Basics of Finite Element Analysis – Part II
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Lecture - 44
Post-processing

Hello again, welcome to Basics of Finite Element Analysis Part II. Today is the second day of this week. Yesterday what we were covering was the details of numerical integration using Gaussian Quadrature approach.

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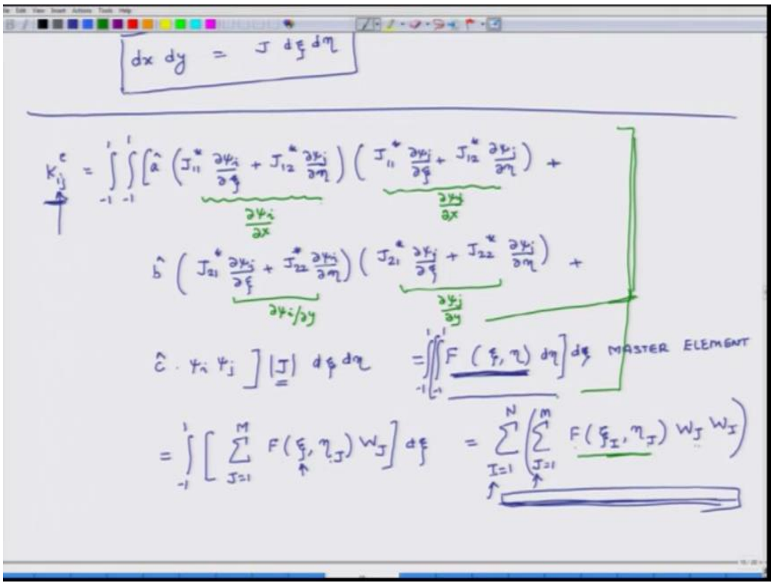

And what we had developed yesterday, was the relation for Gaussian integration. Suppose we have to find the value of K i j and then we have to basically transform the whole integral into Zeta Eta space and once we are successful with that endeavor then we use the Gaussian Quadrature method to find the integral using this relationship.

So, please note that here indices capital I and capital J correspond to the index associated with Gaussian Quadrature point. So, for each K I J, I have to add up several times based on the values of n and M and that is how, we calculate the value of K I j. If K I j, so this index K i the lowercase index each time it changes I compute a different value. So, I compute K 1 1, K 1 2, K 1 3, K 1 4 and so on so forth, but for each calculation of integral I have to do this addition over a different in this capital I and capital J and that is how we go around doing that.

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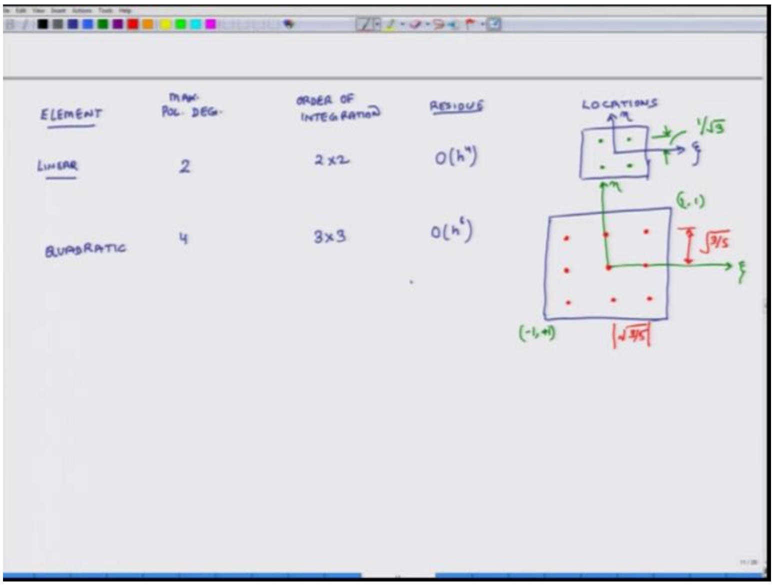

Let us look at some locations of some of these points. So, if I have a linear element then the maximum degree if I use Gaussian Quadrature it will help in integrating it accurately or will be. So, polynomial degree, so if I have to use a linear Quadrature then, it in this case there were 2 Quadrature points in one particular dimension. So, order of integration it is 2 times 2 and the residue or the error as I keep on shrinking the size of my element the order of that residue, it goes down in this fashion where, h is a normalized length of a typical element and what are those locations?

So, this is my element, what is this element on which I am integrating? This is master element. So, I evaluate see, I evaluate F as Zeta I and Eta J. So, I evaluated these points and what is the location of this point? This is 1 over square root of 3. So, the lower point is minus 1 over the square root of 3 below the Zeta x and the same thing is also true for these are the (Refer Time: 04:29). So, what do I do? I evaluate the value of F at these 4 points, I know the coordinates of these points I have specified it in this picture, I evaluate this F because I know all this expression. I know everything in this expression, I know everything.

So, I evaluate this entire complicated expression not the integral, the integrant and the value of the integrant at these 4 points and at each point I multiply it by this weight Gaussian weight and this Gaussian weights we have given to you in our earlier discussion on Gaussian integration. So, these are standard weights. So, I do this and I get my integral. If my expression is more complicated say Psi i is quadratic in nature then Del Psi i will be linear, because Del Psi i over Del Zeta will be linear because, I am differentiating it and instead and let us say that a is quadratic. It could be quadratic, cubic linear constant whatever. So, if is the overall integrant is more complex then I use Quadratic Quadrature. So, this is Linear Quadrature, I use Quadratic Quadrature.

Here, I can handle up to fourth polynomial degrees. So, I can integrate polynomials up to fourth degree and in this case the order of integration is 3 by 3 and here, the residue as I keep on reducing the size of the element, it shrinks faster than, the linear system. So, it is O, it is order of h to the power of 6. So, let us look at the Quadrature points here.

So, once again this is the master element. This is minus 1 1, 0 minus 1 minus 1, 1 1 and I evaluate it at 9 points. These are the 9 points and the spacing between these 9 points is 3 by 5 square roots of 3 by 5. Same spacing is also in the y direction.

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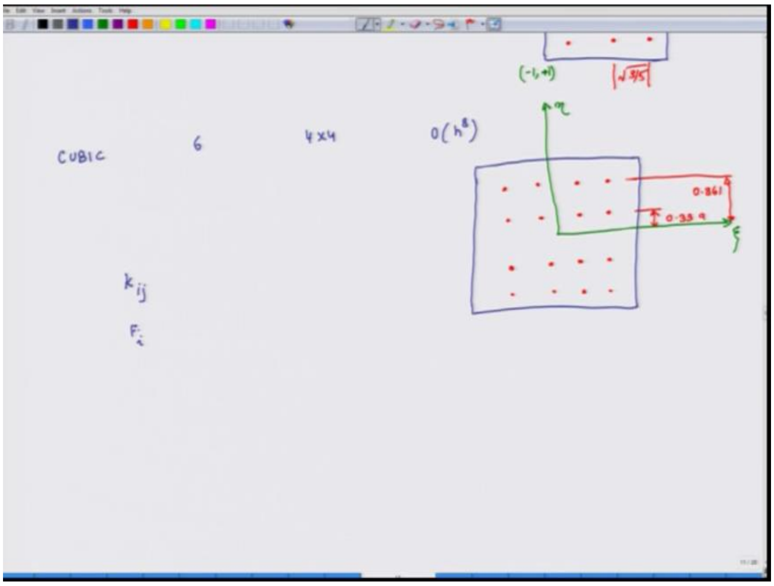

So, here capital M and capital N are 3, these terms and if I have even more complicated or not more higher order polynomial, I can go up to cubic Quadrature. Here, I can handle up to 6 per order polynomials. Here, I have 4 by 4, that is 16 Quadrature points and the residue converges at h to the power of 8, it is extremely fast convergence and let us look at. So, what are the locations of these points? This distance is 0.339 and this distance is 0.861, same distance.

So, same things on the other side of the Zeta and Eta axis also. So, in this case I have to evaluate the function F at 16 points multiply it with appropriate weights. These are not functions these are numbers appropriate weights add them up and I will get the integral.

So, in this way I can calculate any stiffness matrix or any element of this force vector using this Gaussian Quadrature method. So, this completes our discussion on Gaussian Quadrature. So, we will move on to the next topic and we will briefly discuss post processing.

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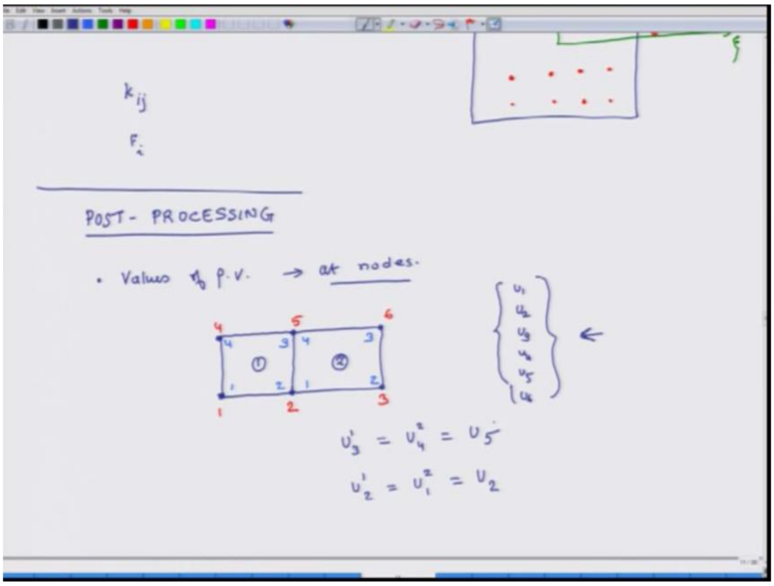

Post processing, so we have learnt how to formulate the problem? How to discretize the domain; break it up into individual elements we have learned. How to develop a weak form? From that weak form if we introduced in the weak form approximation functions and weight functions then we get finite element equations. So, we get equations for each element. We also learnt how to assemble these equations and in that context last week we had discussed the dis-connectivity matrix and we have learnt how to assemble elements when different elements come together and share a particular side or node. We learnt all this and also how to apply boundary conditions and solve the problem.

When we solve the problem, what is it that we get out of the problem? We get values of primary variable right. So, once we solve the assembled equations after applications of boundary conditions, we get values of primary variable and at what location do we get these values?

Student: At nodes.

So, we get these values at nodes. So, the finite element equations once we solve them we get values of primary variables. We do not get values of secondary variables, (Refer Time: 12:10) or stresses or strains or heat transfer rates or flow and all that stuff. We get specific values of primary variables and that too we get it at, specific nodes. Now suppose, there are 2 elements; element 1 element 2 and these are the local nodes and these are global nodes numbers. When we get the solution of primary variables, what we will get? We will get this entire vector right. We will get the solution for all the nodes. Some notes we may already know in the beginning because the boundary condition.

But once we have done the whole finite element analysis we will know this vector completely right and while we are doing that. So, when we are solving this we get this completely and in the assembly process we ensured what? Continuity, Conditions. So we ensured that U 1 at node 3 was equal to U 2 at node 4. Similarly we equated, we actually equated and this we equated through the connectivity matrix right. We equated U 1 at node 2 is equal to U 2 at node 1 and this we called. U 2, no I am sorry. This we called U 3 global node 3, this is U 2 and this is U 5. Now suppose the first thing we may be interested in is we want to find out the location, the value of U at this midpoint or some point, then what do we do? How do we find it out?

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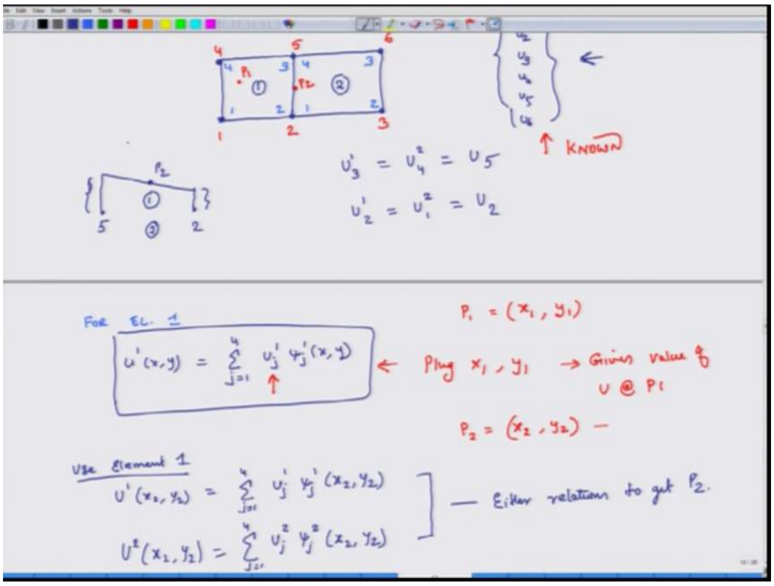

So, we know that for element 1. So, let us say the coordinates of this point are x 1 y 1. So, we know that for element 1, U for first element is what right, U for first element at any location in this element is U J which is the value at J-th times i j evaluated at x y, when I add this up I mean this is how we formulated the problem right.

So, if I have to find the value, suppose I have to find the value here at point P 1 and let us say the coordinates of point P 1 are x 1 y 1. Then how do we find out the value at point P? Suppose I am interested in finding the displacement or value U at P. All I have to do is in this relation plug x 1 y 1 right and we already know U Js, how do we know U Js?

So, U Js are known also Psi Js are known, approximation functions we have already developed those. I can do this either at the global level or the local level does not matter, but I know U Js I know Psi Js. So, all I have to do is plug x 1 y 1 and I will get the value of U at point p 1. So, this will give me point P 1.

Now, here is a question, suppose there is another point P 2. So, let us say point P 2 is x 2 y 2, in global degrees you know the global coordinate system it is x 2 y 2. How do we find out? That is placement at point P 2. Should I use element 1? Or should I use element 2? Or I use both and then what do I do? See if I use element 1 then what is the value U 1 at x 2 y 2. So, I have to put a subscript now I have to be careful. So, at location x 2 y 2 which is point P 2 [FL] J is equal to 1 to 4 U J for the first element Psi J for the first element x 2 y 2.

So, I will get some value at point P 2 right because, this point P 2 is in element 2 as well as on element 1 right. I can also get it U 2 x 2 y 2 is equal to J is equal to 1 to 4 U J for element 2 Psi J for element 2 x 2 y 2. So, these are 2 different formulas. Which formula is correct? Or both are wrong or both are correct? Both are correct. So, why are both correct?

Student: (Refer Time: 19:25) It is continuous.

See we had ensured that U was continuous at point 5. U was continuous at point 2 and between 5 and 2, this is a linear element. How is it varying? If it is a linear element how does it vary? It varies linearly. So, U at 5 maybe the displacement is this U at 2 maybe the displacement is this.

So, this is node 5, this is node 2 these are global nodes, how does the displacement vary? It varies like this. So, it does not matter, if I am looking from side 1 it will still vary in a linear way. If I am looking from element 2, it will still vary linearly because and the value at this midpoint P 2 will be same because why? Why it will be same? Because these values are same, I can use either of these values to get P 2, I can do either of these values to gets P 2.

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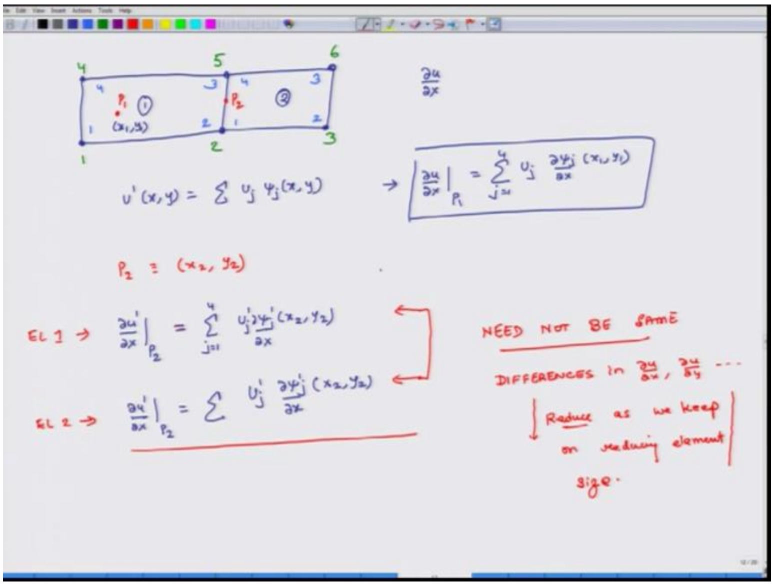

Next we look at derivatives. So, consider element 1. So, this is element 1, this is element 2, local nodes are 1 2 3 4 1 2 3 4; global nodes are 1 2 3 4 5 6. Suppose I am interested in finding the derivative; Del u over Del x and I am interested in finding it at let us say point P 1. So, how do I find the derivative Del u over Del x at point p 1? What is the relation for u for element 1? U for element 1 equals what? U J Psi J x y right. So, if I have to differentiate I have to get Del u over Del x at point P 1. What do I do? I just differentiate this term U J Del Psi j over Del x and where do I value at a type x 1 y 1. So, if the coordinates are x 1 y 1, I get this relation and that will be my derivative.

Now suppose this is point P 2 and point P 2 is x 2 y 2. Now I again have two options, I can compute from element 1 and I can compute from element 2. So, if I compute from element 1 then Del u over Del x at point P 2 from element 1 and this is equal to oh there has to be a summation sign here and if I am computing from element 2.

So, here I am using element 1. So, the superscript is 1 and in the other case, I am using data from element 2, so the superscript is 2. Which one is right? Derivative is did we enforce continuity of derivative When we were doing assembly? Where we did, we enforce that? Where did be enforce the continuity of derivative? Did we ever enforce continuity of derivative? We never enforced the continuity of derivative. Which means that if I calculate from element 1 the value of this derivative that number will be same or different? It can be.

So, these two values need not be same. See when we do beam equation or some other equations there we specify continuity of derivative also, but in this formulation we never specified continuity of derivative. If we specify continuity of derivative that also becomes a variable. So, when beam equation, if you remember go back to Euler Bernoulli beam. Each node has how many degrees of freedom. It has two degrees of freedom right. Here it has only one degree of freedom. So, we only a specified continuity of the primary variable we did not specify continuity of the derivative.

So, these relations need not be same. So, if you use element 1 to calculate the derivative you will get 1 answer, if you use element 2, you will get another answer, which answer is correct? So, the answer is this is a finite element solution. So, these differences and even higher order derivatives, what happens to them? They go down, they reduce as what? As we keep on reducing elements. So, these differences keep on reducing as we keep on reducing the size of the element.

So, what we have to make sure is that we should feel comfortable, that the movement or the change in derivatives as we cross the boundary of an element is sufficiently small only then we can say that the solution has converged, if we are interested in finding out derivatives. Where are derivatives important?

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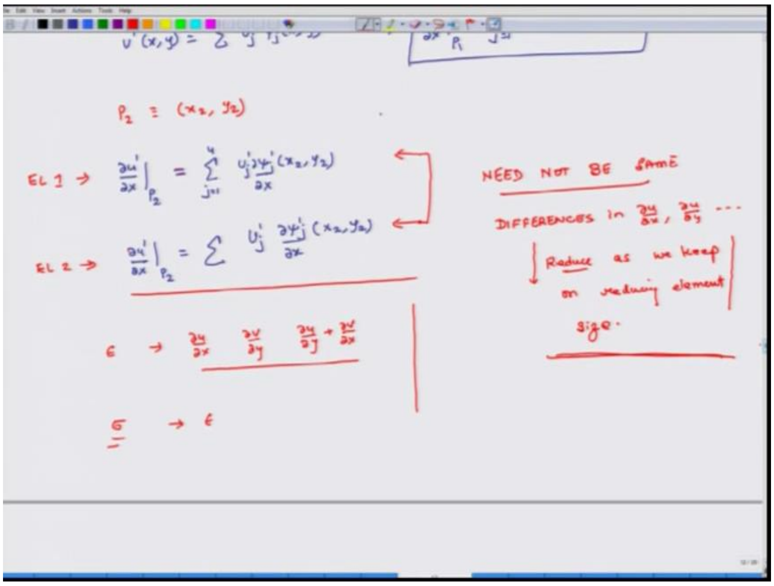

For instance, if I have to find strain, then what is it? It is related to Del u over Del x Del v over Del y Del u over Del y plus Del v over Del x and so on and so forth. So, I am finding interested in finding strains and I am interested in finding strain especially at the point which is at the boundary then, I have to make sure that the number is the difference between side 1 and the side 2 is sufficiently small. Where else we know that stress is what? It depends on the strain, you multiply stress by elasticity constants you get strain stress.

So, stress if I am interested in. The again we have to keep on reducing the mesh size to sufficiently small level. So, that it is converged. So, this is very important to understand that derivatives are not continuous in a lot of FEA solutions, across an element boundary. Even though the displacement or primary variable may be continuous, even though primary variable may be continuous, I think this closes the discussion for numerical integration and also today’s lecture.

We will now move on to the next lecture which is tomorrow and till then have a great day, Bye.