Basics of Finite Element Analysis – Part II
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Lecture – 33
Stiffness and Force matrices for Triangular element

Hello, welcome to Basics of Finite Element Analysis Part II. This is the sixth week of this course and today is the third day on the week. yesterday, we had started developing expressions for K matrix elements and in that context we had introduced some identities which are shown here.

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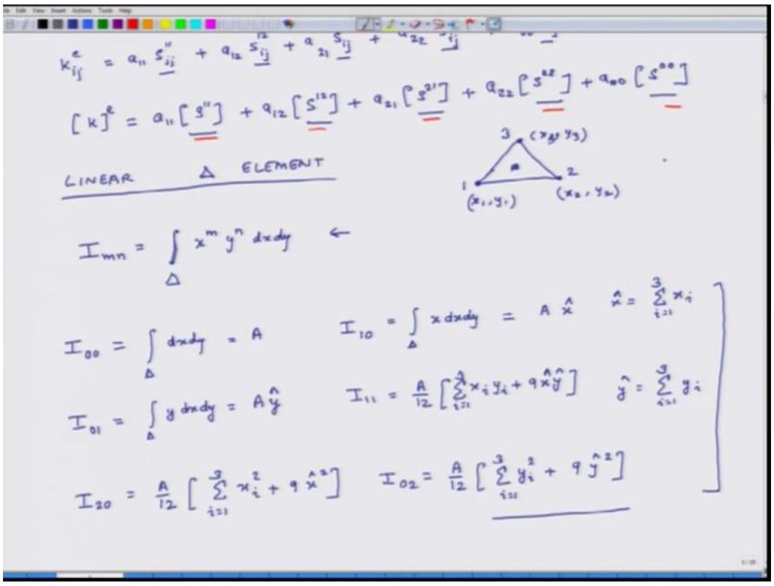

So, these are the identities related to a triangular geometry and these identities are that I m n which is integral of x to the power of m times y to the power of n times d x d y and for different values of m and n, we had written down these identities. What we had not done was, we had not proved these identities because that is not necessarily germane to the theme of this course, but these are standard mathematical identities, if you have interested in figuring out the details you can explore on the net or in mathematical books and you will find it out. So, what we plan to do is we want to use these identities to develop expressions for elements of S 1 1, S 1 2, S 2 1, S 2 2 and S 0 0 matrices. So, that is what our aim is.

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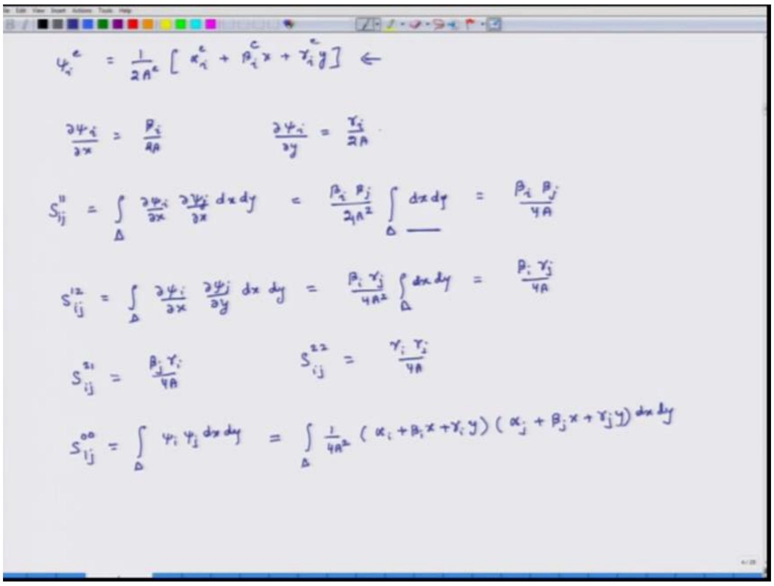

So, before we start doing that we want to recall that is Psi i for the eth element is what? It equals 1 over twice of A e, which is for the element times Alpha i times Beta i x plus Gamma i y and all these are specific for the eth element. Please note that Alpha, Beta and gammas these are constants and they are constants for specific a element. So, for one triangle they may have some set of values for another triangle their values will be different.

So, if I differentiate Psi i with respect to x then, what I get is Beta i divided by 2 A e or what I will do is. So, I will omit this e for purposes of gravity because it comes so again and again and similarly derivative of Psi i with respect to y is equal to Gamma i divided by 2 A.

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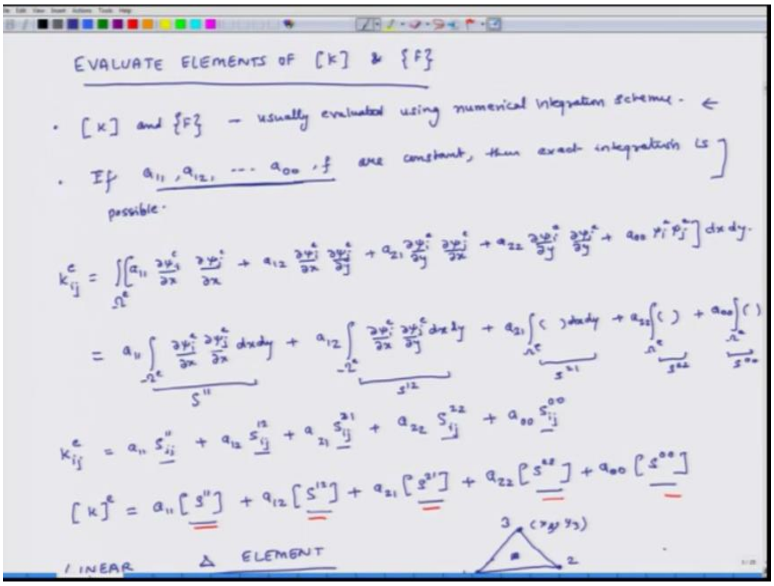

Now S i j 1 1; so, what is S 1 1? It corresponds to Del Psi i Del x times Del Psi i j Del x integral of that. So, for the for S 1 1, the i jth element is Del Psi i over Del x Del Psi j over Del y excuse me Del x d x d y and my domain of integral is in this case triangle, because we are thinking about a triangular element. So, we are integrating it over the triangle. So, this Del Psi i over Del x is Beta i over 2 A, Del Psi j over Del x is Beta j over 2 A right. So, it is Beta i Beta i divided by 2 A or 4 A square, integral of d x d y. So, I have taken these Betas, so, this is Beta j. So, I have taken these Beta out of the integral sign because they are constants. So, I can take them out.

Now this is integral of d x d y and this is same as I 0 0 right and the value of I 0 0, if we go back is A value of I 0 0 is A. So, this equals Beta i Beta j over 4 A. Let us look at S 1 2. So, S 1 2 equals Del Psi i over Del x and because the second index is 2. So, it will be Del Psi j over Del y 2 corresponds to y 1 corresponds to x. So, that is how we figure it out, d x d y this equals. So, Del Psi over Del x is Beta i over 2 A Del Psi j over Del y is Gamma j over 2 A. So, I get Beta i Gamma j over 4 A square d x d y integrated over the triangle and what I get from here is Beta i Gamma j over 4 A.

Similarly S 2 1 i j equals Beta j Gamma i over 4 A and I will also write the expression for S 2 2 directly and that will be Gamma i Gamma j over 4 A. So, what we have done is we have computed S 1 1, S 1 2, S 2 1, S 2 2 and now we will develop an expression for S 0 0. So, S 0 0 equals integral over the triangular domain of Psi i Psi j d x d y.

So, 0 implies there is no differentiation. So, what is Psi i? Psi i is again what is Psi i? We can get the expression from Psi i from this relation. So, this is 1 over 4 A square Alpha i plus Beta i x plus Gamma i y times Alpha j plus Beta j x plus Gamma j y d x d y.

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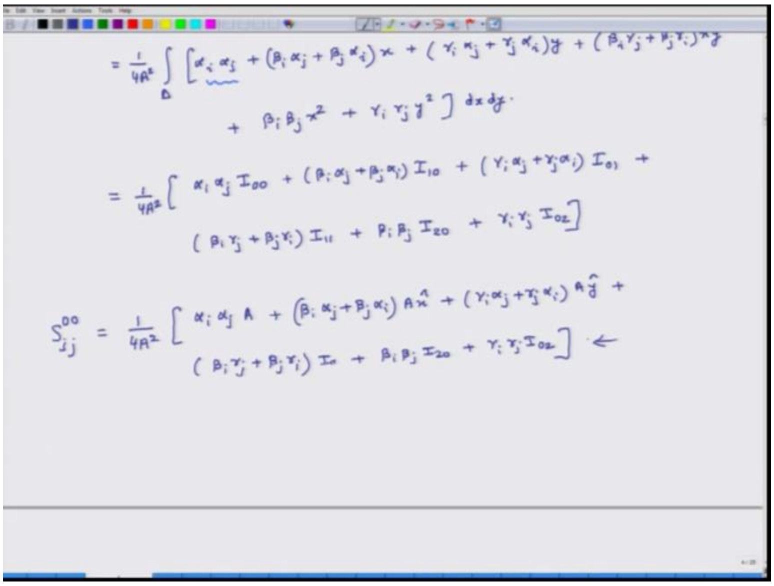

So, this is equal to 1 over 4 A square and I will get. So, this is having 3 terms, this is having 3 terms I will long set of 9 different terms. So, I am going to write down those. So, first the term is Alpha i times Alpha j. That is a constant then, the next term will involve x. So, that is equal to Beta i Alpha j plus Beta j Alpha i x. Next 2 terms will involve y. So, those will be Gamma i Alpha j plus Gamma j Alpha I y, then we will have a bilinear linear term involving x y. So, what will those Beta i Gamma j plus Beta j Gamma i x y then, we will have x square term which will be Beta i Beta j x square and then a y square term Gamma i Gamma j y square d x d y.

So, this equals. So, let us look at the first term Alpha i and Alpha j times d x d y. So, this means, its integral will be Alpha i j times I 0 0 right I 0 0 then, I have this x term. So, this is Beta i Alpha j plus Beta j Alpha i and this constant is multiplied by x times d x d y and it is integrated. So, this will give me I 1 0 plus the third term Gamma i Alpha j plus Gamma j Alpha i and this is multiplied by y times d x d y. So, integral of y tends d x d y is I 0 1, then I look at the fourth thing. So, this is Beta i Gamma j plus Beta j Gamma i and that is multiplied by x y and if I integrate x y d x d y over the triangle I will I 1 1 and then I have Beta i Beta j and integral of x square d x d y will be I 2 0 and then finally, I have Gamma i Gamma j I 0 2.

So, I will write this again 1 over 4 A square I 0 0 is. What is the value of I 0 0? It is A, then value of Beta i Alpha j plus Beta j Alpha i and the value of I 1 0 is A x hat. Then the third term Gamma i Alpha j plus Gamma j Alpha i and its value is A y hat plus Beta i Gamma j plus Beta j Gamma i and the value of A I 1 1 is a by 12 and x i. There is a long expression, but you can plug in here.

So, I will at this stage I will just leave it as I 1 1, but you know what is the expression for I 1 1 plus Beta i Beta j I 2 0 plus Gamma i Gamma j I 0 2. So, this is S 0 0 0 i j. So, using this relation you can calculate S 0 0. So, now, we know how to calculate S 1 1, S 1 2, S 2 1, S 2 two and S 0 0 and once we are able to calculate these, then we can use these things to compute all the elements of the K matrix with the assumption that A 1 1, A 1 2, A 2 1, A 2 two, A 0 0 are constants. So, that is about the K matrix. The next thing is about the element of force vector. So, that is another thing we will see.

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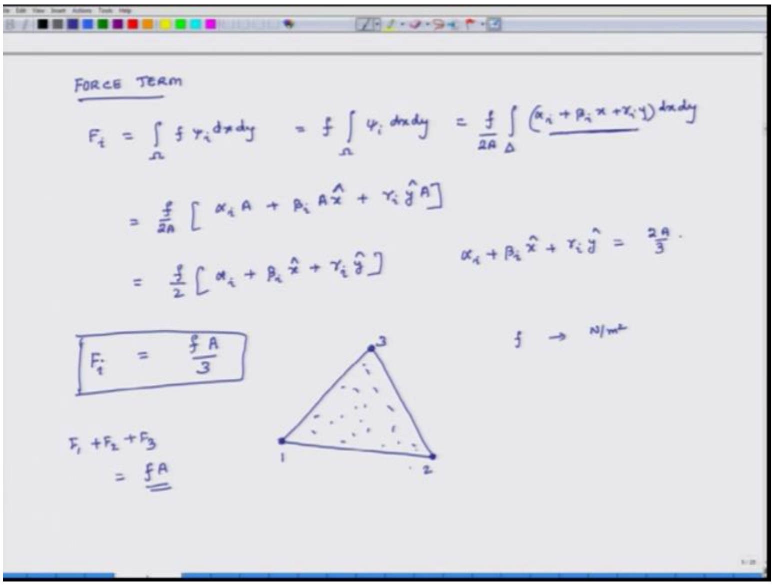

So, force term. So, we had written that F i equals integral over the domain f times Psi i d x d y. Now again if f is constant then, it will be f this small f is constant then I will be integral of Psi i d x d y and this is equal to what is Psi i? Psi i is Alpha i plus Beta i x plus Gamma i y d x d y and this here we are integrating over the triangle. This expression can only report for a triangular domain. So, that why I have to replace omega by triangle.

So, this is equal to and there has to be also a 2 A in the denominator. So, this is equal to f by 2 A, integral of Alpha i d x d y is Alpha i times A integral of the Beta i times x d x d y is Beta i times A x hat an integral of Gamma i y d x d y is, Gamma i y hat A. So, A goes away f by 2 times Alpha i plus Beta i x hat plus Gamma i y hat and there is another mathematical identities which tells us that Alpha i plus Beta i x hat and you can do the mathematics and you will find out that is true plus Gamma i y hat equals 2 third of the area of triangle. So, what I get is f times a divided by 3 F i. So, what does it mean that if I have a triangle.

So what i get is f times A divided by 3 F i. So what does it mean? That i have a triangle and I am putting some force per unit area. So, I am putting some force in the norm. So, so the triangle is let us say here and I am putting some force per unit area and the intensity of that force is F. So, its units are what? If it is force then it will be Newton per square meter. In other problems for instance case of diffusion and other things it will be somewhat different, but if we have talking about mechanics then if it is forced then force per unit area.

So, then the total force will be what? which will be exempted upon the triangle will be f times A right and there are 3 nodes 1, 2 and 3. So, for the ith node, the total force which will be exerted on the ith note will be one-third of that total force; understand that. So, F 1 plus F 2 plus F 3 equals f times A where A, is the area of the triangle and each node shares equal amount of load. So, this is the physical interpretation of this mathematics.

So, now, we know how to calculate the force term and also the terms for K matrix. What we will do in the next class? Is we will continue this discussion, but what we will do is? What we have done till so far is we have calculated these terms for a triangular element. What we will now do is? We will also do similar mathematics for a four nodded triangular element. So, that we get comfortable with this whole idea.

So, that closes the discussion for today and tomorrow we will continue this and we will apply all this understanding to a four noted rectangular element.

Thank you very much bye.