Basics of Finite Element Analysis – Part II
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Lecture – 22
Newton-Cotes Quadrature

Hello. Welcome to Basics of Finite Element Analysis Part II. In today's lecture we will discuss different types of quadrature known as Newton-Cotes Quadrature. And we will also contrast it with Gaussian Quadrature and with that discussion we will close our coverage of numerical integration schemes.

So, before we discuss this Newton cotes quadrature, let see let us very briefly recap the nature of quadrature points for Gaussian quadrature.

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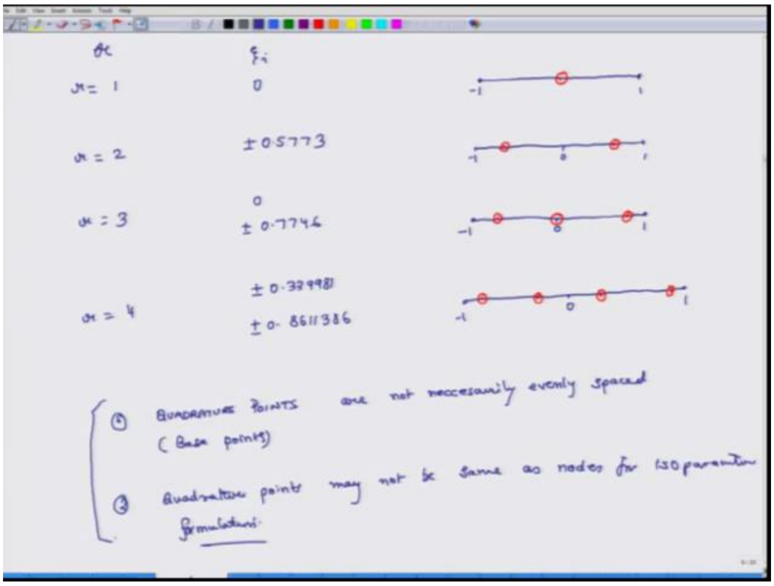

So, it is said that if r is equal to 1 then my zetas. So, these are the values of zetas. Zeta is at 0, if r is equal to 2 then zeta is equal to plus minus 0.5773. If r equals 3 then zeta equals 0 and plus minus 0.7746. I am rounding off if r is equal to 4, then it is equal to plus minus 0.339981 and plus minus 0.8611386.

Now if r is equal to 1 then this is the domain of the element minus 1 to 1 and my quadrature point is located at the center. If r equals to then again this is my minus 1 1 and this 0 and my quadrature is located at 2 quadrature points as located somewhere here a little half way mark between 0 and 1 and 0 and minus 1.

If r equals 3 then I have so this is minus 1 1 0 then I have 1 quadrature point located at center and the other 2 quadrature points are fairly close to the ends. If r equals 4, so this is minus 1 1 0, 2 quadrature points are like this plus minus 0.33 and then other 2 quadrature points are fairly closed to the extreme ends. So, the point what I am trying to say is that between the limits of 0 to minus 1 of minus 1 to 1, the quadrature points and these are also known as base points are not necessarily evenly spaced this is one.

And the second one is quadrature points may not be same as nodes for isoparametric formulations, the second point is valid only for isoparametric formulation what does it mean? In isoparametric formulation if r is equal to 2 there are 3 quadrature points and also the number of nodes will be 3. And they may not be the same the node will be at minus 1 0 and 1, but the quadrature point may not be same is if the formulation is not so that is there. So, these are the 2 important things to note.

But this Gaussian quadrature gives fairly accurate answers and it is performed in the zeta space that is minus 1 to 1 space and so it is very easy to automate. Now, there is another quadrature which is somewhat popular, but not as popular as the Gauss quadrature.

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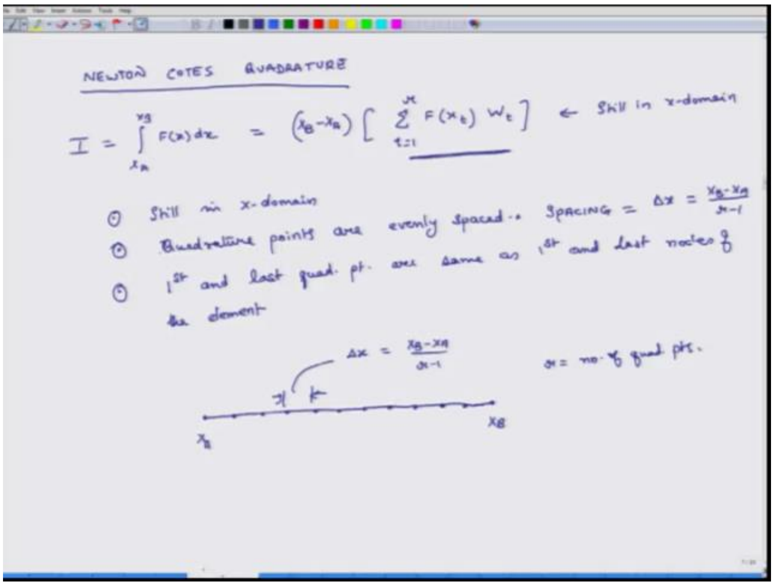

And that is known as Newton cotes quadrature. So, let us look at this relation, suppose I want to compute an integral of a function XA to XB. Then if I have to use Newton cotes quadrature formula then it is nothing, but XB minus XA times the sum of this function and thus I evaluate at quadrature point t-th quadrature times, this weight t-th weight. And t varies from 1 to R. Now note that here we are still in X domain, we did not shift to the zeta domain this is very important to understand so we are still in X domain.

So, some important features are still in X domain. Second feature is these points quadrature points are evenly spaced third first and last point are same as first and last nodes of the elements this was not true for Gauss quadrature, but here first and last nodes points are same as first and last nodes. So, here spacing between points is equal to delta X and this is equal to XB minus XA. So, suppose I have r points then how many spaces we will have r minus 1 if suppose there is r points, r quadrature points then I will have r minus 1 interval. So, divided by r minus 1 and we will through a table tell what these weights are.

So, if you have an element this is XA this is XB then you if you have a lot of 5 10 quadrature points. So, like these are all quadrature points evenly spaced. And this is the spacing between 2 quadrature points and this is equal to XB minus XA divided by r minus 1. Where r is the number of quadrature points. So next thing is that, what are the values of weights. So, for that we will give you this stable.

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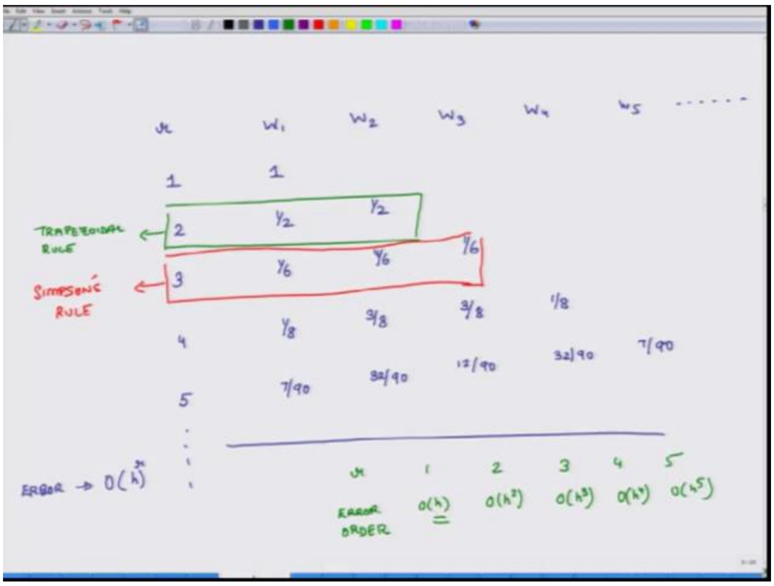

So, you have this column r and then you have weights W1, W2, W3, W4, W5, and so on and so forth. So, if r is equal to 1 then weight is 1. If r is equal to 2 then weight is half and half if r is equal to 3 then weight is 1 by 6, 4 by 6 and 1 by 6. If r is 4 then weight is 1 by 8, 3 by 8, 3 by 8 and 1 by 8. If r is 5 then it is 7 by 90, 32 by 90 oh excuse me 12 by 90, 12 by 90 and 7 by 90 and this should be 32 and so on and so forth. So, I can develop this table further down.

Now, 1 important thing to note the couple of important observations that the case r is equal to two this case this is same as trapezoidal rule. And the case r is equal to 3 this is same as Simpsons rule. Another thing is that the error in the integration error it is order is directly proportional to the bar of O it is orders directly proportional to R. So, what does that mean? So, here I will construct another table.

So, r can be 1 2 3 4 5 suppose then error order. here, the order of the error will be O to h here the order will be h square, here the order will be h cube, here the order will be h 4, here the order will be h 5 and so on and so forth.

So, what does that mean that means, h is a small; so this h is some normalized length it is not exact you know. So, it is normally non dimensional thing, but what the point is that is h is small then as I increase my r things become more and more accurate because if this h is normalized length. So, if it is less than 1 then as r increases I get more and more accurate solutions.

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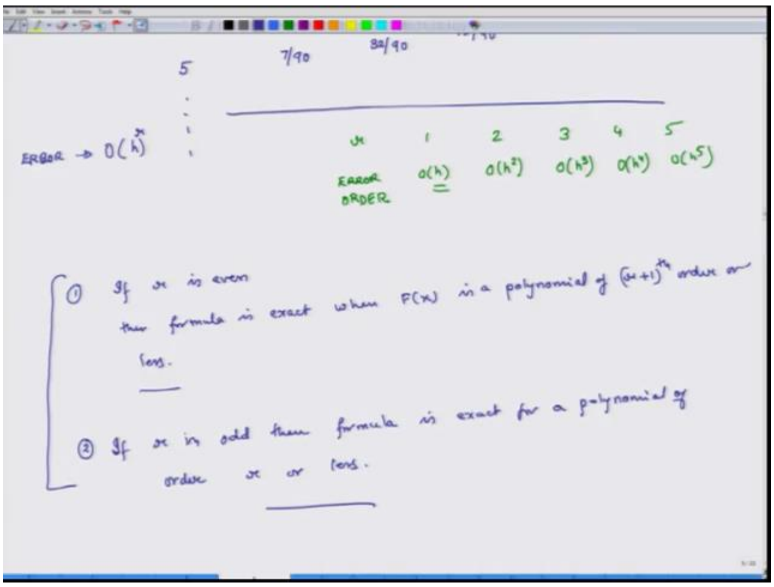

And 2 more observations 1 if r is even. When is r even? That is number of r corresponds to number of intervals or odd number of base points then formula is exact when FX is a polynomial of r plus 1th order or less and if r is odd then formula is exact when this degree. So, this I will erase this is not correct if r is odd then this quadrature is exact for a polynomial of order r or less.

So, these are 2 important things to note. So, this closes our discussion on quadrature and also today's lecture. And with this we close all the discussion on numerical integrations schemes, and from tomorrow onwards which is day 4, day 5, and day 6 we will introduce new topic which is how do we go around doing finite element analysis for 2 dimensional problems with which have a single variable. So, with that I close it for today and we will meet tomorrow.

Thank you very much.