Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology – Part- 2

Module- 42
Yield Criterion used in Metal Forming Processes Edit Lesson

by
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering,
IIT, Kanpur

Hello and welcome to this manufacturing process technology part 2 module 42.

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We were discussing last time about the flow curve and the flow curve equation and how you could calculate the flow stress as well as the average flow stress and various situations of different two strains or true stress values today we are looking we are going to look at different aspect which is regarding the yield and the yield criteria.

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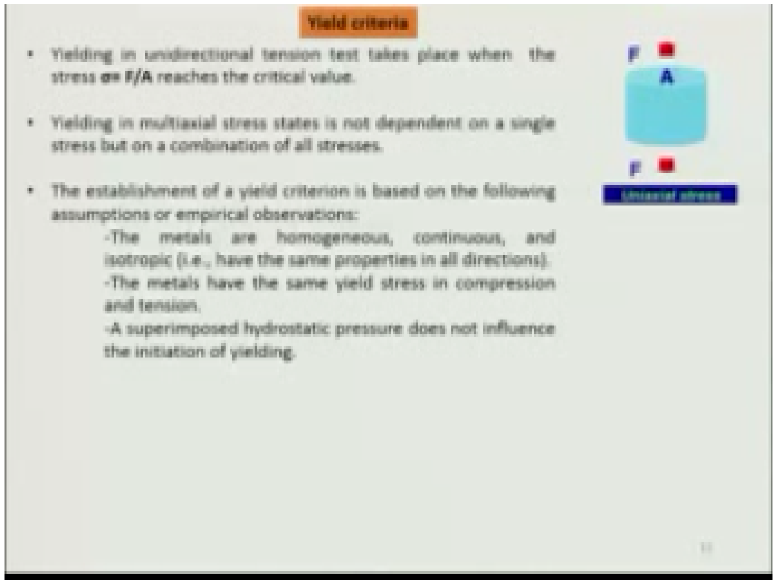

So if we all recall you know earnings from solid mechanics we all sort of recall that yielding in unidirectional tension test takes place when this stress value force per unit area or reaches a certain critical value which is also known as the ultimate yield strength of the material so it was called ultimate yield stress or yield strength of the material so ending in multiracial states particularly when there are more than you know one stresses which are involved and not only that there are also here.

As well as compressive expansive stresses which are invincible stresses which are involved so it does not depend any more on a single stress you know but really on a combination of all such stresses which would act on the material at certain point in space you know at a certain point of time so therefore the general phenomena of idling is really to do with multi axial stress tapes and how-to compute these four states and then the other issue is that this criteria here as you are seeing on the unidirectional front.

Where we are saying that the stress reaches a certain critical value before the yielding starts to take place holds true for the multiracial stress for state of Treasurer stresses as well and the challenge here is really to establish the right yield criteria which would actually cater total this sort of multiracial you know states of stresses where there is combinatorial of several different stresses like the principle stresses along the three direction or even the shear stresses created because of such stresses in a parallel planes.

You know so that there is a combinatorial of all these stresses which would actually meet certain desirable value okay which is a property of the material where he would happen so there are certain assumptions that we may need to sort of make while we are talking about on the yield criteria and the first assumption that we need here is that we treat all metals to be homogeneous continuous and having same properties in all directions.

This material property is also known as isotropic that means you know all the all the material properties are sort of homogenous and not a function of the direction also we treat all metals to have the same yield stress in compression and tension and so therefore the Ile de criterion would sort of be formulated based on some of these assumptions and also a superimposed hydrostatic pressure which does not influence the initiation of the of the yielding.

So we have to separate this component this hydro static component out when we talk about determining the right yield criteria so the first so there are two different criteria which normally are deployed when we talk about metal forming so the first criteria is also very popularly known as the one Mrs. criteria this was a yield criterion proposed back in 1913and the criterion sort of says that the eliding occurs when the work of deformation per unit volume provided by the system of stresses.

Which exists exceeds a critical value for a particular material so essentially it is also sort of talking about the same as we reported here in the in axial unidirectional case here also there is a critical value over and above which the healing would start to happen the only issue is how to resolve the effective stress you know of a certain element provided this element is facing so many different you know states of stresses.

So that we could find out what is the Nets trace of state of stress and whether it has exceeded this critical value so that is the challenge and for doing that we will actually try to formulate a model based on stream energy where we try to look at how this whole complex issue of trying to calculate an effective stress is an effective stress to be over a critical value is kind of resolved.

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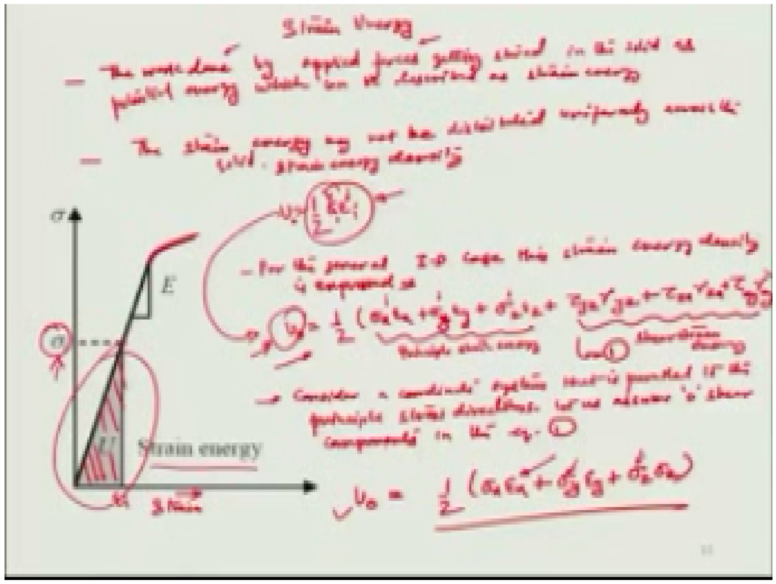

So let us look at some aspects of strain energy we know that when a force is applied to a solid D forms and we can say that the work is done on the solid which is proportional to the force on one sideband also the deformation and also the work done by the applied force is sort of stored in the solid as potential energy which is popularly known as strain energy so it is basically the work done by applied forces getting stored in the solid as potential energy.

Which can be described the strain energy so we need to find out what are the applied forces and we also need to find out what is the work done by these applied forces so we know that the strain energy is necessarily distributed uniformly throughout the solid and strain energy may not be distributed uniformly across the solid and we can define the strain energy density or the energy or the work done per unit volume.

By looking at area under the curve the stress-strain curves particularly so this right here represents the stress-strain curve of a solid this point right about here as you may recall is the proportionality limit of the elastic limit beyond which there is a yield or a flow stress or flow material flow which happens and so wear mostly concerned with the strain energy density as an area under this stress strain curve represented here by this shaded region.

Which can also be recorded at a certain value of stress and strain combination to be half epsilon I epsilon I so if you look at whether it is energy density it is really too because this stress right here is the strain right here is the change in length per unit original length and the stress is dimensionally force per unit area so we can say that is the kind of work done per unit volume which is stored within the material by virtue of these applied forces to deform.

The solid in general so for the general 3d case this strain energy density is expressed as a combination of all the principles and shear stresses and we can simplify this into more like an algebraic expression by looking at half times of the energy in the X Direction + energy in the Y Direction due to the principle stresses + the strain energy in the Direction and + we have several shear components and shear stresses.

So these are all the compressor for the expansive stresses the XY α XY this component being the shear strain energy and these are because the principle stresses and principle strains so you can call it principle strain energy or longitudinal strain energy and that is how you could write the overall strain energy density equation if the materialism elastic then the strain energy can be completely recovered by unloading the body.

So if we are flying in this particular region the moment the forces are eliminated and the stress comes back to zero the state of stress comes back to zero the material recovers back all the way to the initial length okay and there is no such no acid stain which is recorded however as I have repeatedly told in the last lecture as well when it crosses the elastic limit this luxury is no longer available and there is material flow which would happen which causes the material to sort of expand and there is a permanent set or a permanent deformation.

And it is the energy which is stored within the body of which we have concern or of which ware wanting to sort of find out how to calculate that energy per unit volume of the body in question so we can simplify this expression further by considering a coordinate system that is parallel to the let us say only this principle stress direction so let us consider a coordinate system that is parallel to the principal stress direction and in this coordinate system.

The coordinates are defined in a manner so there are no shear strains or shear components so let us assume zero shear components in the above equation let us call this equation number one in the equation one so we get the strain energy density to be simply the summation of all the principles stress-strain products in all the three different directions so weave Σ X epsilon X Σ Y epsilon y+ Σ Z epsilon Z half of this value as the total amount of strain energy density in such a case you know recorded.

Within the system now we can play around with this a little more because we know that you know this strain in the X direction is not only a function of the Σ X but also in some way related to how much is Σ Y and how much is Σ Z and they are finally related with each other with respect to the Poisson ratio so if we look at that aspect here and bring that aspect in here we are left with three different equations.

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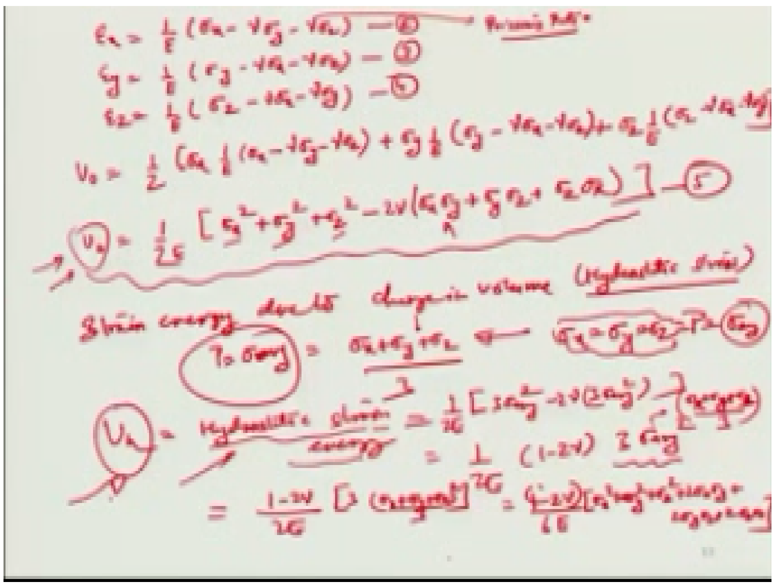

That is epsilon X which can be recorded as 1 by modulus of elasticity e times of Σ X _ unties of Σ Y _ again nu times of Σ Z in the similarly epsilon Y which is 1 by epsilon times of Σ Y _nu times of Σ X _ nu times of Σ Z similarly epsilon Z which is 1by E times of Σ Z _ nu times of Σ X _ nu times of Σ Y so these are recorded as equations to 2 3 to2 4 and we want to substitute these back into the equation 1 at which we have just generated in the last step let us call this one a in order to find out.

What is going to be the final formulation for you 0 okay so if we do that we are left with you zero equals half of Σ x times of again you know I can take this value here 1 by epsilon times of Σ x _nu Σ y _ nu Σ z + Σ times of 1 by epsilon times of Σ y_ nu Σ x _ nu Σ z +again Σ z 1 by epsilon times of Σ z _ nu Σ x _ nu Σ y so on so forth and finally the expression which comes out overall is basically you 0 equals half of or 1 by 2e times of Σ x square + Σ square 2 Σ z square _ twice of new times of Σ X Σ Y + ΣY Σ Z + Σ Z Σ X.

So on so forth so let us call this equation 5 so we have now finally obtained in the case of you know 0shareor only the principal stresses we have been able to in a coordinate system that is parallel to only the principal stress direction of able to obtain the overall strain energy density as 1 by twice epsilon square of Σ X + square of Σ y + square Σ Z_ twice nu Σ x Σ y + Σ Y Σ Z + Σ Z Σ X shaving said that now further it may be pertinent for me to mention here that you know the strain energy density at a point can be thought of sort of divided into two parts.

One is because of change in volume and another is because of the you know the distortion of the element and accordingly we can or better known as change in shape so in the first premise that I had discussed prior to this lecture I had mentioned very clearly that particularly in metal forming it is important that we only consider the change in shape and not the change in volume.

Because we have all the volume all the metal forming operations presuming a constant volume process so somehow the dilatational part or the change of volume part has to be takeout you know in order to formulate a good yield criteria which can be applicable to metal forming processes and we are going to do this here by looking at the change you know the strain energy due to the change in volume.

So let us say the strain energy due to change in volume which normally happens because of let us say hydrostatic stresses what I mean by hydrostatic stresses is that consider this whole object to be placed inside water or any other medium and there is equalized pressure in all the directions by Pascal's law so that it basic it compresses the volume to a lower size or a lower volume okay.

So this component needs to be out of the overall strain energy for us to determine the yield criteria so in that event if we assume the pressure P to be the average stress you know let us say Σ P Σ average is equal to let us say the pressure P and this can be also recorded in our case because we have three components Σ XY and Z as an average of all these three terms together let us say.

So this component somehow needs to you know so basically what I mean here is that a component which is subjected to a hydrostatic pressure should really have Σ X equal to Σ y equal to Σ Z equal to the pressure P or the Σ average so that is exactly what we want to substitute in this equation and try to see what is going to be the hydrostatic strain energy.

So let us walkabout hydro static part of the strain energy and we do some decomposition of the energy here because this is the un required part for our metal forming processes and it has to be taken out so we have this as 1 by twice e price of Σ average square obviously all these3 Σ X Σ Y and Σ Z are individually equal to the average state of stress hydrostatic stress within the material _ off twice me twice new this new basically as I think I had recalled that layer is the Poisons ratio.

So do not forget that this is the effect of the stresses in the perpendicular directions to a certain principle direction so this changes into twice_ new times of again if substitute the value of Σ average into this expression right here we have thrice Σ averages square so we have in fact right this whole expression as 1by twice e1 _ twice mu times of 3 Σ average square and that is what is the hydrostatic component of the strain energy.

Which we need to get rid of so what I am going to do in the next step is to sort of you know try to if I just assume the Σ average to be the sum of Σ 1+ Σ 2 + Σ 3 by 3 as I had done in this Σ X + Σ y + Σ Z by 3 as I had done in the previous step right here so I would like to get a final expression which reads 1_ twice new by twice e times of you have three times of Σ X + Σ Y Σ Z whole / 3 whole square.

So in other words we are left with 1 _twice new by 6e times of Σ x square Σ y square + Σ z square +twice Σ X Σ Y + twice Σ Y Σ Z + twice Σ Z Σ s X so that is how I get the hydrostatic strain energy caused by the uniform hydrostatic stress.

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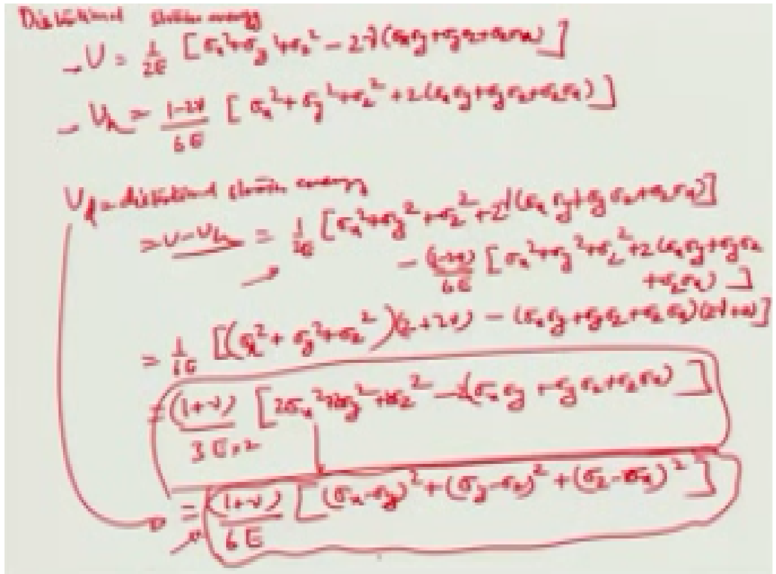

And now we will like to calculate slightly different value which is the distortion all strain energy and as we know that earlier also we had decomposed the system into either a dilatation L component or a distortion Component so in this particular case also if the overall energy that have been reported earlier you was provided by the expression one by twice e times of Σ x square + Σ y square+ Σ z square _ twice nu times of Σ x Σ y + Σ Y Σ Z+ Σ Z Σ X and we have also calculated you H to be equal.

It too or the hydrostatic strain energy to be equal to one _ twice nu divided by 6 e times of component Σ x square + Σ y square + Σ z square + twice Σ X Σ Y + Σ Y Σ Z the Σ Z Σ X so definitely the total distortion strain energy which we also know as UD here let us say would come out to be in this particular case u_ u H because there is no other form of strain energy either it can be hydrostatic or either it can be uniform which corresponds to a volume or it can be because of constant volume.

But a change in shape which is actually our criteria for the metal forming operation so the distortion strain energy here now can be just a difference between these two equations right about here and i can write this down in a more better manner as 1 by twice e times of Σ x square + Σ y square + Σ z square + twice nu times of Σ x Σ y + Σ Y Σ Z+ Σ Z Σ X _ off 1 _twice new by six times of e Σ square + Σ y square 2 Σ z square + twice Σ X Σ Y + Σ Y Σ Z for Σ Z Σ X respectively.

And if I wanted to find out really what is going to be the value of this particular expression right here I can formulate it step by step and calculate algebraically I have you know the total expression written down as 1 over 6 e times of Σ 1 square or Σ x square + Σ y square + Σ z square hole x twice + 2 nu _ of the component Σ X Σ Y + Σ Y Σ Z+ Σ Z Σ X again the component m x twice nu + two or in other words we can have the whole expression as 1 +2divided by thrice e times of Σ x square + Σ y square + Σ z square _ of Σ X Σ Y + Σ Y Σ Z + Σ Z Σ X.

So that is how we have the distortion Elena g overall distortion energy and if I supposing again multiply this whole expression with respect to let us say two so everything is multiplied in the numerator and denominator by two we get an expression 1 by nu divided by 6 e and this right here changes in to Σ X_ Σ y square + Σ Y _ Σ Z square + Σ Z _ Σ x square.

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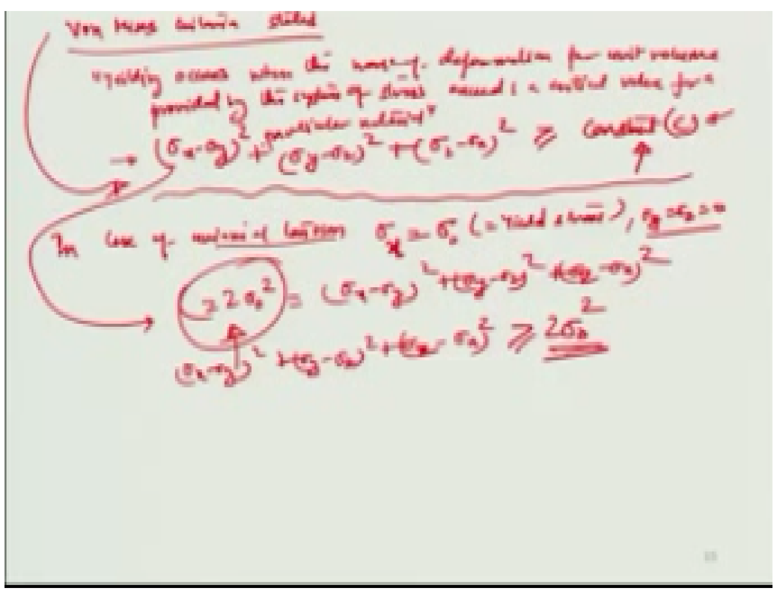

So this is the ultimate state of the distortion energy case you know this transient strain energy case so when we are talking about the coordinate frame which is really parallel to the directional of direction of all the principle stresses so having said that now the if we apply really this criteria this Tress criteria it talks about that you know let us just reframe what we wrote before sorry the one misses criteria we wrote before that this one misses criteria stage that yielding occurs.

When the work of deformation which actually is the distortion of strain energy please understand that in metalworking we are talking about rolling process for example we are not per se you know changing the overall volume by compressing the material we are actually having a volume constancy situation where from one thickness the material is changing to the other thickness and maybe you know there is a spread a more spread in one direction in comparison to the other direction while doing so.

So basically the work of deformation per unit volume provided by the system of stress should exceed a critical value so that is how you know we have defined earlier the one Mrs. criteria and This is also for the metal system of the material system so this is for a particular material we must mention this so for a particular material the property is particularly the elastic constant etcetera do not change in any event the second assumption that we are making in metal forming is that this material is completely homogeneous anisotropic and the elastic properties do not change as a function of direction.

So mathematically as we already have recorded the strain energy particularly the distortion all strain energy which is not the volume you know the change of volume kind of situation so as basically this expression right here and so we are more interested in this expression ΣX _ Σ y square + Σ Y_ Σ Z square + Σ Z _ Σ x square so basically according to this criteria all it means is that this Σ X _ Σ y square + Σ Y _ ΣZ square + Σ Z _ Σ square should be greater than or equal to some constant C okay.

So this is actually a material property which is also known as the ultimate yield stressor the ultimate you know you can say the distortion energy or that critical value for the particular material system which needs to be exceeded when the yielding starts to happen okay so that is criteria here is more commonly known as the one misses criteria for the Ile deco happen.

So whenever we do all these experiment are or we study the processes in details like the rolling process or you know maybe let us say even the bending processes metal forming or the sheet metal forming kind of processes we will actually try to apply this criteria for determining what is going to be the power which is needed for flowing the material.

So that it reaches this criteria when forming actually starts to happen so that is the reason why we are doing this criteria and also another criteria will do in just the next lecture in great details so if let assay there was a case of axial tension so let us say in case of axial tension when we are just concerned with you know the principal stress in one direction let us say the X direction in this particular case to be some value Σ 0.

Which is the eel stress at which the deformation or the permanent set is going to happen other stresses Σ Y or as a matter of fact Σ Z are all 0 so it is on the un axial state of stress so we can we can easily write this formulation here to be if supposing the Σ X Σ 0 so we have twice Σ 0 square equal to Σ X _ Σ Y square / Σ Y _ Σ Z square + ΣZ _ Σ x square which is actually equal to C.

So this criteria in case of in axial tension would work out to be the Ile de criteria for the material to start forming and so the one Mrs. overall criteria can be written down as Σ X _ Σ y square + ΣY _ Σ Z square + Σ Z_ Σ x square to be greater than or equal to twice Σ 0 square where Σ 0 is the eel stress in the uniaxial direction in case of uniaxialtension for the state of pure shear let us investigate this little more appropriately that let us say we are considering this state of stress of particular cube.

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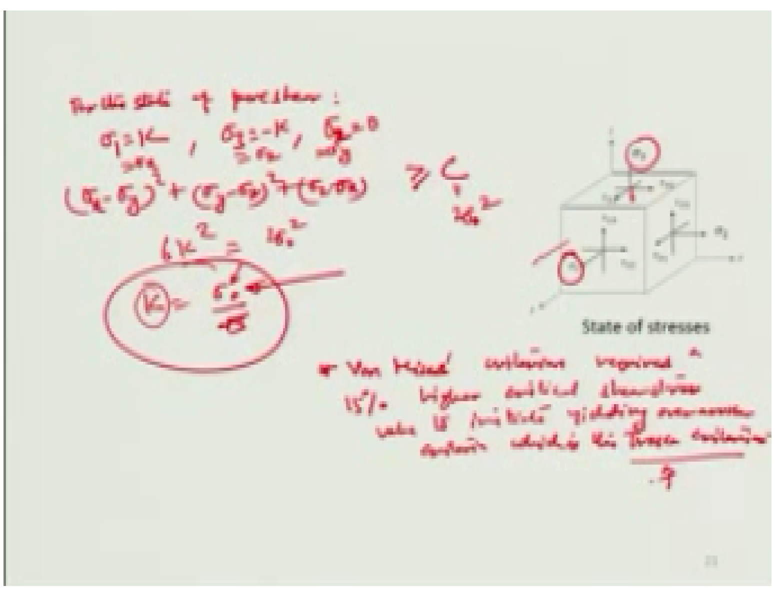

And we wanted to find out for the state of pure shear where you know you can think of let us say for example pushing the material through a set of rolls so you have only Σ 1 as achieving a certain value we call this value k letups say and Σ 3 which is actually_ K so it is pressing so on one hand what the role does is basically impresses the higher section by the role pressure while the friction and pulls ahead pulls the metal ahead.

So it applies a stress Σ 1 so if we assume this Σ 3 to be _ K and relatively very less force in the Direction Σ 2 can be zero in that particular direction so we are left with again if you put this back into the one Mrs. criteria Σ X _ Σ square + Σ Y _ Σ Z square+ Σ Z _ Σ x square is greater than or equal to some constant C so where this is Σ X this is Σ Z and this is Σ Y Σ 2 Σ Y.

So we are left with so the sea is actually twice Σ 0 square from the earlier step and we are left with 6ksquare becomes equal to twice Σ 0square or K can be recorded as Σ 0 x square root of three so this Σ 0 is the you can say the ultimate yield strength of the material or ultimate yield stress of the material particularly what happens in a unit axial tension case which can be recorded and using this all formulation we see that the criteria becomes that the K value should be equal to Σ 0by of three in case of the state of pure shear.

So there are many interpretations how you can apply throne Mrs. stresses to a case of unit axial tension or again state of pure shear and then try to correlate we will look Atmore details when we do actually the farming you know metal forming modeling particularly for individual processes like rolling etcetera generally there are two sort of observations that I would like to share here one is that the one Mrs. criteria requires a fifteen percent higher critical shear stress value to initiate yielding over another criteria.

Which I am going to now do probably in the next lecture which is known as the criterion so what I am going to do no was sort of close this particular module but in the next module we will start to work on this Tresca criterion and try to develop again a condition just as we developed in the one misses you know stress condition where metal yielding can start to happen so with this I would like to close this particular module thank you very much.

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