Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course title
Manufacturing Technology- Part-2

Module-44
Trescas' Yield criteria & Rolling Process

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Hello and welcome to this manufacturing process technology part 2 module 44.

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We were discussing about the Thresh cause criteria and we were also discussing about how to find out the principal stress principal plane angle and the maximum shear stress angle and in context of that we had formulated you know several expressions where you know we found out that the principal plane will be at an angle.

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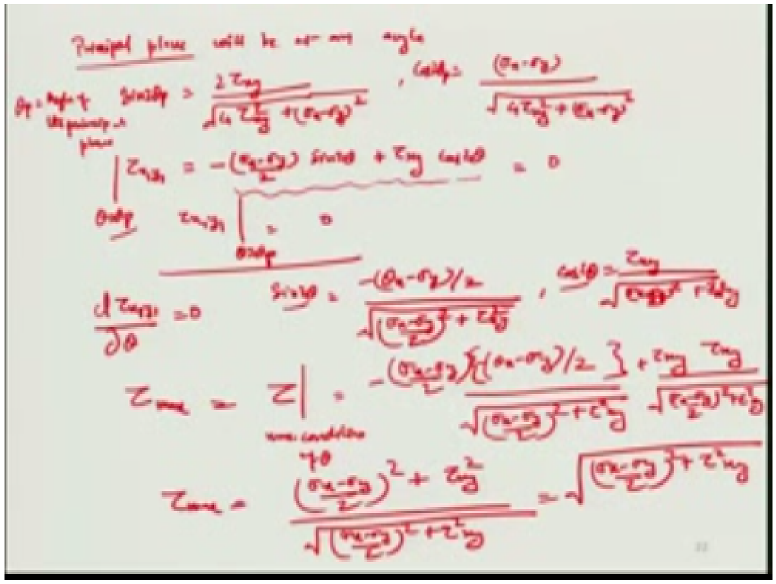

Let me just write this down here principal plane will be at an angle which is actually given by sin 2θp equals twice τ xy/√4τxi ^{2} +(σx-σy)^2 and similarly cause of 2θp should be equal to σx -σy /√4 τ xy^2 +(σx –σy)^2 so having said that this let me just write this down properly cos 2θ P so this is about the principle plane angle θ P so θ P is the angle of the principle plane our learning was that further the sorry error on writing here okay.

So our learning was here that at this particular plane if we wanted to evaluate the maximum shear stress which is which is given by τ x_1y_1 equals - σx -σy / 2 sin (2θ + τ xy), cos (2θ) this actually comes out to be if we put this θ = θ P and try to find out this actually comes out to be 0 okay so this comes out to be 0 so there is actually at the principal plane no shear stress or the τ x_1y_1 component at θ = θ P applies to be equal to 0.

So we will use this criteria to sort of try to determine what is the maximum shear stress we already know that the maximum shear stress is corresponding to again you know differentiating this ∂ τ x_1 y $_1$ /∂ θ = 0 and from there we evaluate and we find out an expression or a condition δ or sin (2θ) =(- σx- σy / 2) / √ σ x- σ y/2)^2 +τ xy ^{2} and similarly cos (2θ) as τ xy / √ σx- σy/)^2+ τ x^{2}y so if you put this values of sin (2θ) and cos (2θ) back into this expression here.

Which is actually meant for determining the maximum shear stress so we can get τ maximum let us say just putting it this way that the τ maximum here should be equal to the value of the τ corresponding to you know if we just substitute these maximum conditions of θ okay so that should be by=- σx- σy /twice sin θ which is again you know =(- σx- σy/2 so this goes in bracket

$(-\sigma x- \sigma y / 2) / \sqrt{} \sigma x- \sigma y/2)^2 + \tau xy^2$ and plus τ XY times of again τ xy divided by the same term here $(-\sigma x- \sigma y / 2) / \sqrt{} \sigma x- \sigma y/2)^2 + \tau xy^2$.

So we can find out the τ max or the maximum shear stress to be equal to $(-\sigma x- \sigma y / 2) / \sqrt{} \sigma x- \sigma y/2)^2 + \tau xy^2 / \sqrt{} \sqrt{} \sigma x- \sigma y/2)^2 + \tau xy^2$ which is actually again under $\sqrt{} \sigma x- \sigma y/2)^2 + \tau xy^2$.

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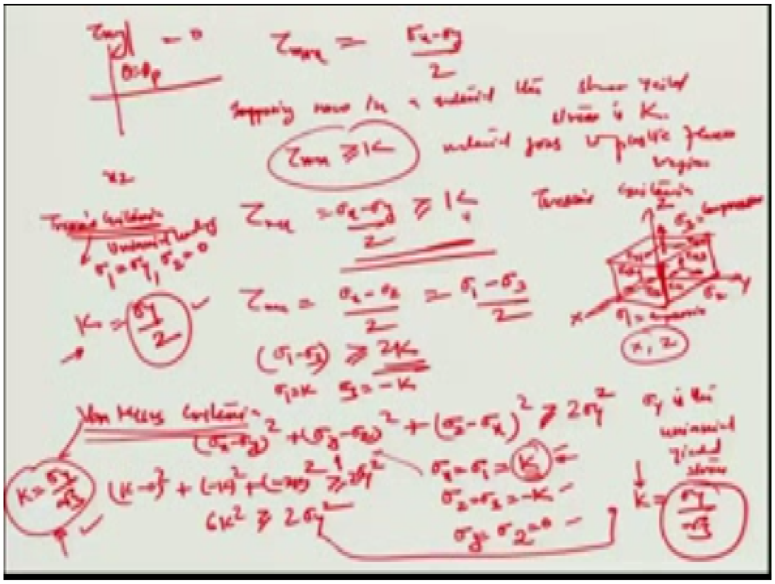

so we already are aware that the T XY at the principal plane corresponding to = θBP should be equal to 0 and if we substitute this value back into the expression for τ_{max} we get τ_{max} equals you know σx- σy / 2 so and this basically would be defined as the limiting value of the shear yield stress k so supposing now in a material the shear yield stress is recorded as k then obviously if τ_{max} >= K the material will go in plastic flow so material goes to plastic flow range of plastic flow region okay.

So in this particular expression therefore the τ_{max} = σx- σy / 2 should be greater than or equal to K and that is what is the Thresh cause criteria for plastic deformation of plastic flow plus cos criteria now if we apply to specific forming problem which is normally a prince plane strain kind of plane strain kind of problem let us look at this you know element of a material it is a cubic element of a material and you know we have various states of stresses in this particular element.

Let us call this x y and z so there are the three principal stresses and associated also shear stresses corresponding to all these three directions let us call this σ 1direction σ2 and σ3and let us also call equivalent you know shear stress directions as well as sorry this is this particular direction so you have here τ_{12} τ_{13}similarly there are two shear stresses in the principle plane to which is τ_{21} in the σ2 direction τ_{21} and τ_{23} so τ_{21} τ_{23} and similarly you have τ_{31} and τ_{32} in the Z direction.

So these are the total state of stressors and if I may so recall that any forming processors let us say for example if you are considering only the rolling process the states of stress which really matter are the XZ plane you know if we look at it in that manner and why typically the dimension is so big that there is hardly any for mobility or that is hardly any deformation or any let us say strains which are of considerable nature in the in the third Y direction.

So in all these cases we would really consider rather than the XY plane the XZ plane and so τ_{max} in this case would actually happen to be σ x- σ y/2 okay and along the principal direction and so therefore this can be recorded in any event of σ1- σ2 / 2now if supposing you know if we wanted to consider the plus cos criteria we should actually have the σ 1 σ3 component at least greater than equal to twice the shear yield stress of the material for the material to come in the plastic range in a similar manner earlier.

If you may recall we had pointed out about how using the strain energy per unit volume or work of deformation concept we had arrived at the one versus. criteria so the one versus criteria also we had a relationship between $\sigma x- \sigma y^2+ \sigma y- \sigma z^2 + \sigma z- \sigma x^2$ to be greater than or equal to you know at least $\sigma y^2$where σy is the uniaxial yield strength uniaxial yield strength okay so this is actually Y let us not confuse it with this Y right here.
So this is Y the axial yield stress of the material okay and so if we wanted to follow the same principle here where let us say you know generally in a farming process the σx becomes σ1 and let us say this is equal to some value K corresponding to which the material will come in plastic flow or this is actually the you can say the constant material property flow property which is also the yield property of the material and similarly we have σz= σ3=-K.

Obviously for example if we consider a rolling process let us suppose this Y will be compressive in nature while this σ1 would be expansive for the you know the material to get strained along the jet on the x axis so that rolling can happen okay so having said that now in the plane strain case we will not consider any kind of stress along the Y direction because obviously the dimension is too big for any marginal strain to be recorded in this particular direction.

So we can k call $\sigma y = \sigma 2$ in this particular case to be equal to 0 okay so having said that if I calculate the criteria corresponding to which the one versus. would actually yield or the one versus criteria would hold or plastic flow would actually start happening we substitute all these values back into this equation here we have $K- 0^2+(-k^2+(-2k^2))>=2\sigma$ yield stress square or in other words $6k^2 >=2\sigma y^2$ which gives you an overall you know sort of condition that K should be at least equal to $\sigma y/ \sqrt{3}4$ plastic flow to happen.

So that is what one versus criteria says that the maximum value of K should be at least equal to the yield value divided by root of three for the plastic flow to happen in the trespass criteria or the thresh cause if we apply the same formulation we have this shear so in the plane strain case you simply can have that you know one of these $\sigma 1$ are equal to let us say the uniaxial stress σy okay.

And you could actually have this you know $\sigma 2$ to be equal to 0 in uniaxial loading and you could have the K to be equal to $\sigma Y/2$ okay for the plastic flow to happen so the criteria are completely different in terms of what would be the that critical limit beyond which the plastic flow would take place and one mrs criteria is slightly better than the Thresh cause criteria it is more accurate and so we will basically now apply these criteria both this criteria to real you know forming processes like rolling or forging processes and try to see how plastic flow can be initiated and what is going to be the mechanics of such process so let us now apply some of these yield criteria is to real processes metal forming processes and in context of that the first process that I will start with is rolling process okay.

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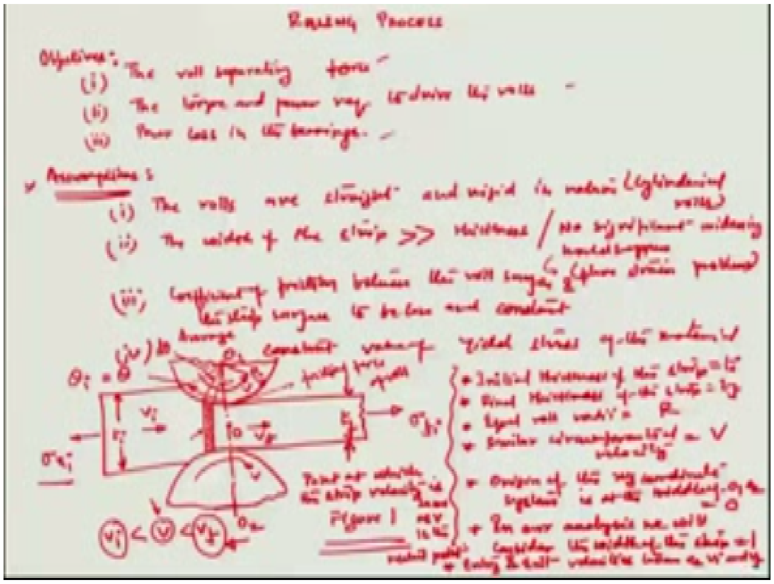

So we will just do some modeling related to rolling try to find out what are the kind of forces which are involved in doing rolling process and we can also probably talk about some mechanics which is involved you know for obtaining the pressures which are there at the rolls or for instance the driving torque which is needed for driving the rolls so that rolling may happen so all these different models can be represented here.

So the basic objective of the analysis here is to really determine number one the role separating force hmm number two the torque and power required to drive the rolls and maybe the power loss that is therein the bearings we would try to make some assumptions and this is normally done for simplifying the analysis or simplifying the real situation you know actually the situation would be quite different but at least you know from first principles can we arrive at a model where we can estimate all these things related or all these parameters related to the rolling process is going to be the scope of the teaching here in this particular lecture.

So we assume that the roles are straight and rigid in nature the cylindrical roles okay we also assume in our analysis that the width of the strip is much larger than the thickness and we further assume that no significant widening would happen on the width side because of the rolling process on we just wanted to so basically we are just converting this or simplifying this into a plane strain problem.

And because now it is only about the change in the thickness reservist the change in the length the width is so large that it is sort of neglected quite a bit in you know all the operations related to the rolling process further it is important to assume the coefficient of friction to be low and for simplicity sake although obviously friction would depend on the temperature to some extent but in this case we assume the friction to be constant okay.

So we assume the coefficient of friction mu between the roll surface and the strip surface to below and constant further we assume that the yield stress of the material remains constant for the entire operation we are not having a changed yield stress because of strain hardening or other issues and we would just like to have average constant value of yield stress of the material now these are presumptions which actually in the reality may have to be significantly different.

Because of the fact that you know there is going to be a strain hardening effect and there is going to be a change in the right area sorry not the criteria but the ultimate yield strength of the material because of you know strain hardening and so on so forth but these we simply assumed to be negligible in our model and with these assumptions if we start the model let me just draw how this rolling app operation is happening or as a matter of fact what are the different kind of stresses which are there on the material element which is being for seed here.

So let us assume that we have a metal strip which is being changed cross sectionally by two row set of roles and it changes its thickness of the material this is the sort of distance between both the roles let me just draw it little more appropriately so that there is a change in the thickness of the material emanating from this particular portion and I would assume certain parameters here let us say the final thickness of the sheet that is coming out of the roles is TF the initial thickness which was there right at the beginning of the process is it.

Obviously there is going to be a velocity at which this sheet is being fed let us consider that velocity to be VI this is TI write it down a more appropriate manner velocity here is VI we have a final velocity somewhere here VF there is a velocity of the roles both roles are moving at a velocity V and obviously the two centers around which this role is moving are represented by O_1 and O_2 so this right here let me just pull this term out here and write it's lightly over here average.

So this is 01 and this is 0 2 similarly we also have an angle which is defined by the start of the rolling process all the way to the center here and specifically we are talking about this angle right here which is actually θ so let me just write this out here as angle θ okay similarly we estimate or we just write down the radius of the rolls here as our and further we assume that there is a small strip where we want to really obtain the mechanics of all the forces or the interplay between all the forces in this rolling process.

I am representing this step with a shaded region and let us suppose there is an angle Dθ from the center of the rolls over which this step is being considered so I would like to consider the angle here between the dimensions of the shaded step and I would consider this angle as dθ okay further I will consider the angle θ to be equal to the initial angle θi okay because that is what the initial angle is at the instance of time T =0 of the rolling process.

We are assuming the process to be in a manner that rolling is already happening and we would look into such a formulated region right here and try to investigate the balance of forces in this particular region obviously we are having stresses in this particular metal role in both these directions this is the reaction stress σxi. I can call which is trying to push the thicker material away from the roles in the reverse direction.

Whereas this is the forward stress σfI which is trying to pull the material into the thinner zone okay so they are opposite to each other and obviously the balance is being played by the frictional force which is actually frictional force of the role which creates a situation which drives the thicker sheet forward okay and therefore subjects it to stresses and strains which changes the sectional area of the sheet.

So having said that now we have in general written down so I will just write down all the geometric details that I have planned in this figure let us call this figure1 okay so the first thing i have written down here is that you know the initial thickness of the strip is going to be TI the final thickness which it gets rolled into of this trip is going to be TF we assume equal role radii of R and similar circumferential velocity to be equal to V further the origin of the coordinate system.

So let me just mark all these geometric details by different stars so we assume also the origin of the coordinate system at the midpoint of the line joining centers O_1 and O_2 so somewhere around here maybe at the very middle if I were to extend this middle so this is the point 0 so origin of the XY coordinate system which would be used for doing our analysis Force analysis coordinate system is at the middle of O_1O_2 and we call this oh right.

So this is the origin O in our particular analysis we will consider the width of the strip to be unity to one and why we do that is that because obviously we do not have any influence in the direction outside the page okay so therefore because it is a plane strain case and so therefore it is you know per unit width we can actually report whatever we are doing on this so-called xz plane and obviously the entry and exit velocities of the strip which are taken to be VI and VF.

So I will write this down here further the entry and exit velocities taken as VI and VF respectively and in actual practice I would like to assume this in a manner that the VI is obviously lesser than VF we assume that V is really the role velocity which is somewhere along the point of contact of the sheet metal with respect to the role you have to understand that the role at some point of time is in direct proximity or indirect touch and kind of flows the material at the same velocity as its own circumferential velocity V.

And so therefore you know the VI is obviously lesser than V is obviously more than F less than less than V or V F is greater than v is greater than VI because there is a change in the thickness of the material and the thickness is the change in the thickness is always in the forward direction so therefore the final velocity of exit of the heat is the initial velocity plus the velocity at which the metal is getting formulated and therefore VF is greater than VI and VIII is obviously somewhere in the center.

So V has to be greater than VI and less than V F so the point where the velocity of the strip is equal to the role velocity we will refer to that point as the neutral point okay so the point at which the strip velocity is same as V the role velocity is the neutral point so obviously now we will do all our analysis in context of all these different criteria is which has been laid down and I will close this module here but in the next module we will look at the various forces the various you know the pressures which are generated the power which is consumed at the Bering etc in one crisp form by assuming all these simplifications and going ahead with modeling the rolling certainly until then goodbye thank you so much for being you.

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