Basics of Finite Element Analysis – Part II
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Lecture – 12
Convergence
(Part - II)

Hello. Welcome to Basics of Finite Element Analysis Part II. Today is the last class of the current week, which is the second week of this course. In last class we had shown that energy convergence from the top in a very large class of problems when we use finite element method and this freshly true for linear and conservative systems, it is all true for systems which are non-linear in nature and they are also non-linear but conservative.

So, this is I have very important conclusion that energy con convergence happens from the top. So, finite element solutions tend to over predict the overall energy of the system compared to the actual energy which is present in the system.

Today we will extend this discussion further and we will understand this concept in couple of other context. So, will. So, we will be extending this discussion further.

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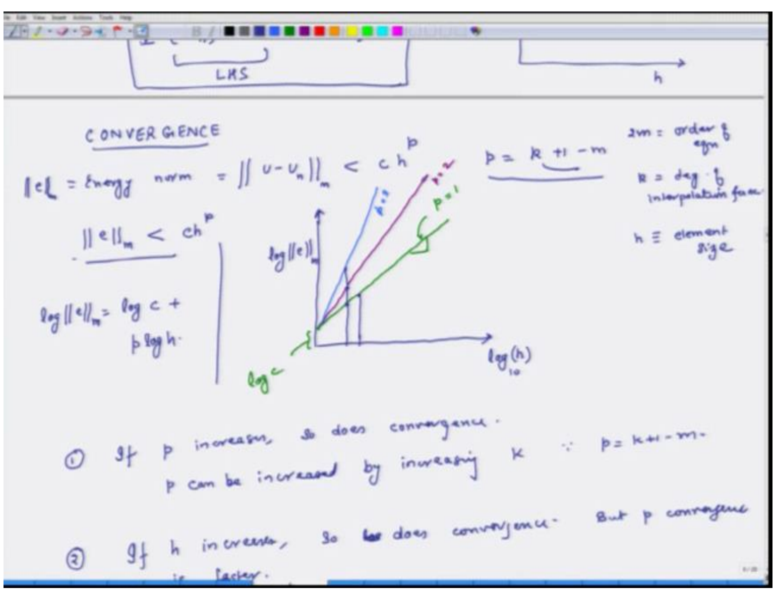

So, the theme of the discussion still remains convergence. Now earlier we had shown that energy convergence right or energy norm. Energy norm and we had designated as express designated rate like this u minus u h m.

So, this is the energy norm and this is less then c h p and I can also call this e m double bar both sides. So, this is the energy norm and we had said that p depends on interpolation order and also the degree of equation right.

So, now I will give you expression for P. So, p is equal to k plus 1 minus m; how did I get this number? This would base on several theoretical studies. So, if you want to understand the theoretical principle underline this definition of p you will have to go and do some literatures away.

So, this is this here 2m is the order of equation. And k represents the degree of interpolation function ok. So, what this? So, E energy norm is less then c h p and C is a constant we have discuss this earlier

So, if I plot this e n on a large scale and of course, h it relates to element size. So, on the x axis I am going to plot log of h based and on the y axis I am going to plot log of e m.

So, what will it look like if I take log of both sides of this equation of this equation, I get log of log C plus h log excuse me p log h. So, my lines we look something like this right. This number will be log c and the slope will be p, if my p is equal to 1. So, let us say this is p is equal to 1, then the slope will be 45 degrees right on a log scale. If p is equal to 2 then slope will be sharper, but the y intercept will still be the same which will be at log C. So, what does this show? So, if p is equal to 3 we can make 1 more line. Slope is even steeper. So, we make some very important observations.

First if p increases so does convergence right. If p increases, so does convergence what does it mean. I mean here convergence happening faster it has must faster phase. Here, p is less it is 2. So, it is somewhere slower convergence here p is 1 it is a slowest convergence correct. And, we can increase p. p can be increased by increasing k because p is equal to K plus 1 minus m.

And we cannot change m is the order of the differential equation that we cannot change that is the nature of that is that depends on the physical system so that we cannot change. So, we can increase p by increasing k second observation if h increases so does convergence, but it depends on, you know but p convergence is faster. Because here, we are changing the slope by increasing the p in case of h we are just moving from here to here.

But when I increase the slope so if I increase let us say h it increases from here to here right if I increase p this changes much larger. If I increase p further it becomes even more vertical. If I increase p is further in that keeps on growing much larger.

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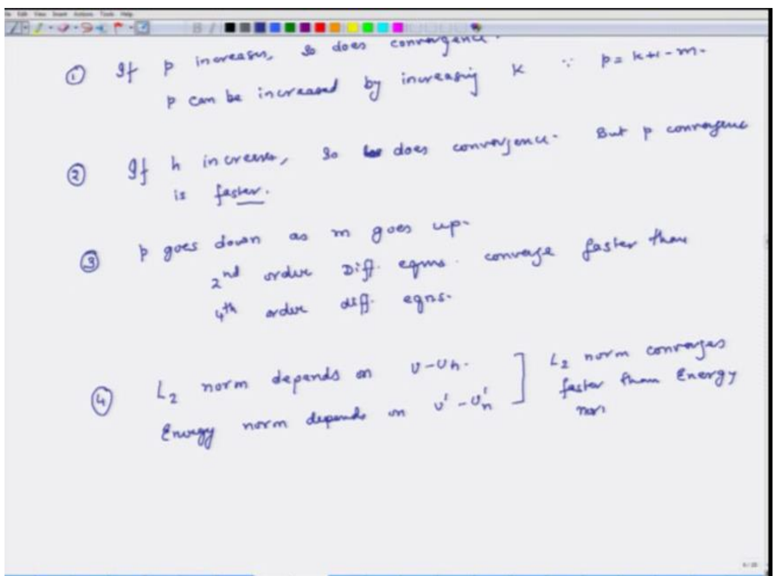

So, p convergence is faster. Third p goes down as m goes up. So, second order differential equations converge faster than fourth order differential equations. Agree because for fourth differential equation for the same value of k for same value of kp will be smaller.

So, forth order so the equation for being will convergence slowly compared to equation for the bar because there second order equation. Fourth L2 norm depends on u minus un. And energy norm depends on u prime minus uh prime right. That is how we have defined it energy norm. The energy norm is dependent on the derivatives and the L2 norm is dependent on the displacement themselves ok

Now, and given the fact that p is equal to k plus 1 minus m. m is what the order of differential equation. Now, if I am computing energy norm the order of differential equation may be same, but essentially I am computing the norm on derivatives.

In differential equations also if m goes up what is happening we are computing higher order derivatives right. If m is less we are computing lower order derivatives in energy norm we are computing the derivative in L2 norm we are not computing the derivative, but the original variable.

So, with that understanding L2 norm convergence faster or slower faster than energy norm this is very important to understand.

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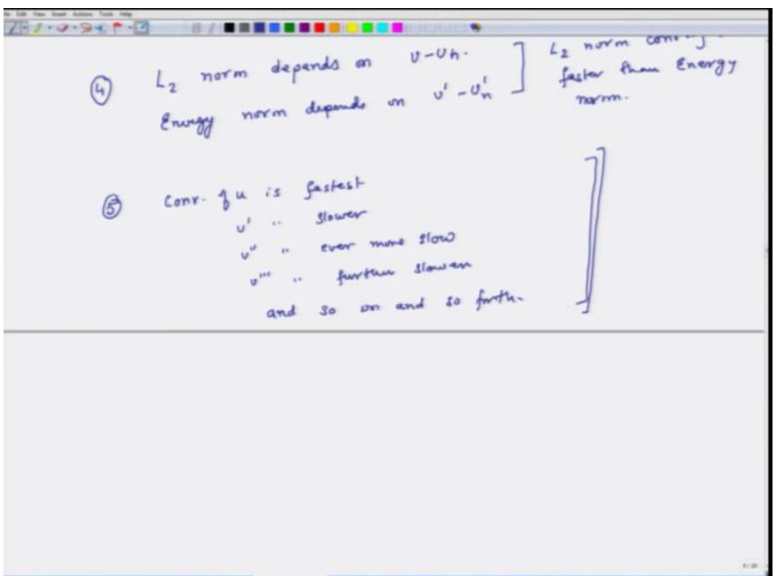

So, the final conclusion is convergence of u is fastest of u prime is slower of u double prime is even more slow of u triple prime is further slower and so on and so forth. So, in any appear problem if you are just interested in the displacements the prime we variables they will converge very fast.

But, you are interested in strains even though the displacement may have converged there is strains may not have converge because they are dependent on what the derivatives of you if you are interested in bending movement. What is bending movement E times I times second order derivative of W that will converge even slower.

So, it is very important to understand this that if you are interested in derivatives and higher order derivatives. Then maybe you have to make even more fine mesh, because even though displacement may have converged that does not mean that derivatives have converged. So, this is very important to understand.

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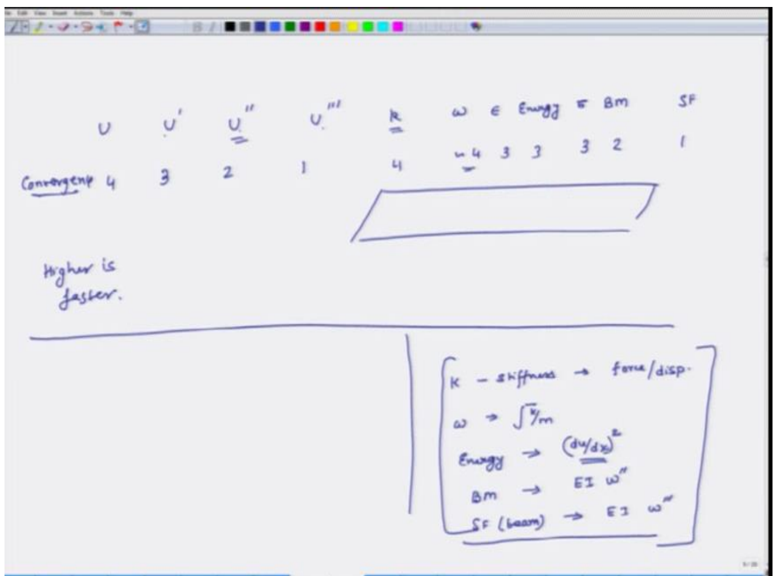

So, with this understanding we will just make a small table. So, suppose so you does not necessarily mean displacement only in XL direction here I am talking u in general it is displacement or primary variable this is it is first derivative this is it is second derivative this is it is third derivative right.

Let us look at k. k is the stiffness. So, this is force by displacement right. Then look at omega national frequency this is it is what k over m. Then let us look at energy it is related to du over dx square. Square of strain then look at bending moment it is related to el w prime. Second order derivative of w and then let us look at shear force in a in a beam. What is shear force in a beam? It is related to third order derivative right.

Let us say that convergence so amongst u. U prime, u double prime, u triple prime which one get be fastest u prime is fastest. So, I give it on number 5 some number this will be lower I give it a number 4 I am not saying that this will be in this proportion this some ranking. This will be even slower. So, I give it a number 3 actually I will make it 4 3 2 1. So, this is convergence higher is better is faster. So, 4 is highest so it you converge fastest. Now let us look at stiffness.

Stiffness depends on displacement. So, maybe it will be somewhere close to 4 omegas it depends on displacement, but square route of displacements. So, may be somewhere around 4 I do not know a square route whether it will converge faster or slower trebly faster. If I square route to the difference of you get amplified right 99 and 100. 1 percent error and 99 square and 100 square the error have gone up or errors gone down. So, you can check that similarly, 99 square root and 100 square route what it will be close to 4. Energy it is related to derivatives so it will be slower. Bending movement it is related to second order derivative.

So, it will be even further slow and shear force it will be slowest. So, this is something very important which we have discussed. So, the point of all these exercises that when you are engaged in conducting finite element analysis first thing is you have to make sure that your results are converged and when I said that your results are converged it is not a general statement you have to look at which entities you are interested in.

What are the entities in which you are interested in terms of their convergence? If you are interested in convergence of bending movements then we have to deeper you know because even though you may be the primary displacements may be converged that may not necessarily ensure convergence of bending movement right. I forget to add 2 more things. So, strain. Strain is dependent on derivatives. So, 3 and stress it is E times strain. So, again it is 3. So, based on what you need are you have to figure out how much convergence you want and you have to exactly investigate whether your solution has converged in the context of what your actual need is.

And based on this understanding you will have to do more convergence studies if your result entities involve derivatives of primarily variables and the higher you go in terms of their derivatives the more the higher the number of elements or the higher is the value of p have to ensure that the results have converged.

With this we close the discussion for today, and also for this week. We will start a new topic in the next week and till then thank you very much, and have a great weekend bye.