Basics of Finite Element Analysis – Part II
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Lecture – 23
Two dimensional FEM problem

Hello. Welcome to Basics of Finite Element Analysis Part II. Today is the fifth day of this week and going forward we will be introducing a new topic and which deals with, how do we go around doing finite element analysis of problems, which are governed by partial differential equations in two dimensions and these differential equations could govern the processes, where one single variable is involved or multiple variables are involved, but for starters we will be doing two dimensional problems, involving a single variable.

So, with that let us first lay out the overall nature of problems we will discuss in context of 2D problems with one variable.

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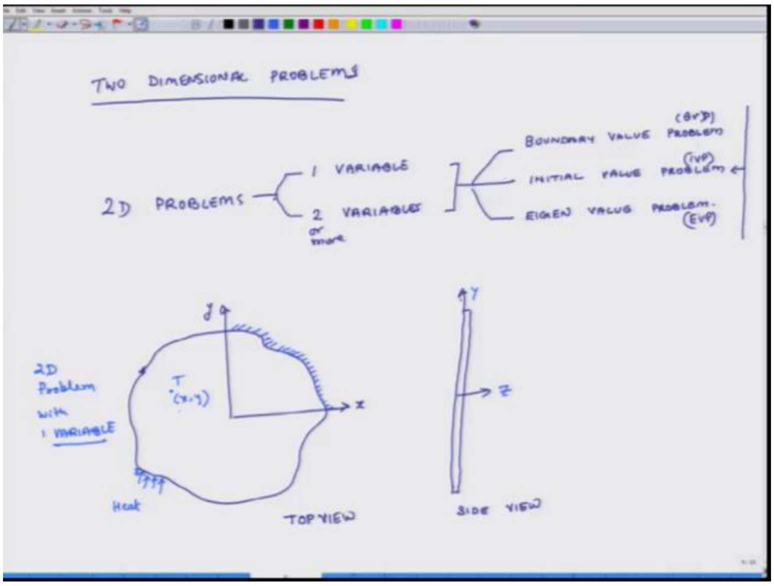

So, two dimensional problems so, first we will classify these problems. So, you can have these problems involving one variable, you can have problems involving two variables or you can have even more, two or more what would be the example of a one variable problem; suppose I have a plate of this shape. So, if I look at this plate. So, this is x and this is y, and this is the top view of the plate and if I look at it from the side, then the plate is it looks like this. So, this is top view and this is side view, this is my x direction and this is my z direction and this is the plate and let say that this plate is insulated is that sorry.

So, this is z and this is x, y, y. So, suppose I have this plate and let say part of this edge of the plate is insulated and then, I have natural convection in other parts of the plate and in some part I am injecting some heat. So, I am heating from the portion of the edge the other part is open. So, it can have it is open, and this other edge is insulated and if I am interested in finding the temperature at any point. So, here there is one single variable t, but it varies with respect to x and y.

So, it is a two dimensional problem involving, one single variable which is t. So, this is the 2D problem with one variable, there could be other problem I could have a similar plate. Where I am fixing one part of the edge and the other and the other part remaining part I am just pulling it and because of that, pulling there will be displacements in the plate and let say those displacement in x and y directions, are u and v respectively then it will be a two dimensional problem involving two variables.

So, you can have for 2D problem with one variable, two variable or n numbers of variables. So, these are two broad categories, 2D problems in one variable and 2D problem at two variables and then these things can be reclassified into three types of problems; one is Boundary value problem, then you have Initial value problem and the third one is Eigen value problems. So, we had described these three different types' problems in part one; but, this is abbreviated as BVP, this is IVP, this is EVP. So, in boundary value problem, we do not consider time effects, so there could be a plate and whatever solutions we are getting with the steady state solution it does not change with time that is a boundary value problem.

In initial value problem time effects are also incorporated. Suppose I have a plate and I just started heating it from a part of the edge and then, it takes may be ten minutes for the plates to reach a study stage temperature, but how is temperature changing with time will depend on the initial state of the plate also and also the boundary conditions. So, those two problems are initial value problems and then the Eigen value problems. We calculate the Eigen values of the system and do not in that case consider the role of initial conditions and also the role of force related boundary conditions.

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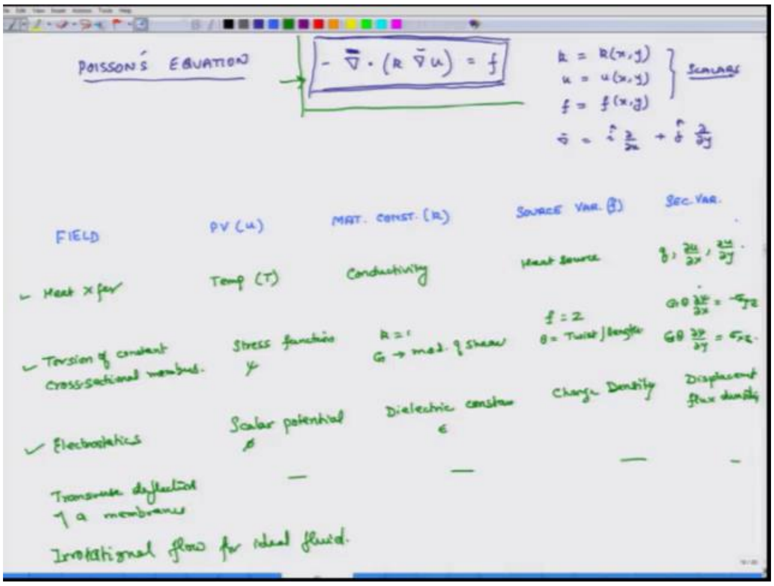

So, these are three categories of problems and what we will be discussing as we go forward there are 2D problems with one variable. BVP the boundary value type initial value type and Eigen value type. All these three categories we will now look at an equation and this equation is called Poisson's equation and some important equation because if we solve this particular equation we can solve a very large number of different types of problems, this is written as minus and is this inverted triangle symbol known as gradient operator and then I take a dot dot products.

So, it is a vector operator. So, and then I take it is dot product with k a constant or function of x and y it could be times gradient of a variable. So, u is not displacement it is a variable and it is a scalar variable it does not change with direction and that equals f. So, here k could be a function of x and y, u is certainly a function of x and y and f could be a function of x and y and then of course, all these are scalars. So, they are not changing with directions and the other thing is that the gradient operator is defined as del over del x, plus unit vector in y direction which is j times, del over del y.

So, this Poisson's equation is having a lot of applications, where is large number of applications. So, let us have a review of what different type of applications is applicable to? So, we will construct a table field, then p v stands for primary variable and in this equation the primary variable is u then, we have material constant which is k, then we have source variable which is f and then, we have secondary variable which could be either q or del u over del x or del u over del y. So, again remember u is not displacement it is some generalized scalar variable, it could have lot of definitions, but it certainly does not have a direction it is a scalar and the same thing is true for f, f is not force f is some function it is a again a scalar variable.

So, the first thing where we can apply this is in the area of heat transfer. So, if I am trying to solve a two dimensional heat transfer problem then, the primary variable is temperature. So, the governing equation for two dimensional heat transfers is this equation. So, temperature lot of times we expressed temperature is T. So, u becomes T then material constant is conductivity then, we have heat source and finally, we have sorry this will be here. So, he and the secondary variables could be q or del u over del x which is the gradient of temperature, in x direction del u over del y, which is the gradient of temperature in x direction and del u over del y which is the gradient in y direction.

So, this is from heat transfer then if, we look at mechanics this is torsion of constant cross section cross sectional members. What does this means; if you have a shaft it could be a circular shaft or square shaft and I am trying to do torsion on it then, it is this problem this equation defines the solution for that. So, here what is u?U is the stress function and we will discuss this later. So, it is a stress function it is not stress it is a sum function called stress function and lot of times in literature. It is designated as psi it does not have a direction. So, scalar variable k is equal to one unity and here we use a shear modulus or shear f equals to and theta equals twist per unit length and what are the secondary variables g theta del, oops del psi over del x is equal to minus sigma y z and g theta del psi over del y is equal to sigma x z.

So, this is thermal this is mechanics, we will look at maybe 1 or 2 more electro statics. So, here when electro statics it is a two dimensional problem, this u represents scalar potential designated as phi material constant is dielectric constant epsilon this is charged density and this is displacement flux density. So, this is from electrical engineering and then we have several others I will just list down 2, but I will not give all this details one could be transverse deflection of a membrane, another could be irrotational flow for ideal fluid and this list is not just limited to that we can also use it in magneto statics and several other areas.

So, the point what I am trying to make is that one single equation Poisson's equation, if we know how to solve? We can solve a whole list of problems because, the same finite element code can be used can be used to solve heat transfer problem, torsion problem, electro statics problem deflection problems the only thing which changes is that in one case, you call u as temperature and in another case you call as a stress function and so on and so forth. This gives you the over view of what the landscape is.

In the next class we will start developing some of the details of this problem. So, tomorrow we will develop we start developing details of this problem and this discussion will go on for at least one week. So, that we become fairly comfortable with how to solve two dimensional single variable problem using finite element analysis so.

Thanks a lot. And we will meet tomorrow, bye.