Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology-Part-2

Module-43
Concept of Principal stress, strain

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Hello and welcome to this manufacturing process technology part 2 module 43. We just discussed in the last module about the one misses criteria which is based on the work of deformation. And what it assume is that the strain energy generated per in volume should be above a certain critical energy for the yielding to happen okay. Now this criteria came out in 1913.

But there was a criteria earlier to that which in fair and I should have done earlier, but because of this, because of the sequencing of the lectures it was like this. And way back in 1864 which is also known as the Tresca criteria. So we will try to now analyze that and it is based on sheared stress basically okay.

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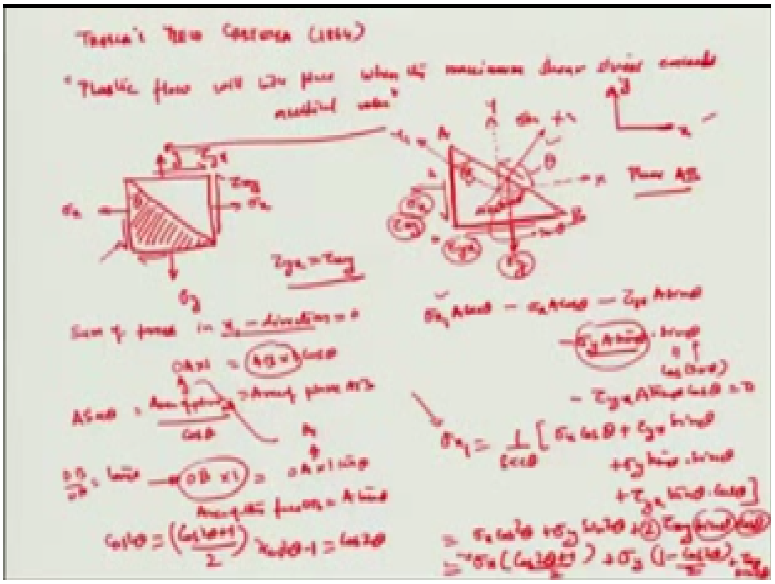

So we will call this Tresca yield criteria and I will first write down what the criteria is and then we will try to see how the proof exist or how we can actually find out the maximum shear stresses etc, particularly in a system of stresses which has both shear as well as principle stress is acting together. So this was in 1864 okay. The criteria says that the plastic flow will take place when the maximum shear stress exceeds a critical value.

So here this is actually more like the shear stress model earlier it was the energy model. So the energy model really yields a more accurate destination I would say of the shear, you know maximum shear stress in a comparison to the discussed criteria. But still it is important to have a look at what is this maximum shear stress criteria and for that we need to understand certain things related to how, you know in a multi axial state of stress when there are sheared stressors and there are principle stressors.

What are the kind of issues which are involved in a block elements. And let us consider the block element here, where there are principle stressors as well as sheared stressors. So let us say it is a plankston problem that we are wanting to evaluate. And we have the principle stressors σ_x and σ_y and also the sheared stressors yx and τ_{xy} and similarly the balancing stressors on the other side of the block here.

We consider a plane along the angle θ represented by the diagonal with respect to one of the sides where we try to evaluate what is the effective stress which is there. So here the goal really in this whole block element is to be able to find what is going to be the maximum principle stress on some any one plane at a certain angle θ and what is going to be the maximum shear stress actually. So our final goal in the discussed criteria is to have an estimate of when will the plastic flow start by looking at that maximum sheared stressor.

So because of all the different principle stresses and the shear stresses which exist in the system there has to be a certain plane at a certain angle θ where the shear stress would be maximum. And so, we will try to obtain that plane mathematically and try to find what is the maximum value of the sheared stress and then equate that value in a computed manner. So that this can be equated to be ultimately the stress of the material.

And that will fulfill our eating criteria for the stress case. So that is exactly what we are trying to do here and for that we would look at particularly this shaded triangular element as I have drawn in this half of the block okay, and let us actually look individually at the different stress components and what is going to be the overall stress in a plane which is at an angle θ with respect to the side of the block.

So let us say one side faces a principle stress σx and other, there are two components of shear strain here, one is τxy and other is τyx okay. And then there is also another principle stress σy that this block faces. So let us find out what is going to be the overall stress, you know the principle stress as well as the shear stress in this plane, so we are more interested in obtaining the stress, you know in this direction as well as in this direction owing to this particular, you know plane strain the problem.

And then we would like to do some optimization here for different values of θ where in one case the σ would be the maximum and another case the shear stress would be the maximum and then we will take that particular angle where the shear stress will be maximum try to compute what is the sheer force done so once we have done all that then we arrive at this e criteria of the shear stress value existing a cretin critical value for the material for the plastic flow to happen or the.

So that is the basically the hole strategy that we are going to develop here so here let us look at you know a coordinate frame let us say this is actually the intestinal x, y coordinate frame and we are talking about just changing the coordinate frame into another frame which is perpendicular and parallel to this plane this plane so let us record this element as OAB so we are talking about the plane AB and we are talking about a coordinate system which would exactly perpendicular so let us call that x1 direction and parallel to the plane AB is called that y1 direction.

We have the normal coordinate so the normal plane again represented here as X and Y just as I did here okay and the idea is that this can be thought of as if the normal coordinate as moved by an angle θ to generate this x1, y1 and modified coordinated etc and then let us now try to evaluate the sum of forces in the x1 direction okay so we assume that this particular block element here is unit length into page and therefore the area of cross section that this block limit would have this unity in this z direction times the whatever is the length component AO or AB okay OB.

Or as a matter fact KB so this is how we will try to formulate the problem so let us say if we want to compute in the first instance the sum of forces in x1 direction so we are left with a balance of forces in x1 which is due contributing due sigma x1 form example okay so let us say you know the total amount of force on the plane AB should be actually equal to the if I look at this area because of OA so OA times of inanity is the area.

And we want to see what is the area AB times of again utility because this block is unit length into the page okay so the relationship which would exists is that OA should be = AB times of cos of θ for the area of this plane A B so the area of the AB okay it actually it is = to the area of the O A / COS of the Q and the it is = to the area and the so we want to find out the overall force B to D and the principal stress in the S 1 direction and in the β X 1and the V Q is the one of the forces and then there are the other many other forces on this particular point because of the several component like β X and the β Y and the X Y let us record individually forces.

So the force applied here in the direction X1 more actually we to so we assume that we block does not change in the paper sixe and the and the torsion so far and the balance ordered here till the viscous level happen in the X 1 happen in the direction and y the forces and the X 1 direction should be = to 0 so there are some of the forces in the direction on the particular point and it is = 0and the – and the _of β A COS of the Q so again the X is the forces acting on the plane O A and the angle Q is the exactly the opposite direction and so if I translated that is this will come in the direction and here and okay this can be X and the A C O S of the Q is the negative in the direction of β X 1 is the – of the β X 1 – of the cylinder we have Y.

X times of a Sin θ – of σy times of a 10 of θ Tan of times Sin θ is please understand this we are talking about the case where we are want to measure the contribution because of σy and this being the area a, a component ab obviously with the OB into one may be is equal to OA into 1 times of Tan of θ, so because OB / OA should = 10 of θ okay so therefore the area of the phase OB let say area of the phase OB should be equal to the area of the phase OA which was actually represented at a in last case.

Times of one of θ so we are talking about total amount of force σy equate Tan θ on this particular phase AB and this force is in this particular direction and we are simply trying to take the component of this force in the x1 direction meaning there by now this angle is θ and obviously this is 90 – θ okay so this is 90 – θ so we are taking the component of this force meaning there by this times of Cos Sin 90 – θ and cosine 90 – θ is basically sin of θ okay, so that is why this term exist – of τyx times.

Of a tan of θ in a similar logic in a similar manner times of Cos θ should be equal to 0, so having sad that now I know that I mean we already know that in this particular case of the blocked does not this plot of deviate both these components τxy should be = τyx because they are like cancelling components, so the respect each other so therefore I should be left with an expression for σx1 which is equal to 1 / second θ times of σx Cos of θ + τyx Sin of θ + τy Tan of θ Sin of θ + τxy times of you can actually replace this.

With yx because obviously they are equal and opposite to each other, so you have τyx times of Tan of θ times of Cos of θ and this can also be the presented as θ I am sorry this is σ, this can be also represented as σx times of Cos^2 θ + σy times of Sin^2θ + twice τxy Sin θ Cos of θ similarly we will do the sum of forces in the x1 direction in the y1 directions is I am going to now force try to the present there is in a little more you know better manner so we going to substitute for the various values here so we already know that this Cos this Cos^2 θ can be represented as so twice Cos2 θ – 1.

Is Cos^2 θ so we just represented in two θ terms so therefore Cos^2 should be equal to Cos^2θ + 1/ 2 okay, so that is I am going to represent so this is Cos of 2θ+ 1 / 2 plus again similarly I have 1 –

Cos of / 2 1 – Cos of 2θ / 2 as Sin^{2}θ + τ_{xy} times of Sin of 2θ okay so because twice Sin θ times Cos θ is Sin of 2θ and this is how we should be to represent.

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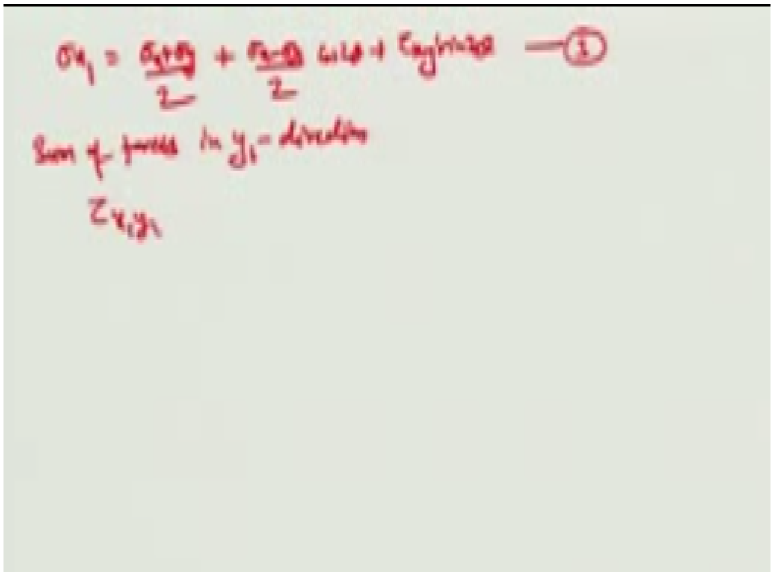

Manipulate our manipulate the different terms of this particular expression we have $\sigma_{x1} = \sigma_x + \sigma_y$ by 2 + $\sigma_x - \sigma_y$ / 2 Cos of 2θ +τ_{yx} Sin of 2θ which is the effective shear or effective principles stress in the plane by in the principle plane you know which plane is actually the plane ab is known as the principle plane so the effective principles stress in the principle plane okay, now whether this is maximum or not is actually depended on a condition where we would like to differentiate the overall stress in the x1 direction with respect to θ and try to say and try to equate it to 0 and say what is that angle.

Corresponding to which the principles stress in that particular plane would be maximum and in fact that angle is generally taken as the angle of the principle plane okay how the particular block, so when we try okay, so now before proceed I would also like to sort of go ahead and do the similar summation of 4 says of that particular block in the last slide in the Y1 direction just as we did in the x direction and here in the y1 direction I would record τ_{xy} or you can say x1 y1 which is the shear stress along the AB plane, okay. So we just write it down here for little more clarity.

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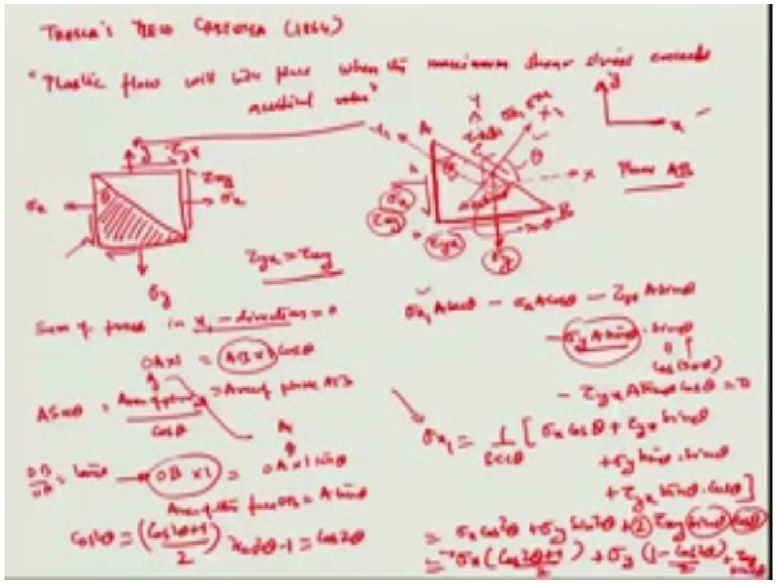

So we were talking about σ x1 in this direction and this particular is you know stress is the shear stress in that x1, y1 plane which I am recording as τx1 y1.

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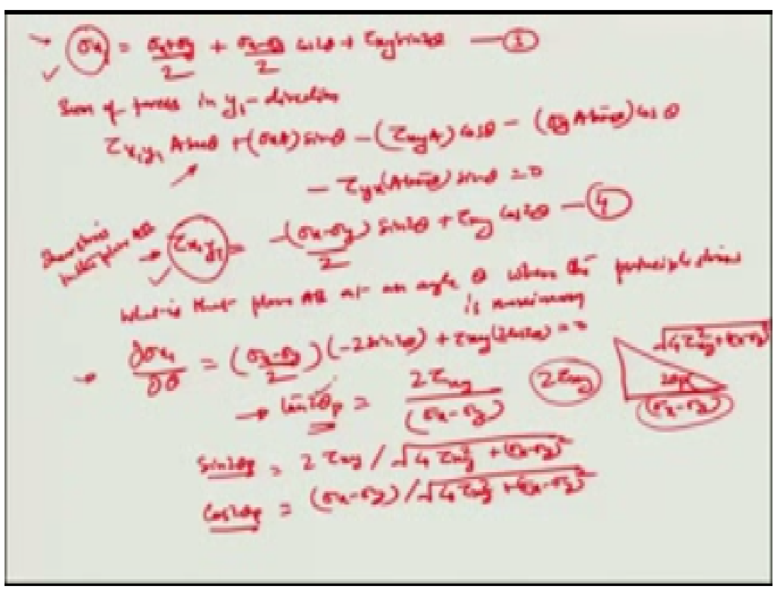

So here the τ x1 y1 times of A sec θ + σ x A time the same manner in the same logic that I have done in the last you know module σx A times of sin θ – (τ xy A times of cos θ) – σy times of A tan θ times of cos θ – τ yx times of A tan θ times of sin θ it should be equal to 0 and from here I can obtain the τx1 y1 or be the total shear stress in the plane AB so shear stress in the plane may be to be equal to – (σx – σy/2)sin 2 θ + τ xy times Cos 2 θ.

You have to just do in the same manner as I did before in order to obtain this expression here let us call this expression f4 which defines the shear stress in the principle plain so we have now one expression which is the finding the maximum principles stress in the principle plain or sorry the principle stress in the principle plane or the let us say the plain AB right now let us not confuse that with the principle plane.

Because it will correspond to that that value of θ where this space is maximum so let us say in plane AB so we are finding out the principle stress along the plane AB and the shear stress along the plain AB and these are represented by equations 3 and 4 and now we have to look at the maximization problem here where we try to actually find out what is that plane AB at an angle θ where the principle stress is maximum.

So let us look at this now, so here for example in this case if we differentiated in the first term σx1 with respect to d θ you would obtain σ x – σy/ 2 times of -2 sin 2 θ + τxy times of 2 Cos 2 θ = 0 for the maximization condition and this happens when the tan 2 θ and let us call this difference with plane okay because here as where the principle plane is really the plane where the principle stress is maximum, okay.

So that comes at an angle θ p = or given by $\tan^{-1}$ 2 τyx / σ(x – σy) okay. That is how you are recording the principle planes that actions so we can think of it as a you know an angle here right here twice θP where one side as a stress σx – σy in other is twice τxy so that you have a you know and tan(tan 2θP) basically given by this twice τxy/ σx – σ y and the overall value or the let us say the hypotenuse value here is 4 times of(τxy^2 + σx – $\sigma y)^2$ whole under the root okay.

So I have now I can record what is sin θ and Cos θ, though sin (2p) and Cos (2 θp) can be recorded as perpendicular by hypotenuse and base/hypotenuse respectively so this becomes equal to 2τxy / this whole term √4$\sqrt{}xy^2$ + σx – σy^2 and σx – σy/√4τxy^2 + σx – σy^2 whole under root, okay. So that is how you record σsin2θpm cos2θp. Now therefore by putting one value of θp you know we can try to find out what is the maximum sort of σx1 at this particular plane you know which is corresponding to this 2θp to find out what is the maximum principle stress coming.

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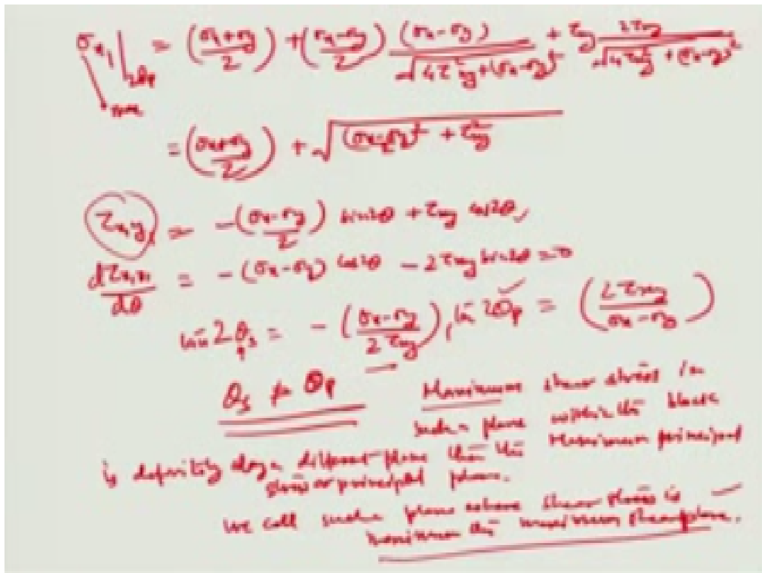

So let us evaluate the value of σx1 at this particular θp in 2θp plane which is corresponding to the principle plane so we have here the expression written as $\sigma_x+\sigma_y/2+\sigma_x-\sigma_y/2\cos 2\theta$ which actually can be recorded as $\sigma_x-\sigma_y/\sqrt{4\tau^2 xy+(\sigma_x-\sigma_y)^2}$ plus again τxyxsin2θ which is can be recorded again as $2\tau dxy/\sqrt{4\tau xy^2+(\sigma x-\sigma y)^2}$ in a manner so that you know will be compute all this together the value would actually come out in this case as finally $\sigma_x+\sigma_y/2+\sqrt{(\sigma_x-\sigma_y/2)^2+\tau xy^2}$ so that is going to be the maximum value of the σx1 at the plane or the principle plane at which is at an angle θp with respect to the sides of the block, okay.

Similarly if I want to look at what is going to be the maximum value of the shear stress and try to find out whether it is going to be in the same plane or a different plane you will record that in this case the plane completely changes in the maximum shear stress does not come in the same plane as the principle you know the plane actually. So we call that the maximum shear plane and let us actually look at that value of θ.

So we already have recorded that the τx1y1 which is the value of the torque or value of the overall shear stress at the plane AB okay, of our geometry so the already you, we have described the τx1y1 to be equal to $(-\sigma_x-\sigma_y/2)$ sin 2θ+τxy cos 2θ we have described this in the earlier section here when we talked about the shear stress in the plane AB.

So let us now look at the maximized condition of the shear stress and find out whether the θ would be the same θp or some other θ based on just these derivative equal to 0, so we do ∂ or dtxτx1y1 by the θ here which is actually equal to again $–(\sigma_x-\sigma_y)$cos 2θ –(2τxy sin 2θ=0 and the condition that emerges here is basically corresponding to tan2θ let us call it θ_s which is actually the shear plane is equal to $–(\sigma_x-\sigma_y)$/2τxy if you record tan 2θp had earlier been found out as a completely different plane which is you know 2τxy/σ_x-σ_y.

So basically you can think of it that the it is actually the inverse you know if I look at value wise it is really the inverse of the principle plane, so θ is definitely not equal to θp and it basically means that you know the maximum shear so there is a interpretation here that the maximum shear stress in such a plane within the block may not be or is definitely on a different plane or along a different plane then the maximum principle stress.

In fact I mean we can show here that if I substituted the value of this 2θm and 2θp back into the, of the total amount of shear in the plain you see that in particular plane the shear does not even exist okay I mean I can probably put this value here and show let me do that in the next step but the conclusion we have is that the maximum shear stress in such a plane within the block is definitely along a different plane then the maximum principle stress or the principle plane okay.

So we call such plane call such a plane where shear stress is maximum the maximum shear plane in fact what we are more concerned with is this maximum shear plane because here is where the stress criteria needs to applied for any kind of stress condition multi excel stress condition to a particular block system corresponding to a angle θ = θ as these stress kept on the shear stress kept maximum along this plane a maximum shear plane okay.

And so that maximum stress should be able to cross over the yield stress for the yield criteria to satisfy or the plastic flow to begin basically. So that is what the whole purpose of this exercise is of the stress guy is, so let us look at that what is criticality or what is that condition corresponding to which this will happen. So before doing that I w would like to just try to evaluate certain issues here one is that what is going to be the state of you know let us say if let us talk about the principle plane so what is going to be the state of stress or state of shear stress on the principle plane. So as we have all recorded that corresponding to let us say tan θ.

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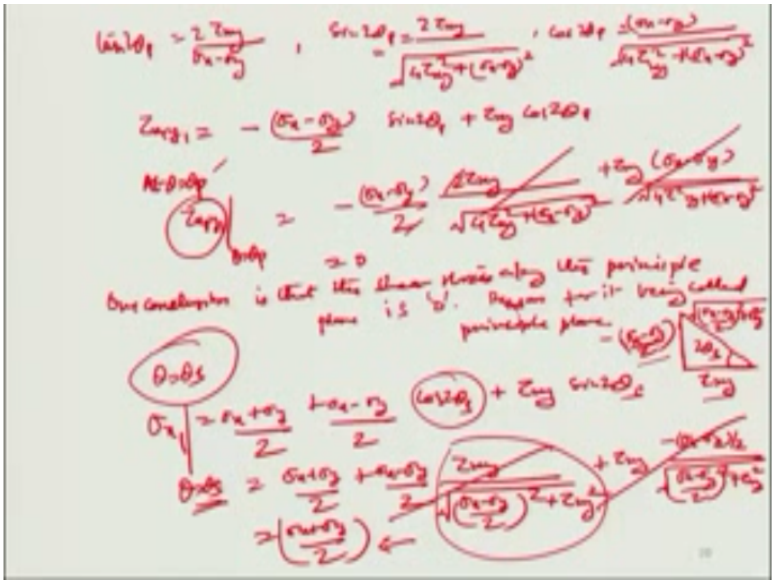

Tan 2 θ p which is actually equal to 2 xy / σ x – σ y other words let us just also write down both the causes and the signs of 2 θ p so 1 is 2 τ xy / √4 τxy ^{2} + σ x – σ y ^{2} and the other is basically σ x – σy / √ 4 τ xy ^{2} + σ x – σy^{2} okay so having set that if I put this value of the principle plane in the shear stress equation which is again you know given by the τ x1 y 1along the plane ab is – σ x – σ y / 2 sign of 2 θ okay + 2 or τ xy times of cross of 2 θ so I add let us say θ = θ p along the principle plane if I calculate what is going to be the shear stress θ = θp this comes out to be equal to σx – σ y / 2 times of this sign value 2 τ xy/ √ 4 τ x y ^{2} +σ x – σy^{2} okay + τ xy time a cos of 2 θ p values.

So I am just subtitling θ = θp here σ x – σ y / √ 4 τ x ^{2} or τ xy^{2} + σ x – σy ^{2} so obviously these are equal and opposite to each other and the whole shear stress at the principle plane becomes equal to 0. So one conclusion that we have is that along he principle plane there is no shear stress component. Conclusion is that the shear stress along plane ab shears stress along the principle plane. In face that is the reason why it is called principle plane you know, so one that the shears stress along the principle plane is 0 okay.

So reason for it being called principle plane so I need the principle stress exist there. Now the plane of maximum shear will not shoe this you know this kind of situation because the plane of shear we do have average force and average principle stress which exist which is actually equal to σx+σy/2 and e can see that by substituting the value of θ equal to θp or θ equal to θs in the equation for the principle stress σx1.

So if I say σx1 equals as I said earlier recorded σx+σy/2+σx-σy/2 times of cause of 2θ+twice yx times of sign of 2θ and then substitute the value of θ equal to θs that means along the maximum here plane what is going to be the value of the principles stress will find there it is a non zero value so if I just let us say put θ equal to θs if we calculate.

So we have σx+σy/2+σx-σy/2 times of cause of 2θs which is again as I said illustrated earlier equal to xy/root over σx-σy/2^2 +xy^2 remember we had called about this angle twice θs you know the maximum shear plane angle is the xy and – of σx-σy/2 okay so basically these are you know the efficiency angle is described by the triangle were this specific σx-σy/2 and twice or xy or both of this triangle.

The sin and cos terms can be again found out as so we have to calculate what the hepatic news here which is is σx-σy/2^2+xy^2 so that is how we arrive at this term here for cos of s2 sin of 2s so let us put that value as well sin of 2s becomes equals to – of σx-σy/2 divided by root over σx-σy/2^2 so that is how you substitute and try to calculate actually calculated how these values theses two terms eliminate out okay.

And we are left to only σx+σy/2 this is actually the state of an average principles so in this maximum here it plane it is very wise to conclude at an angle corresponding to θ equal to θs the principles stress does not go down to 0 but definitely this is the plane along with the shear stress can be maximum so then you know we will may be in the next module take up this issue that what is the criteria which I will allow the yield or plastic flow to happen given all this calculations so far.

So that the maximum stress exceeds beyond the certain ultimate yield stress criteria so with this I will like to close this module and in the next module we will discuss more on the stress criteria thank you thank you so much.

<u>Acknowledgement</u>

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