Basics of Finite Element Analysis – Part II
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Lecture - 34
Stiffness and Force matrices for Rectangular element

Hello. Welcome to Basics of Finite Element Analysis Part II. This is the 6 week of this course, today is the 4th day of the week in the last week we had calculated elements of K and F matrices, and these calculation for specific for a 3 nodded triangular element, that is a linear triangular element. What we will do is something very much similar to what we did in the last class, but with the difference that we will be using all that knowledge to 4 nodded rectangular elements.

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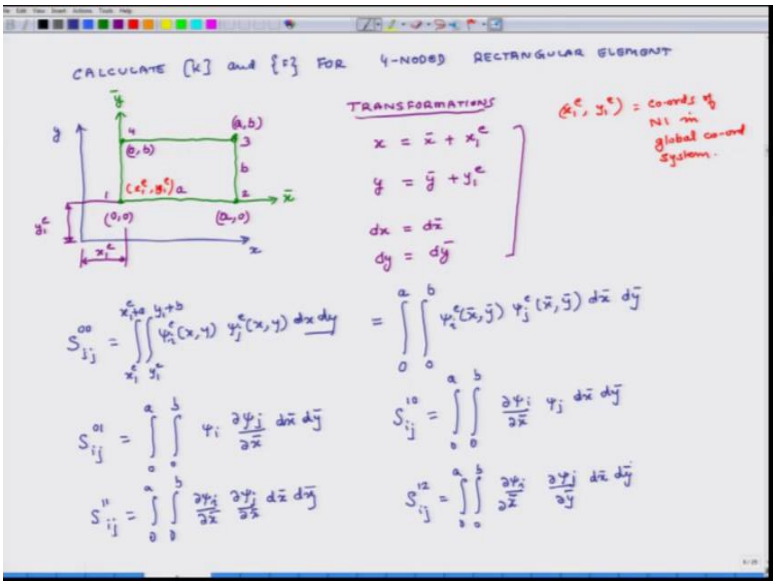

So, what we plan to do today is calculate elements of elements of K matrix and F vector for a 4 nodded rectangular element. So, these are my coordinates x and y and we know that my rectangle was such that it is coordinates are aligned to the axis x and y axis and then I had developed approximation function for this 4 nodded element in local coordinate system x bar and y bar and the 4 coordinates are 1,2,3,4. So these are the 4 nodes and the length of 1 side of the rectangle is a the other side is b. So, these nodes are 0,0 a,0 a, b and 0,b.

So, now first let us look at the transformations. So, this is a point x in the global coordinate system, that can be written as. So, the x coordinates of any point in a global coordinate system is equal to x bar plus x 1 e. What is x 1 e? So, x 1 e is this distance, it is the x coordinate of the first node in the global coordinate system. So, any point in local coordinate system can be mapped to that in the global coordinate system for the x direction using x this relation, x is equal to x bar plus x 1 e agree. Similarly y is equal to y bar plus y 1 e. What is y 1 e? Y 1 e is this in.

So, in the global coordinate system the coordinates are x 1 e and a y 1 e. x 1 e, y 1 e are coordinates of node 1 in which system global coordinate system. And in the local coordinate system the coordinates of first node are 0, 0. So the other thing is dx if I differentiate first equation is same as dx bar and dy is same as dy bar. So, these are the transformations and using these transformations I can calculate values of S metric and different S matrices for instance. We have developed that S00 ij is equal to psi I. This is the original definition for elements of S00.

Now, here we note that psi i is expressed as function of x bar psi j is for expressed as function of x and y and same is psi psi i also right, but I have safe function or interpolation functions in the local coordinate system which we had developed earlier.

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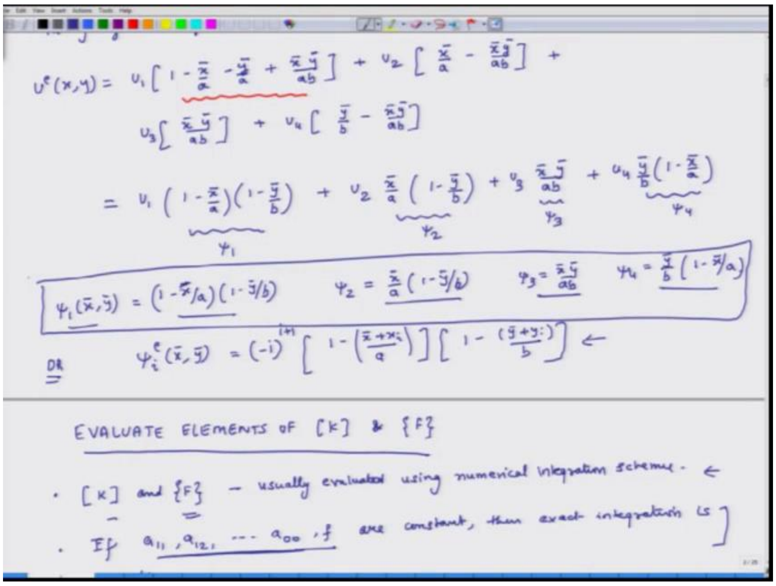

See these are the interpolation functions. We had developed our interpolation functions using the local coordinate system, but my expression for S is in the global coordinate system. So if I have to use these expressions for these approximation functions using the local coordinate system, then I have to transform dx into dx bar and I also have to change the limit is right. So this I can re write in the local coordinate system as psi I, x bar, y bar psi j, x bar, y bar, dx bar, dy bar. Why can I write straight away d x, dy bar because I know the dx is equal to d bar and dy is equal to dy bar. My limit is in the local coordinates system what? So, my initial limit is so for a rectangular element if I have to integrate I have to integrate it 2 times.

So, the integration were would have happened from x 1 to x 1 plus a and y 1 to y 1 plus b and because this is the eth element I have to put superscript e everywhere. Now in the local coordinate system this limit is change. So, x 1 becomes 0 and x 1 plus a becomes a, y 1 becomes 0 and y 1 plus b becomes b. Similarly Sij 01 I can write it as a double integral 0 to a, 0 to b.

So, now I am directly writing, the expression in local coordinates system it is psi I, del psi j divided by del x bar dx bar, dy bar, I can directly write that. Similarly Sij 1 0 equals double integrals 0 to a 0 to b del psi i over del x bar psi j dx bar dy bar and then S1 1 ij equals del psi i over del x del psi j over del x bar x bar is in both places dy bar. So, what is S1 2 it is the first derivative psi derivative of psi ij with respect to x bar and derivative of psi j with respect to y bar dx dy.

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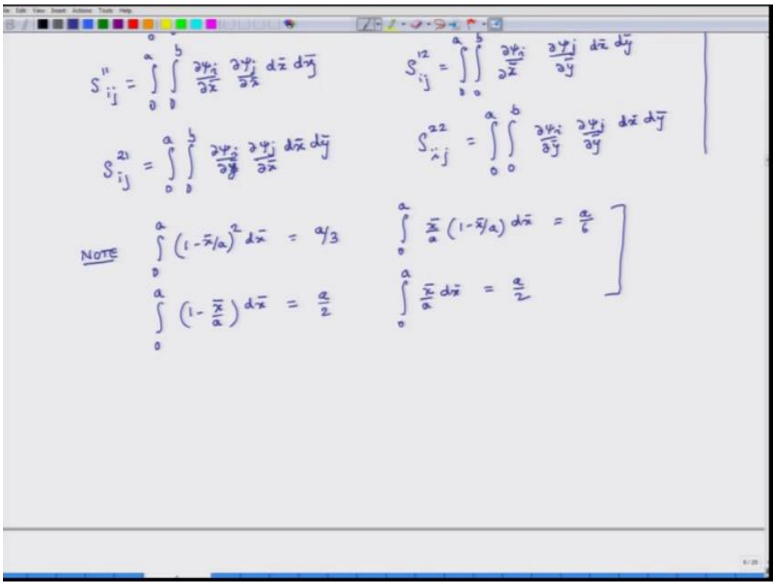

And I have 2 more expressions S2 1 that equals 0 to a, 0 to b and finally, S2 2 ij that equals partial of psi i with respect to y bar, partial of psi j with respect to y bar dx bar y bar. So, these are the formulas and my psi i and psi j the expression for those psi are already given here psi 1, psi 2, psi 4. So for different psi s i can compute all the individual terms of the S matrix. So, while we are doing this computations, we must just note some facts that when I integrate from 0 to a 1 minus x by a square dx bar.

What i get is a by 3, an also if I integrate x bar by a into 1 minus x bar by a dx bar I get a divided by 6, so because we will encounter these integrals. So, I can directly replace them and similarly 1 minus x bar by a integral from 0 to a dx bar equals half of a an integral from 0 to a, x bar over a dx bar equals a over 2. So, we can use these standard mathematical facts when we are computing integral for S1 1, S 1 2 so on so forth.

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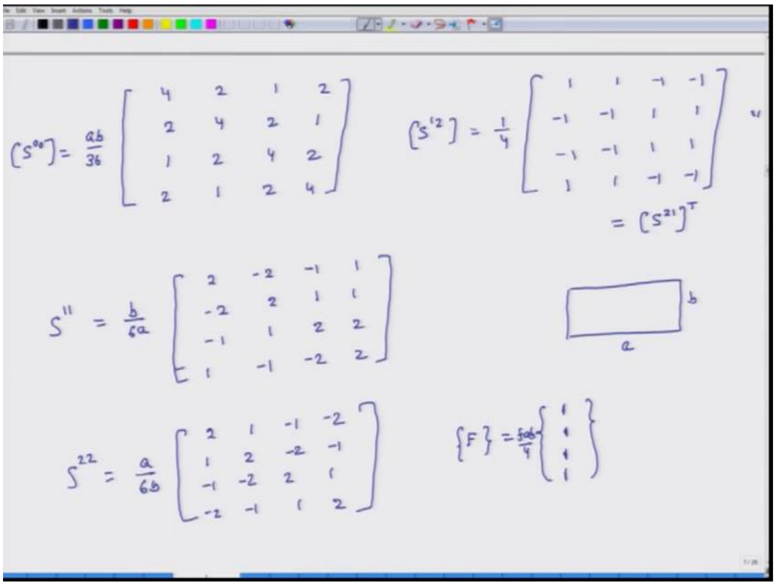

So, what I will do is, I will write down expressions for S00. So, S00 ab is ab over 36 this is S00. 4 2 1 2, 2 4 2 1, 1 2 4 2, 2 1 2 4, then S 1 2 equals 1 over 4, 1 1 minus 1 minus 1, minus 1 minus 1 1 1, minus 1 minus 1 1 1, 1 1 minus 1 minus 1, this is also equal to S 2 1 transpose of S 2 1 and then S 1 1 equals b over 6a and that is 2 minus 2 minus 1 1, minus 2 2 1 1, minus 1 1 2 2, 1 minus 1 minus 2 2 and then finally, S 2 2 is equal to a over 6b, 2 1 minus 1 minus 2, 1 2 minus 2 minus1, minus1minus 2 2 1, minus 2, minus 1 1 2 and finally, the force vector.

So, here is a question, if there is an element a times b wide and force per unit area is F, then what will be these 4 values? What by 4 F times and what is a a b. So, it will be F ab divided by 4 and these values will be 1, 1, 1, 1. So, this completes are coverage for linear 4 nodded rectangle elements also. So, what we have done. So, far in these 4 lectures is that we have computed elements of K and F matrices for a triangular element and for a 4 nodded rectangular element.

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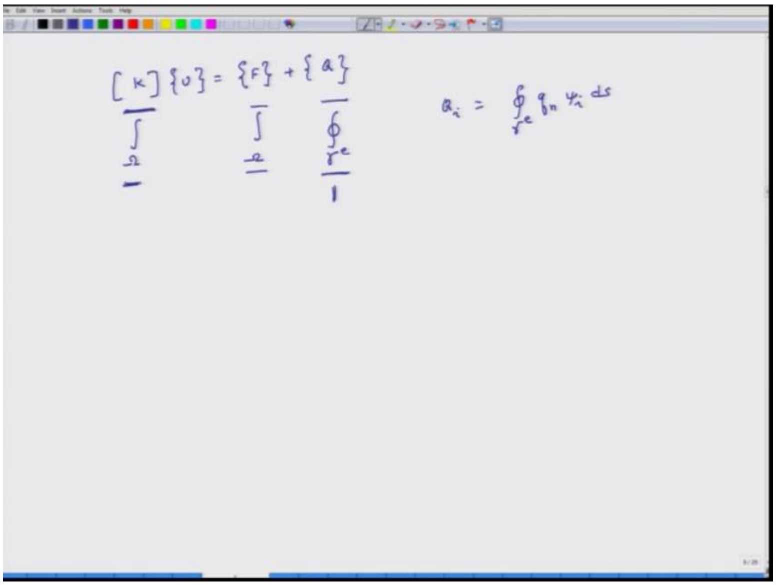

Now, in our finite element equation, which that is the equation at the element level the overall equation, is what? K times u equals and F vector plus there is another vector which is q vector. Elements of this matrix are calculated by a surface integral. Elements of this vector are calculated by a surface integral. So, we learned how to compute surface integrals. What we have not done what are elements of this matrix? They are computed through what through a boundary integral right because Qi equals Qn psi i ds, we have developed this expression earlier. So, we have learnt how to calculate learn surface integrals for in context of stiffness matrices and force vector, but we have not figured out or I am not discussed it till so far, how we calculate this term.

So, in the next 2 lectures, we will cover this term the boundary integral and that will complete our coverage of 2 dimensional finite element analyses at the element level. So, once we are comfortable at the element level then we will learn how to do assembly of element level equations. So, that is what I look forward to talking about tomorrow and we will meet tomorrow.

Thank you.