Basics of Finite Element Analysis – Part II
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Lecture – 02
Fundamental principles

Welcome to Basics of Finite Element Analysis course, this is the MOOC course Part II. And today this is the second day of the first week of this course and what we will do today and also pretty much throughout this entire week is quick review of whatever we discussed in detail in Part I of the course which was Basics of Finite Element Analysis Part I.

So, the first thing we are going to review is get a feel of what is the fundamental principle which underlies finite element analysis. And we will start with an analogy and then we will actually develop the finite element equations and can solve it.

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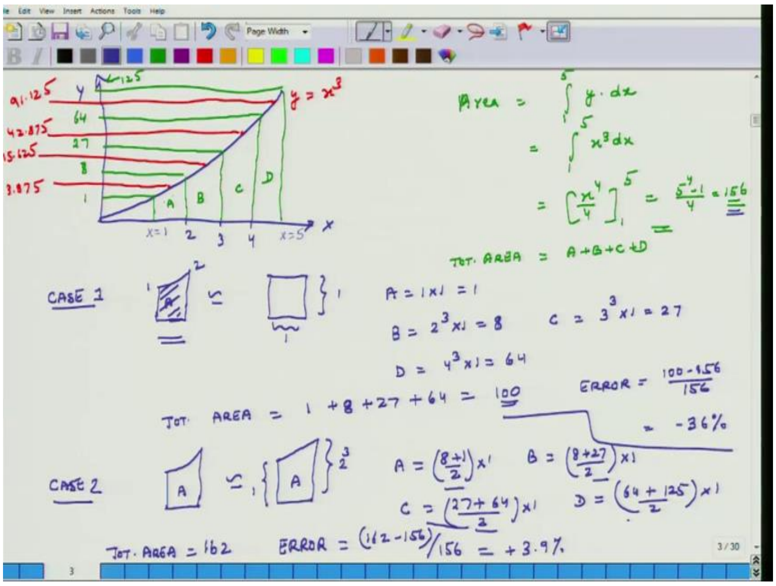

So, the analogy this is x axis and this is y, and this is suppose I have a curve let us say this curve is y equals x cube. Let us say I am interested in finding this area under the curve for the region x is equal to 1, 2 x is equal to 5. This is the area which I am interested in finding out.

Now, one way to find this area is because I know this function very nicely so I can integrate this equation, because integral of this function represent the area under this curve way is. So, area equals integral of y times d x, that is an in the limits 1 to 5. So, I am going to integrate this x cube, d x and that equals x 4 by 4 evaluated between the limits 1 to 5. So, that is equal to 5 to the power of 4 minus 1 divided by 4, and this value turns out to be 156. So, this is one approach, but if this curve was very complicated and if we cannot calculate the exact integral of this function then this approach will not be efficient.

So, the other approach is that I break this area into small pieces. So, let us say here x is equal to 2, 3, 4 and I am going to break this area into small pieces. So, what I have done is my domain is that is the range of a x is from x is equal to 1 to x is equal to 4. And I have broken my domain into four elements. First element is x equal to 1 to 2, second element is 2 to 3, third element is 3 to 4, and fourth element is 4 to 5. And, now I have four areas and I will call them area A, B, C, D, so total area is equal to A plus B plus C plus D. And this if we do our math correctly it should be close to the actually area which is 156.

So now, we are actually going to compute the values of this areas in several ways. But before we do that let us calculate the value of y. So, here when x is equal to 1 because x cube of 1 is 1, when x is equal to 2 then y equals 8, when x equals 3 then y equals 27, when x equals 64, and when x equals 5 then y equals here 125; 5 cube is 125 and what we will also do here is also evaluate the values of middle points. So, the first middle point is 1.5 cube, so that is 3.875. The second middle point is 2.5 cubes, so this is 15.625. The third middle point is 3.5 cubes, so that is 42.875 and the forth middle point is 4.5 cube which is 91.125 and we will see why we need this bit points.

Now, what we do is we start calculating the values of area A, B, C and D, and we do it in two or three different ways. So, in the first case; I will assume that this area, so this is area A is approximately equal to this area. So, what I am assuming that this is point 1 and 2. So, the variation of y between 1 and 2 is constant. I am assuming that value of y between point 1 and two is nothing but constant. This is I am assuming, in reality it is not true but I am assuming that it is constant and it is value is same as that which is at 1. So, this height is 1 and this width is 1.

So area A is equal 1, because this height is cube of 1 is 1. Similarly for area B it is 2 cube into 1 is equal to 8. Similarly area C is equal to 3 cube into 1 is equal 27 and area D is equal to 4 cube into 1 is equal to 64. So, here area A, B, C and D we have evaluated assuming that these areas are nothing but rectangles. Or in other words, the variation of the y across each element is constant. It does not vary so it is constant. In such a case total area is equal to 1 plus 8 plus 27 plus 64 and that equals 100. So, my error is equal to 100 minus 156 by 156 because the actual area is156, so this is equal to minus 36 percent. If I break the whole domain in this case into four individual units and assume that y is constant across each element then I have an error of 36 percent.

Now, I do another approximation. Case 2; here I assume that this area A is approximately is equal to this kind of a trapezoid and here the height of trapezoid on this side is 1 and the height of the trapezoid on this side is it will be 2 cube. In this case A is equal to 8 plus 1 by 2 times 1. Area B using similar method is 8 plus 27 by 2 into 1. Area C, C equals 27 plus 64 by 2 into 1 and area D is equal to 64 plus 125 by 2 into 1.

So, in the first case I am assuming that the value of y is constant along each element. In the second case I am assuming that do the value of y is linearly varying across each element. And in this second case total area, if I add up all these values I get it has 162 which means my error is equal to 162 minus 156 divided by 156, and that comes to be plus 3.9 percent.

So, what we immediately see is that because I changes the nature of function from constant to linear my errors significantly got reduced. So, this function which we assume that it is constant in first case and it is linear this is called an interpolation function. What we are seeing is that the accuracy depends on our choice of the interpolation function very strongly; we will do one more case.

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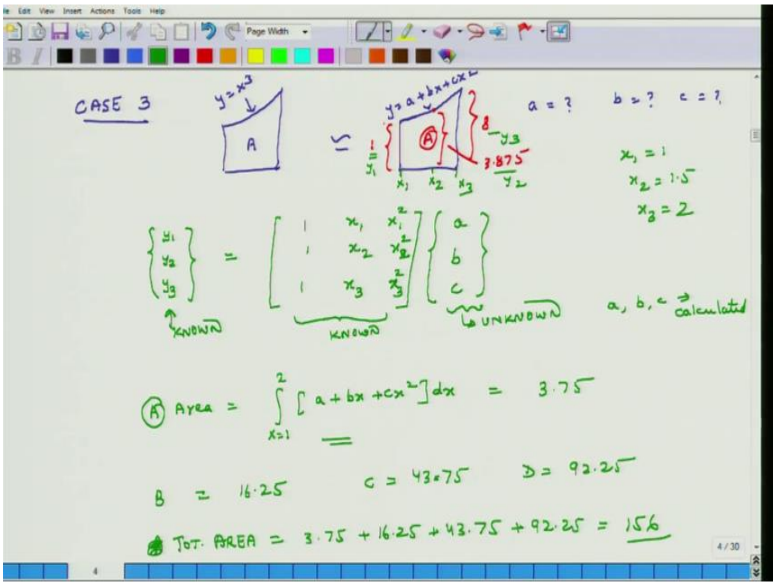

Case 3 and in this case, I assume. This is here in reality Y is equal to X cube, that is the actual function, but suppose we do not know this function then, we assume in first case we had assumed that y was constant. Second case, we were assuming that the Y is the linear function. We assume that it is not linear function, but it is having some quadratic relationship Y equals A plus B X plus C X square. We are assuming that this curve or at the top edge is varying in a quadratic manner.

We are assuming this in reality it is cubic, but here we are assuming that it is quadratic because the mathematics becomes a little simple that is all. If we do this, then we can again calculate the values of different areas, but then, this case, we have to find, what are the values of A, B and C. Unless we know A B and C, we cannot find out the area under this curve.

First we will see how to find out the area under this, the values of A B and C. At this point, we know that the value is this height, is 1, this is area A and this height is 8 or (Refer Time: 14:46) cube, but to find A B and C, we need a third condition. We should also know the value of B, this height at 1 more point and in this case, we selected the midpoint and this height we said that it is 3.875, this is 3.875. Now, we find out the values of A B and C by writing out 3 different equations. The first equations, this is Y 1, this is Y 3, Y 2 this is Y 3. We write Y 1 Y 2 Y 3. These are known.

Y 1 equals A times B times X 1 plus C times X 1 square. I can write it as this. This is my first equation Y 1 equals 1 times A plus X 1 times B plus X 1 square times C where X 1 is equal to 1 X 2 is equal to 1.5 and X 3 is equal to 2. This is X 1, this is X 2 and this is X 3, similarly Y 2 is equal to A times 1 time plus X 2 times B plus X 2 time X 2 square times C and similarly Y 3 is equal to A plus B X 3 plus C X 3 square. This Y vector is known, this is also known and this is unknown. This is these are 3 questions 3 unknowns. I can calculate, A and B can be calculated, once we know A B and C, then area A is equal to whatever A B and C. We calculated, we can calculate from X is equal to 1 to 2 A plus B X plus B X square D X in this way, we can calculate area A by integrating A B A plus B X plus C X square in the limits X is equal to 1 to 2, we can find out area A understood and if we do all this math we find that this area comes to be 3.75.

Similarly, we go to the next element, there again we assume that Y is equal to A plus B plus C X square and we again re compute A B C because our Y vector and this metrics has changed. We have to again re computing A B C and then once we re-compute A B C, we again use this type of a relation to find the second area. B is we find that area B is equal to 16.25, area C is equal to 43.75 and area D equals 92.25 and if I add up, all these areas then total area I find that it is equal to 3.75 plus 16.25 plus 43.75 plus 92.25 and that comes out to be 156. In this case, it comes out to be the exact area which was computed earlier.

From this, what we see something very significant that, the accuracy in this approach where we are breaking up the domain into smaller and smaller units it depends on 3 things.

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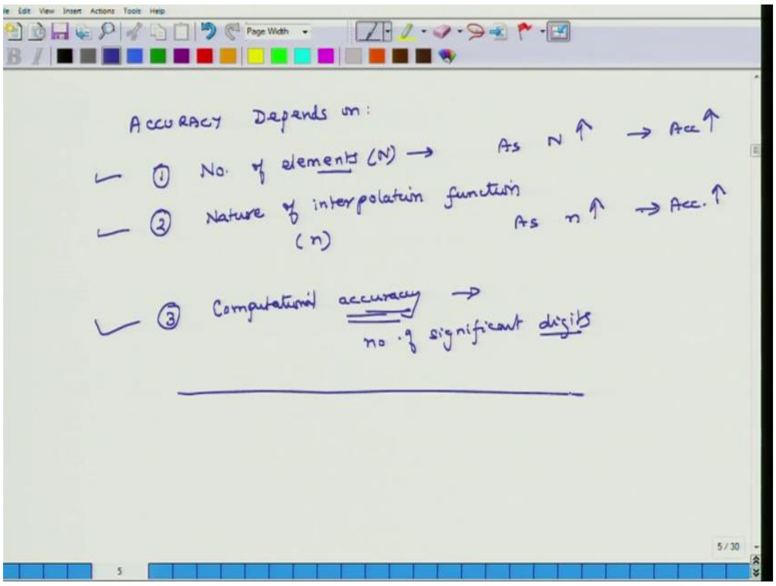

First is accuracy, depends on first thing it depends on number of elements. In this case we had developed 4 elements, if we had created 20 elements then even with the constraints Y function, things would have been much more accurate. Second thing is accuracy depends on nature of interpolation function. In the first case if, let us call this number of elements as N. As N goes up, what happens accuracy also goes up, nature of interpolation function.

This can be measured in terms of N, which is the order of polynomial. So, as N goes up, so does accuracy. This is what we saw just now and the third thing is computational accuracy and what does this mean that, suppose you are dividing 4 by 3, you can have several answers 1 would be 1. Another answer would be 1.3; another answer would be 1.33, another answer would be 1.333. It depends on how many places of decimal you are evaluating each number.

The accuracy of the overall finite element solution also depends on this computational accuracy. This can be measured as number of significant digits. As this number goes up, so does accuracy. The accuracy in a finite element solution, it depends on these 3 things, number of elements, nature of interpolation function and computational accuracy. This completes today’s lecture.

In the next lecture, we will actually start talking or reveal the mechanics of finite element method that what are the different steps and then later as we progress. We actually do 1 example that whatever we have learnt in the last course, we are able to capture in full in this course.

Thanks a lot, and we will meet tomorrow. Bye.