire thing is integrated over the domain d omega. I still have not taken care of W 1 f x and this boundary term.

So, I will move my W 1 f x on the right side, integral over the domain h e W 1 f x becomes Psi i f x d omega plus. So, sorry, because now I am shifting it to the other side I will remove this 0 and h e. So, the other term becomes integral over the boundary of W n gets replaced by Psi i times t x times d s. So, we have to be a little careful because they are lots of terms and we have to keep our, we have to do the book keep in correctly. Now this term involves U J, this is multiplied by U J. This entire integral of this entire term is multiplied by U J. When I integrate this over the domain U J does not play a role because U J is clearly a function of time.

So, it does not matter what U J is? When I am going this integration it is this thing which matters. So, this term is multiplied by U J. This term is multiplied by V J. So, I missed out V J here where, does V J come from? See these are all red terms. So, in red terms we have V x and V y.

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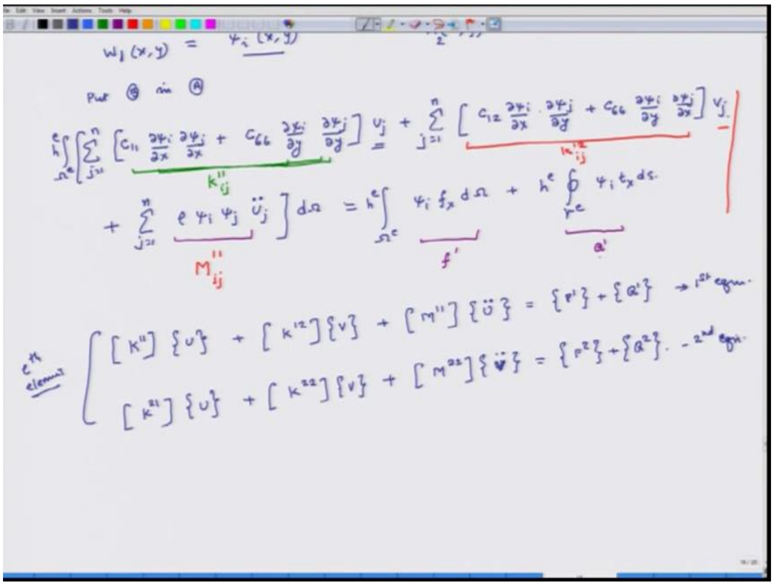

So, this term involves is multiplied by V J and the third term is this, it involves the second derivative of U J. So, I can call this green term as K 1 1 i j. I can call this red term. Why is it 1 1? Because it involves the first equation, so the first index superscript corresponds to the first equation. The second superscripts correspondence to U, which is the first primary variable, then we call this K 1 2 Y i j. Why is it K 1 2? The first superscript corresponds to the first equilibrium equation right which is for the first x direction. This entire equation is for x direction.

So, the first superscript corresponds to the x direction equilibrium and the second superscript corresponds to the second primary variable, because it is multiplied by V J and this I call it element of a another matrix, because it is involving a density term. So, it is related to the mass and it is multiplied by inertial term, this acceleration U double prime. So, this is the mass related term. So, this will be what? M 1 1, 1 2 2 1 what? First superscript corresponds to first equilibrium equation and it is multiplied by U second derivative of time derivative. So, it is my mass matrix M 1 1 i j and this is f 1 and this is Q 1. So, in matrix form I can write this equation as K 1 1 U. So, K 1 1 is multiplied by U vector plus K 1 2 and that is multiplied by V vector plus M 1 1 U double dot is equal to, if I add f and Q I can call it f 1 plus Q 1. What you see is that in this equation there is no M 1 2, there is no M 1 2.There is only M 1 1.

Similarly, for the second equation, so, this is from first equation for the second equation I will get K 2 1 times U and please note this is for which element? This matrix will change from element to element right. So, this is for eth element, but I am just ignoring this e for the time being for simplicity purposes, but all these are for eth element. So, this is K 2 2 V plus M 2 2 times the second derivative of time derivative of V equals F 2 plus Q 2. So, this is the second equation understood.

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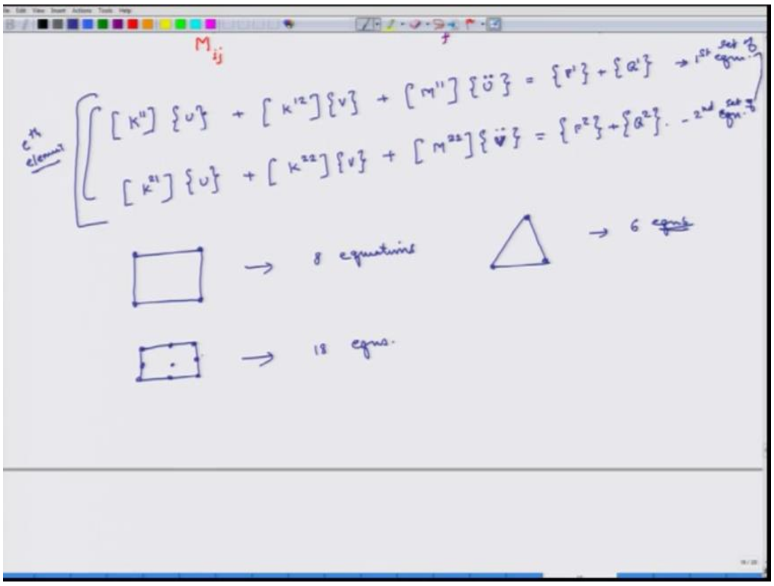

If I do the assembly so how many equations the first so this is first set of equations; this is second set of equations. How many equations do we have in the first set of equations? Suppose so it depends on what type of element we have. Suppose I have a rectangular element which has having 4 nodes then how many equations we have from the first set of equations? 4 right and from the second set of equations we again get 4 equations. We have one equation corresponding to each node.

So, for this thing for such an element we will have 8 equations. If it was a linear triangle then we will have total number of 6 equations. If it was quadratic rectangle then this has 9 nodes. So, it will have 18 equations and so on and so forth. So, I can now club these both set of equations into one big set of equations and that are what we will write.

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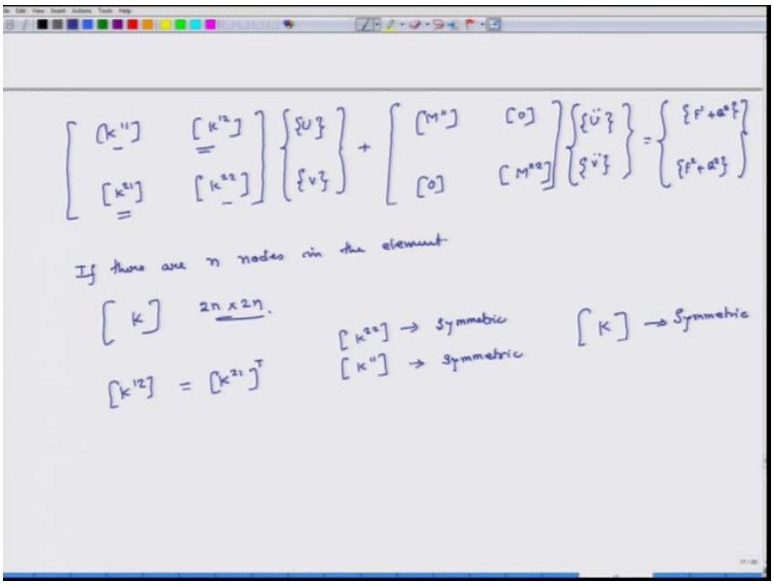

K 1 1 so this big matrix has 4 sub matrices. If there are n nodes in the element then this big K matrix, what is the order? It is 2 n by 2 n.

The other thing is that earlier we had said that C 1 2 was same as C 2 1. This is true for isotropic materials and orthotropic materials. If you do the math and you work out all the details because of that equivalence and for these equations K 1 2 is same as k 2 1 transpose. Also when you will do the math you will see that K 1 1 is symmetric and also K 2 2 is symmetric. So, because this is symmetric, this is symmetric and this is transpose of this, the overall K matrix is symmetric.

So, if you can compute only the top half of the matrix you do not have to compute the lower half because all those numbers you can directly assume through notion of symmetric. So, I think this completes our discussion for today. Tomorrow we will continue this discussion further and we will close this topic and also bring this course to a logical end so.

Thank you very much and have a great day bye.