Basics of Finite Element Analysis – Part II
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Lecture -16
Approximations - Part II

Hello. Welcome to Basics of Finite Element Analysis Part II. Today is the fourth day of this third week of this lectures, and in the last lecture, which was yesterday we had started discussing about approximation of geometry. So, we will extend that discussion today and with that discussion, we will introduce concepts of Jacobian and the Jacobian entities and Jacobian transformation shear functions and Legragian Interpolation functions and some other related concepts. So, what we had mentioned in the last classes that in FEA, we make two important approximations.

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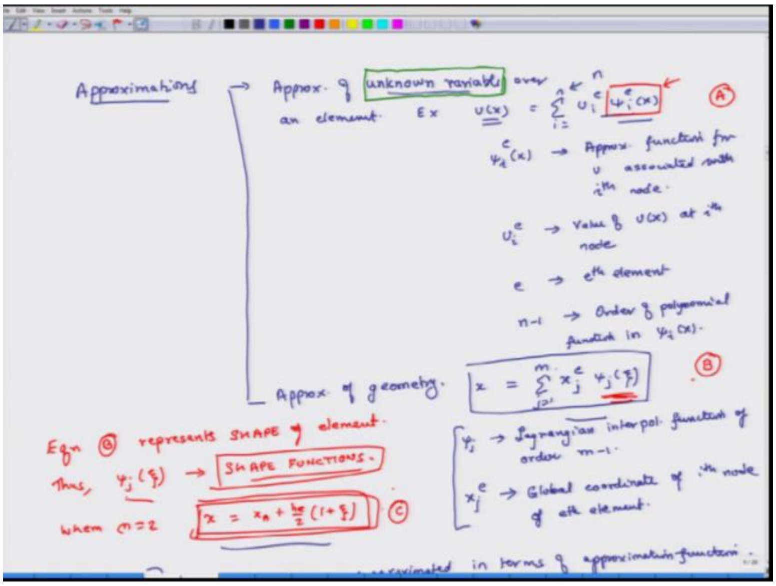

The first approximation we make, and which we have discussed again and again is, approximation of unknown variable over an element. For instance, in a bar problem there is a bar which is under tension, the unknown variable will be the displacement and this displacement is a function of x. So, so example would be that over an element u of x which is the displacement function and we say that it varies in certain, we assume something. So, we assume and we say that over an element it can be expressed as u i psi i x and this is for the eth element and i is equal to one two n.

So, this is the approximation function. So, these are approximation functions. So, here psi i is approximation function for u which is the unknown variable and associated with ith node. And then ui is value of u x at ith node and e is the eth element. And in the last thing is n minus one. It represents order of polynomial function in psi i x. So, this is one approximation which we make, which is approximation of unknown variable. This is approximation of unknown variable.

But then we make another approximation and that is approximation of geometry, because when we go from x domain to zeta domain, there is some transformation and that can be also, that transformation can also be an approximated. Transformation from x domain to zeta domain that can also be a approximated. So, what do we do there? So, there what do we do? We say that x. So, here u was a function of x, but here x itself it getting transformed. So, x is equal to summation of. So, i equals one to n and remember m is different in this here it was n here it is m. So, there is a distinction. So, this is equal to x i. So, this is for the eth element x i sub superscript e times. So, here I will do a different, it is just to differentiate, otherwise it just generates confusion.

So, instead of i I am using j. So, this jth node jth node and this i is the function of zeta. Here zeta is the natural coordinate. So, here psi j is Legrangian Interpolation function of order m minus one. So, if I am summing up m parts, then the order of the function would be m minus one. And x j is the global coordinate. Global coordinate of ith nodes. So, it is not in the natural coordinate. It is the global coordinate of ith node of eth element.

Now, let us call this equation a. Let us call this equation b. Now equation b, what does it represent? It represents the shape of the element. If I plot, it represents the shape of the element. If this is a linear element, then it will be just a straight line. The mapping will be linear if it is a quadratic, then it will be a curve right. So, equation b represents shape of element thus this psi j functions which are functions of zeta and they may not be necessarily identical to this psi i's. We can make them same, but there is no rule which says that psi j and psi r are same. I could have used actually a different symbol here.

Maybe c to make it more clear; Psi i and psi j are not same. They cannot. They need not be same. They can be, but they need not to be same. So, psi j, these functions psi j's are known as shape functions.

They are approximation functions, but because they influence the shape of the element they are also known as shape functions. If you call them approximation functions that are fine you will not be wrong, but because they influence shape of the element they are also known as shape functions. These functions are not called shape functions. These functions, psi i, because they do not influence the shape of the element, they only influence the unknown variable. So, that is why we call them interpolation functions and thus you know approximation functions. You can call these psi j's also approximation functions or interpolation functions, but they also have a special name known as shape functions right.

So, you can call these psi j's shape functions, interpolation functions, approximation functions. Psi i you can call them approximation functions or interpolation functions, but not shape functions. So, this is important to understand. Now I will once again say when n is equal to two, then x is equal to x a plus h e by two one plus zeta. So, in this case and this equation become similar they become same. Equation c and b become same, but when m is three this equation is not valid. This equation is valid, but then you have to construct those different shape functions using this formula, and plug-in there and you will get this relation between x and psi or x and zeta sorry, x and zeta. So, this is important to understand.

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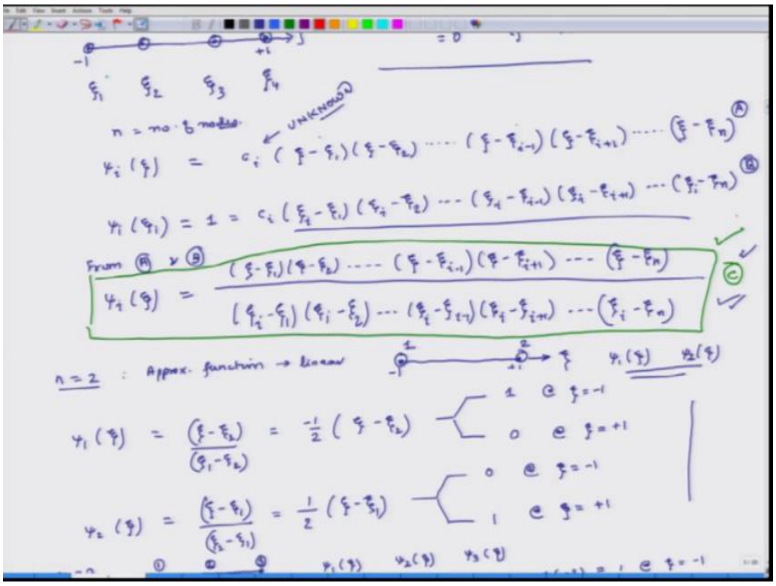

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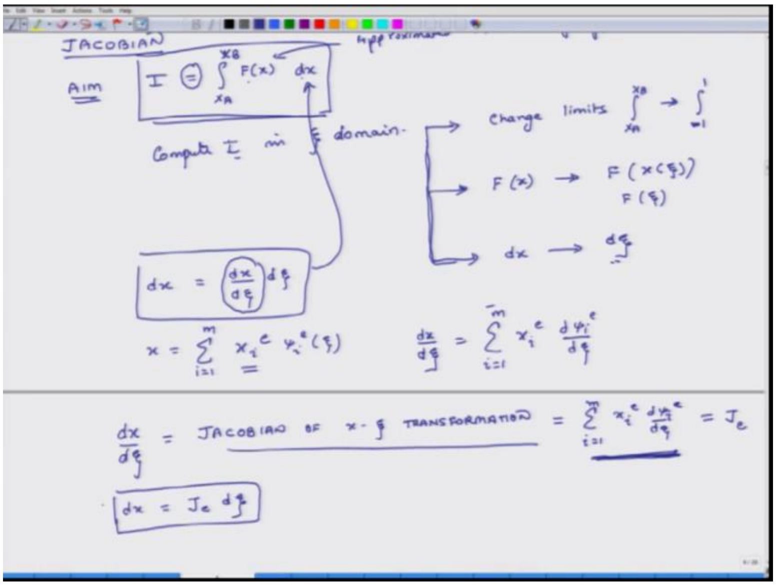

Now, we introduce the system called Jacobian. So, I will go back to our original aim. What is our original aim? To find out the integral, so our aim i is equal to F of x dx xA to xB and because we are going to do an approximate integral, strictly speaking, this equal to sign should be replaced by an approximation sign because I am going to approximate. See this is exact, but this will be approximated as F of zeta. I am sorry; it will approximate as in terms of some interpolation functions right. So, the moment, I this was exact, but if I approximate in terms of some approximation functions, then I becomes an approximate number.

Now, our aim is, original aim is to do this integration, but we have said that we will rather integrated to what limits? Minus one to one, so we will do so aim is to do this integral, but compute this integral i in zeta domain. So, to do this, we have to do three things. We have to change limits. Whenever, we do a transformation, we do three things. We have to change limits. So, x A to x B becomes minus one to one.

The second thing we do is, initially we have fx and we have to change it into f of zeta. So, we transform it into F of x, which is a function of zeta or I can call it F of zeta. And then, right now, I am integrating with respect to dx. Somehow, I have to change dx to d zeta. So, I have to do these changes and then I will be able to compute i in the natural coordinate system. So, what is the relationship between d x and d zeta? How do I transform that? dx is equal to dx over d zeta times d zeta.

Physically, what it means? What does dx mean physically? It is a, dx represents, a small change in x. df means what? A small change in f. dy means a small change in y. How small? Extremely small, infinite abysmally a small change, so dx means position is changing by very small amount in x domain because x represents position right. So, dx means an absolutely small infinite as well as small change in x. That small change in x, what was it? I can divide it by any number. So, I am dividing it by d zeta which is a small change in zeta and I multiplying it by small change in zeta. That is what it means physically right. So, this is what it is.

Now, we have to calculate this thing if I have to change the integration parameter dx to d zeta. This is going to come in right. So, this has to go into this thing and once I fit this relation here, I should know what the value dx is over d zeta or what is the relationship between for dx and d zeta? Now we know that x is equal to. In general, see for a two nodded element, the mapping was this thing, equation c. But general mapping is this thing. This is the general mapping. This integration; this is the general mapping. This is for two nodded element only. So, we will use a general mapping scheme. So, we know that x is sum of x i e psi i and psi is a function of zeta and I am as summing it over a, in this i equals one to m, m, yes.

So, what does this mean dx over d zeta equals what are x size? They are constants, right? They are constants. So, when I do that differentiation, these are constants. These are the values of x at specific nodes. They are constants so they do not change. So, when I do the differentiation with respect to zeta, they do no influence the differential, right. So, this is equal to sum of x i for the eth element d psi i e and psi i is a function of zeta, i equals one two m, hm. Now this term dx by d zeta is called Jacobian of x zeta transformation called Jacobian of x zeta transformation and it can be calculated as this thing. This is how we can calculate. This is the Jacobian of x to zeta transformation and you can actually calculate this.

In this expression is there anything which you do not know? Do you know psis? Do you know psi functions? Psi functions do you know or you do not know? We can calculate psi from these relations. So, if this is a three nodded element, we know psi's from here right. So, psi's are known what about x i's? X i's are coordinates of nodes in the x system. So, if there is a three nodded elements, then I know what are it is coordinates.

So, I know xi's and I know psi. So, I can calculate d psi over to d zeta and multiplied by xi, add them up and using this approach I can calculate dx over d zeta which is the Jacobian of x to zeta transformation. So, it is important to understand. Here in this class, we will write it as j and because it is for eth element we can also call it j subscript e. So, d x is equal to je times d zeta.

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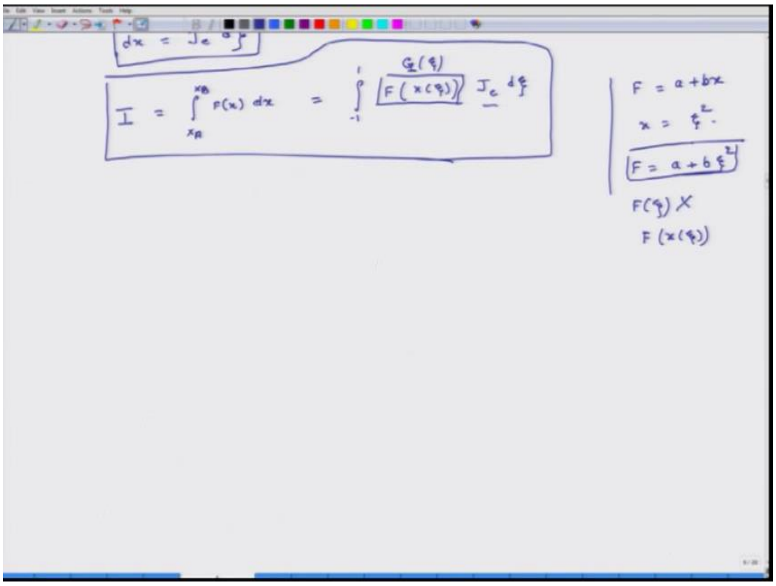

So, our original goal is to do this integration. So, now, I will plug in. Instead of dx, I will plug in the value of d zeta. So, i integral from Xa to xB f of x dx is same as minus 1 to 1; F of x which is a function of zeta times dx and dx's je times d zeta. This, I can also write it as some other function - g of zeta. But g and f are not same; f, I will give you an example. Let say F is equal to a plus bx and x is equal to zeta square. X is equal to zeta x square. Now this may not be valid in an interpolation function but I am just trying to illustrate.

So, what does this mean? F is equal to a plus b time's zeta square right. So, the nature of function has changed when I am expressing it in terms of zeta, so I cannot write this as F of zeta. This is wrong; I can write it as some other function - G of zeta. So, that is why I have written g of zeta but I can write it as F of x which is a function of zeta this is ok, but I cannot write f directly f of zeta that I cannot do.

So, I think earlier also this is, should be G. So now, in this integral if I know the expression for G, I already or know how to calculate je, then I can integrate it in the natural coordinate system and my answer will not change. My answer will not change. So, now, we have developed a methodology to integrate any function, in the domain x domain from xA to xB. I can do the exact integration in natural coordinate system between limits minus one to one, if I have knowledge about je and if I can correctly calculate this function G.

So, this concludes the discussion for today. We will continue this discussion on Jacobian's and numerical integration tomorrow.

Thank you.