Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology-Part-2

Module-39
Heat conduction and Temperature rise during LBM

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Hello and welcome to this manufacturing processes technology part 2 module 39. We were talking about laser machining in the last module and we were also discussing about what happens to a semi-infinite surface and cylinder boundary conditions, when there is a constant heat flux being added to the surface from one of the sides. And in that we had also talked about that if, you know the condition is that the laser beam is of circular nature in the spot sizes of circular nature what is going to be the criteria.

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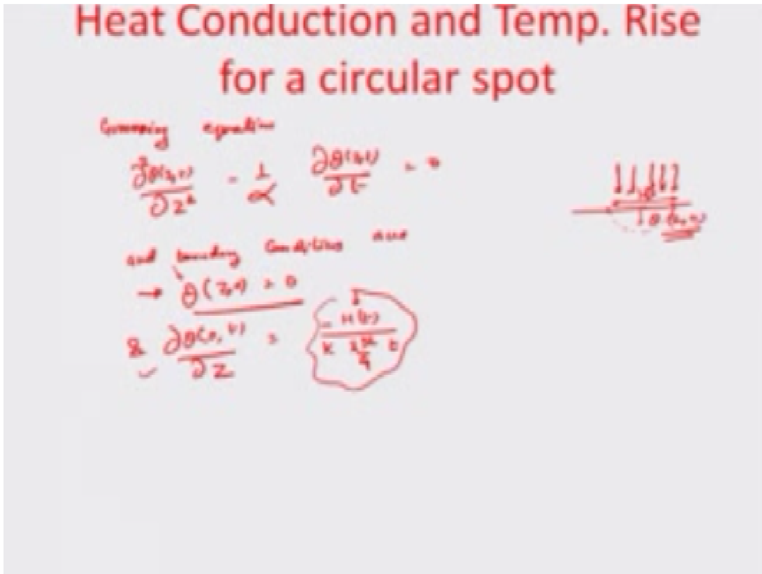

So for that just a quick recap of, you know heat conduction and temperature rise of a circular spot, we wrote the governing equation in this case as $\delta 2\theta$ zt/δz2-1/α times of δzt/δt=0 and the boundary conditions were again written down as θz0=0 that means the temperature at all the different values of z assuming that there is only a circular region of diameter let us say 2D over which there is a constant heat addition or a heat flux through this particular area.

So as a result of which the variation of temperature in the Z direction at point of time 0 when no heat is being added is considered to be that of room temperature, room temperature is considered to be 0 here which is the base line temperature, all the temperature measurements are relative with respect to the room temperature.

And also it was further assumed that the temperature gradient at z=0 that means on the surface right about here as a function of time okay, with respect to the Z direction was actually again determined by –H(t) where H(t) would be the quantum of heat as function of time coming from the circular spot on to the surface divided per unit conductivity per unit the area here which is $\pi D^2/4$, $\pi D^2/4$ okay, and per unit time.

So this essentially is, you can say the heat added per unit or the rate of heat added per unit area, per unit thermal conductivity of the material which is actually nothing but the negative of the temperature gradient corresponding to the z value 0 at the surface okay. And time at all instance of time when the laser beam is operating or interacting with the particular surface. So having said that we would now.

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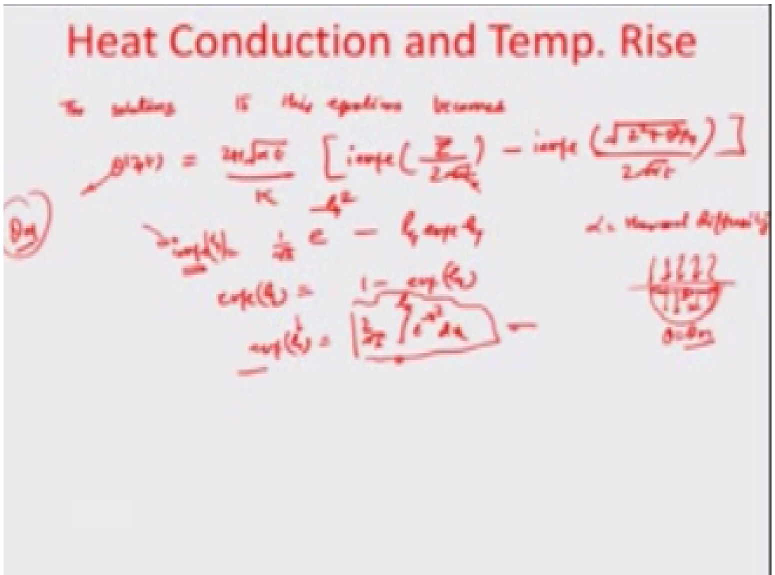

We also wrote, you know what are going to be the general solutions for this particular expression and as I told you that, you know because this is not really a heat transfer class, you would rather prefer to utilize the solution and can be done in any manner that the partial differential equation the soil using either variable separation method or any other method. So this, the solutions, the final solutions of this PD which would really need for investigating what is going to be the depth of melting temperature with respect to time.

So the solutions to this equation becomes θzt=2H √ over αt/K times of the ierfc z/twice √ of αt – ierfc √ over z^2+D^2/4/twice √ αt okay. And this as I think I have already pointed out the, its only an expression of the error function how this ierfc is sort of treated. So ierfc is a function ζ can be recorded as 1/√ of π e^{-x2} of ζ by ζ ierfc ζ where the ierfc ζ is again called as 1 minus really the L function of ζ.

And I think we all are aware that the L function is nothing but a numerical integral represented as 2/√ π 0 to ζ e^{-x2}dx okay. So we calculate the numerical value or estimate the numerical value of this integral for a corresponding variable ζ and then estimate this ierfc ζ which you put back here the ζ and one other cases is the ζ the z or the depth of the melting temperature divide by √ ϕ α t α is again the thermal diffusivity which is we calculated you know in one of the last steps as k / ρ c the conductivity per unit the volume specific heat.

And t really is the time solution up to which you have to really weight as this θ for a circular spot comes to the melting point of that the material which is being heated up on the by the laser so in a way for solving for t you will have to really equate this θ to the θ melting assuming that there is a constant heat addition the surface up to an extent of a diameter let US say capital D you can try to find out would be the z value okay as function of let us say all these different parameters and you know at a certain point of z the melting would stop to take place and that is really the boundary where θ = θ m okay.

So if θ is θ m that is going to give you a solve of depth you know up to which the melting temperature reach beyond which the surface would be stabilized so that is how you find out the value of the z or the shape or size of the way that the melting temperature reaches onto the material okay so if we looked at what would happen at.

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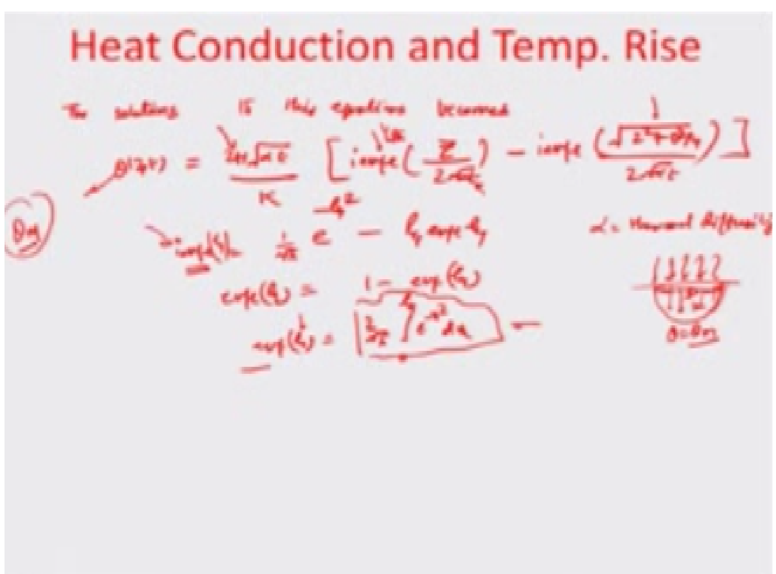

Let us say the value of z = 0 so if I want to calculate z = 0 so obviously the ζ and the first case which is the z / twice √ α d becomes 0 and you know obviously you can always look at it by looking at the IERFC which is nothing but $1/\sqrt{\Pi}\, e^{-\zeta^{2}} - \zeta$ times of ERFC ζ where again ERFC ζ is nothing but 1- the error function of ζ so this value then corresponding to this 0 value of the ζ comes out to be 1/√ Π this climates away this is 1 again this is 0 times where a function of ζ so this climates a away so we are actually left with only 1 term here corresponding to z = 0 which is α √1/1/√ Π or √ Π inverse.

So if would look at the temperature on the surface as a function of time at surface the equation would really change into twice h1√ α t / k which is brooded from the first part here right about here times of this value here which is 1/√ Π times of whatever valuation ahs to be assigned to this particular you know entity here I which I am going to calculate eventually.

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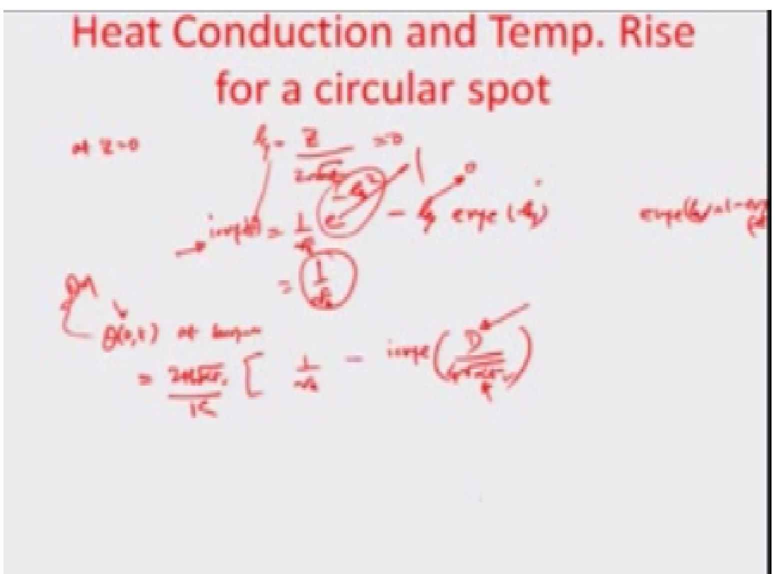

So this would lead to a expression which you know is 1/√ Π- the I ERFC d / √α t / 4 already the two factor is there and we are trying to you know talk about the d^2 / 4 √ or just D / 2 which can be brought in here I am sorry this is D okay so that is how you represent the θ on the surface that is corresponding to z = 0 as a function of time and you know this is how you really have to evaluate with eths solution so at a certain beam diameter.

Let us say d = 2 meter 2mm sport size or 1mm sport size or even few 100 micros spot size for a certain α value that is thermal diffusivity value what is going to be the time in which the α reaches the α m is really questioned here and through which you know you could actually get a sort of indication of the machining time so here let us say we try to address a numerical problem.

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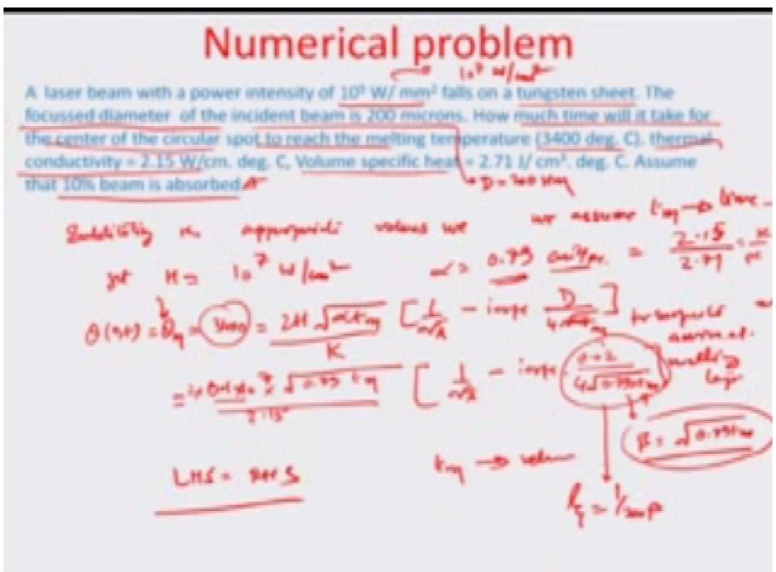

Where there is a laser beam with power intensity 10^5 vot^e / mm^2 again 10^7 vote / cm^2 which wed are heating up on and it falls on a tungsten sheet and we take the focus diameter of the incident beam in this particular case to about 200 microns so this is what the diameter d value is = 200 micro meters we need to a certain how much time will it take for the center of the circular spot to reach the melting temperature of the tinstone which is 3400 ^{0} C using the presumption that we have just arrived for a circulars part size okay.

And given some of the you know properties of the material like the thermal conductivity for example of the volume specific c for example and also given the fact that 10% of the beam is absorb okay, so let say if we substitute the appropriate values we get the value of h the power density has 10^7 Wcm2 and α is approximately 0.79cm2 / s and we calculated this to be the ratio between the thermal conductivity.

2.15 to 2.71 okay the volume specific heat this is ∂C this is K okay and later to the materials so this is material property aspect here, 0.79cm2/ s and if we wanted to solve for what is going to be the time of the machine into start with so if let say θ at 0T as we know is going to be θm fort this material to be melting temperature where the process will start removing or a blading away the material, as you know there is always going to be momentum transfer as part by which melting material is pulled out.

Of this zone machining so this is going to be 3400^0C and we can relate this to twice H √ the formulation is twice h √ of α times of let say Tm we assume Tm to be the time that it takes for the surface to arrive at the melting point so time for surface to arrive at the melting temperature, so if there is EM so twice H / times of √αTm / K times of 1 / √ of π – I ERFC diameter d / 4 √ αT there is how it is a Tm again, okay so that is how you go for a the value of θ0T okay and so if want to add the, the beam power.

Assuming 10% absorption is happened to beam power in this particular case would be about 0.1 times of 10^7W/ cm2 only 10% is couple the remaining 90% is reflected from the surface time the two times of tis H value times of √α which is 0.79 times of EM and this divide by thermal conductivity 2.15 times of 1 / √ π – we have value irfc = diameter d which is in this case about 200 microns, so we have the diameter here is 0.02cm and divide this by 4 times of √αTm α is again 0.79 times of Tm okay.

So we do not know the Tm value and now it is actually upon a to sort of it rate of the value of tm here to have the complete balance between the LHS and RHS of this particular equations, so that is how you actually arrive at the Tm value okay so we would now let say like to have the value here for example or we consider as β √0.79Tm just want me to write it in a proper manner so that the zeta value in this particular case happens to be 1/200β so this whole thing can be expressed as 1/200β okay so let us say this is zeta and this particular case and ψ already have sort of illustrated earlier that you k know if I really wanted to write or find value of the ζ here as 2001 / 200β.

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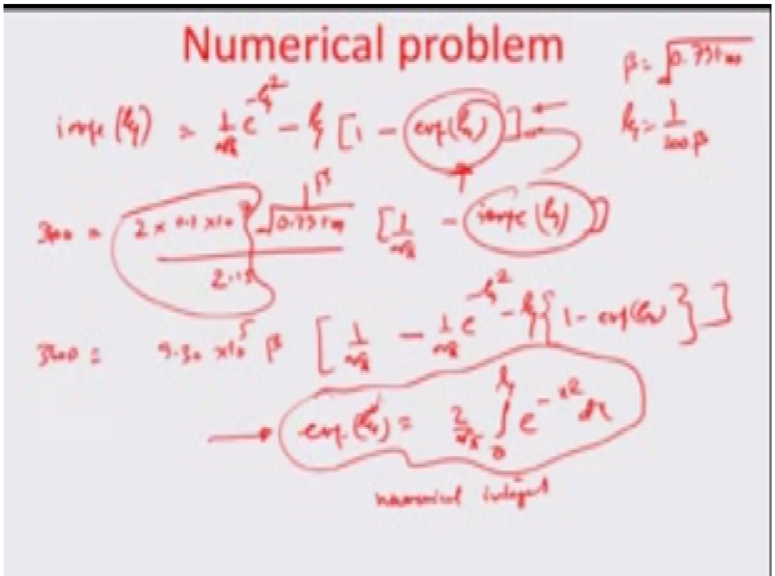

I would like to write the ierfc of some values zeta to be equal to again 1/ √ of π times of e – ζ2 – ζ times of now this is 1 – of the error function okay of the value ζ okay so it also starts from this error function the numerical integral value of the ζ term okay so if I want to substitute this here and try to look at what is going to be or how this is going to look like so like version we had earlier derived was 3400 = 2 x 0.1 10 7 times of √ of 0.79 Tm / 2.15 times of 1/Γ Π – IERFC (ζ), ζ was give as 100 or 1/200 ß here and ß again I think ahead sort of assumed as 0.79 Tm all under the root okay. So I am left with a formulation okay where I say 3400 = actually 9.30 10^5 times of ß you know this value here is 9.30 10^5 this is ß okay.

Times of 1/ Γ Π minus of instead of this IERFC representation we put this particular value which is 1/ Γ Π $e^{-\zeta 2}$ – ζ times of 1 – the error function of ζ here, okay. Where we already know that the error function of ζ is actually a numerical integral of the type 0 to 2 Π or 2/ Γ Π 0 to d time ∫ e^{-} sort of you know x^2 dx, so as I already had mentioned that you know the error function is a numerical integral and there are values which are arrived at approximation. Corresponding to the various values of ζ you will have the different values of error function.

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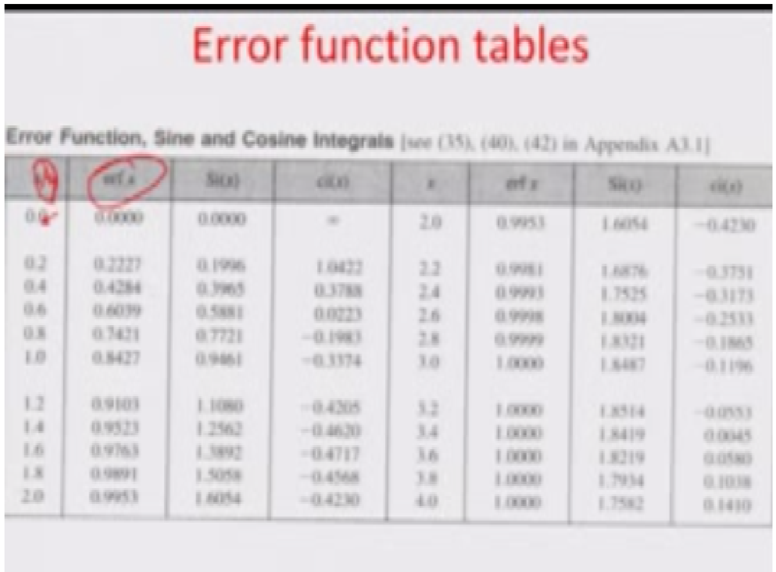
Error function tables

Error Function, Sine and Cosine Integrals [see (35), (40), (42) in Appendix A3.1]

x	erf x	Si(x)	ci(x)	x	erf x	Si(x)	ci(x)
0.0	0.0000	0.0000	∞	2.0	0.9953	1.6054	−0.4230
0.2	0.2227	0.1996	1.0422	2.2	0.9981	1.6876	−0.3751
0.4	0.4284	0.3965	0.3788	2.4	0.9993	1.7525	−0.3173
0.6	0.6039	0.5881	0.0223	2.6	0.9998	1.8004	−0.2533
0.8	0.7421	0.7721	−0.1983	2.8	0.9999	1.8321	−0.1865
1.0	0.8427	0.9461	−0.3374	3.0	1.0000	1.8487	−0.1196
1.2	0.9103	1.1080	−0.4205	3.2	1.0000	1.8514	−0.0553
1.4	0.9523	1.2562	−0.4620	3.4	1.0000	1.8419	0.0045
1.6	0.9763	1.3892	−0.4717	3.6	1.0000	1.8219	0.0580
1.8	0.9891	1.5058	−0.4568	3.8	1.0000	1.7934	0.1038
2.0	0.9953	1.6054	−0.4230	4.0	1.0000	1.7582	0.1410

So the standard tables actually which looks at what is going to be the error function for x given a certain x parameter so x is ζ in our cases and we have to really iterate now from here that what is that ζ value corresponding to which if we outlay the value of the error function in our solution right here, okay. And we already know the value of ζ from which this thing would happen okay that would result in this could be this value could be treated as just ζ/200 as you probably may aware where ß is actually nothing but 1/200 ζ right. So sorry 1/200 ζ I am sorry.

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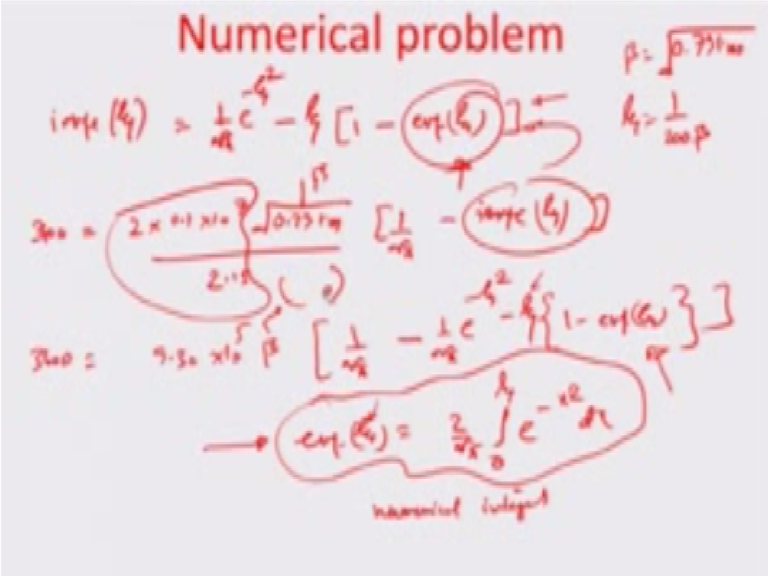

1/200 ζ okay from this expression here so we have everything and ζ on the right and for a corresponding value of the numerical integral I should have parity here, for you know if we do like this by looking at the expression in the table.

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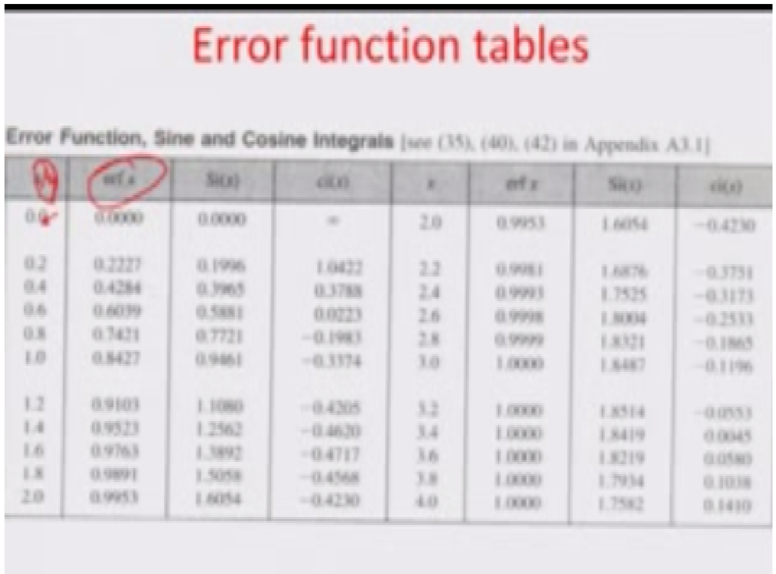

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0.6	0.6039	0.5881	0.0223	2.6	0.9998	1.8004	−0.2533
0.8	0.7421	0.7721	−0.1983	2.8	0.9999	1.8321	−0.1865
1.0	0.8427	0.9461	−0.3374	3.0	1.0000	1.8487	−0.1196
1.2	0.9103	1.1080	−0.4205	3.2	1.0000	1.8514	−0.0553
1.4	0.9523	1.2562	−0.4620	3.4	1.0000	1.8419	0.0045
1.6	0.9763	1.3892	−0.4717	3.6	1.0000	1.8219	0.0580
1.8	0.9891	1.5058	−0.4568	3.8	1.0000	1.7934	0.1038
2.0	0.9953	1.6054	−0.4230	4.0	1.0000	1.7582	0.1410

It so happens that corresponding to a ζ value of around 0.5 okay somewhere between this and this we would like to have.

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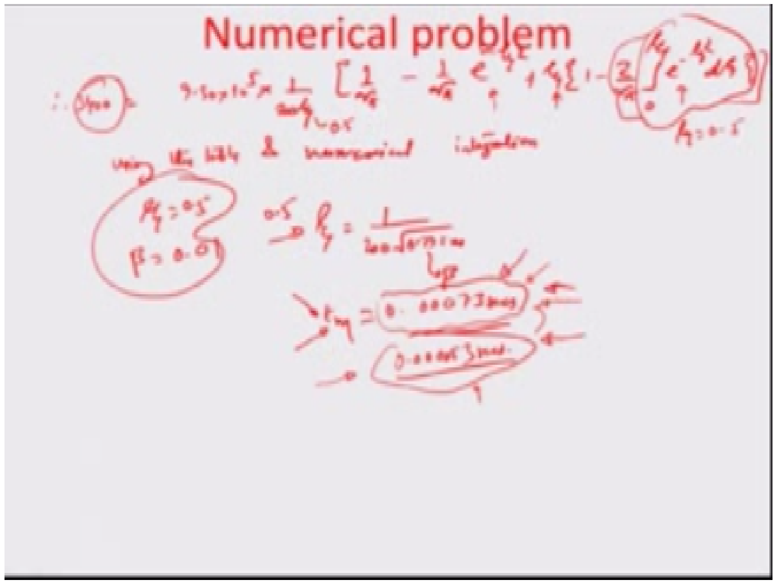

You know if you wanted to just outlay this expression once more 3400 = 9.38 10^5 times of 1/200 ζ times of 1/ √ Π – 1/ √ Π e^- √ $ζ^2$ + ζ times of 1 – 2/√ Π ∫ 0 to ζ $e^{-ζ2}$ d ζ okay we have using the table and numerical integration the ζ comes out to be 0.5 ß comes out to be 0.01 and if this satisfies do you put this ζ value here 0.5 and you know you can calculate this value similarly this can be obtained from the table corresponding to a ζ value of 0. I am sorry this particular value 0.5.

You find out a parity between the left side and the right side okay, so this whole expression becomes equal to about near about 3400 I am not going to the calculation details here but the ζ is actually nothing but in this particular case as you have taken ζ to be 100 or 1/200 times of √ 0.79 Tm on this ß value actually so that Tm value with ζ of 0.5 okay or a ß of 0.01 actually calculated by multiplying you know various factors.

So the Tm actually comes out to be from this expression as 0.00073 so if you look at the expression that we had earlier and the way that we have predicted earlier for a say mean finite case you had a situation where the actual value which was obtained came as 0.00053 okay another 0 so 0.000053. So with the condition or with an assumption that the surface say mean finite obviously it was it resulted in a much smaller time frame for the machining to have happen in comparison to you know the case where there is a circular heat beams obviously heat conduction plays big role here and obviously the heat affected zone is very, very small in this particular case with cylindrical coordinates defining the zone which is affected by the laser machining process.

And one of the reasons why the time scales come out to be quite different okay, from the ideal case of assuming as a mean finite surface. So as we see here this is more accurate this is more appropriate in terms of when the machining should happen and typically and the start of machining really is when the center of the spot on the surface of the material that is being machine comes to melting point and beyond that it is almost the way that heat conduction would happen from the surface onwards.

And how the melting zone would recede you know so that there is a cavity formulated in case you have a through cut there is always a question of how much material needs to be bladed out till and until the melt zone can go all the way up to the depth of the work piece that we are being considering.

So the idea is that the basic essence of the start of machining and the weight period involved there in for the temperature to reach the melting point is a very key thing in the whole laser machining process. Although it is not that big number but still it does matter particularly in micro machining using laser etcetera where the samples that you are trying to make are really very thin and there maybe sometimes and 100 of microns and so there may be a possibility of you arriving at a substantial amount of time needed to sort of weight till the melting zone or the melting temperature is hit up on, okay before the machining can start to take place.

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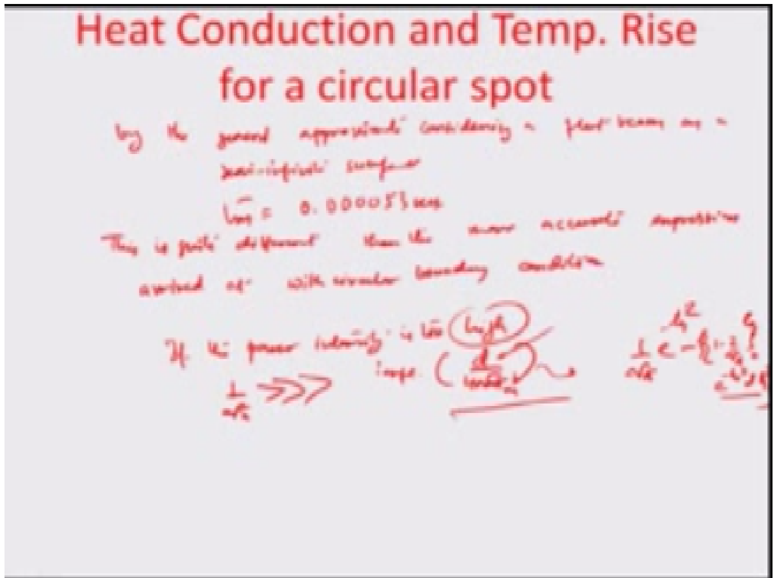

So let us just now sort of compare that by general approximation considering a flat beam on a let us semi infinite kind of surface the total time scale which we actually saw was very, very small 0.00053 seconds okay, this is quite different than the more accurate expression arrived at with circular boundary condition. So just assuming here that supposing if the power intensity is too high, so that you know the term on the right side of the expression satisfies a condition $1/\sqrt{\pi}$ very, very greater than the let us say $i_{erf}Cd/4\sqrt{\alpha t}$ obviously if power is very high this t is small and this whole term is big okay, and so if I want to look at you know this expression it is really $1/\sqrt{\pi}-^{\zeta 2}-\zeta x1-2\sqrt{\pi}$ integral 0 to $\zeta e^{-\zeta 2}d\zeta$.

So as you may recalled here the so let me write this a little more appropriately so this is integral of e to the power let me write this here $-\zeta^2 d\zeta$ okay, so as you may recall because just because the error function is a sort of you can see say an increasing function.

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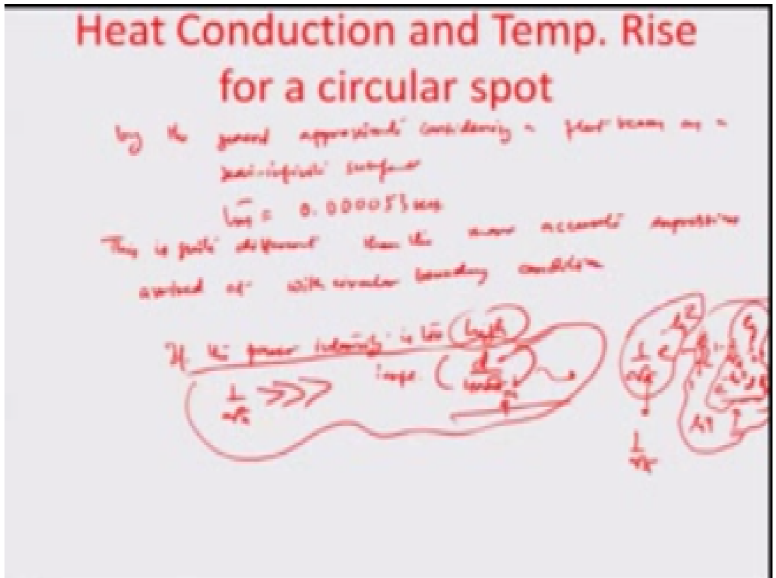

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Error function tables

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x	erf x	Si(x)	ci(x)	x	erf x	Si(x)	ci(x)
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0.8	0.7421	0.7721	−0.1983	2.8	0.9999	1.8321	−0.1865
1.0	0.8427	0.9461	−0.3374	3.0	1.0000	1.8487	−0.1196
1.2	0.9103	1.1080	−0.4205	3.2	1.0000	1.8514	−0.0553
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1.6	0.9763	1.3892	−0.4717	3.6	1.0000	1.8219	0.0580
1.8	0.9891	1.5058	−0.4568	3.8	1.0000	1.7934	0.1038
2.0	0.9953	1.6054	−0.4230	4.0	1.0000	1.7582	0.1410

You can see this table to gage is the ζ increases the error function would kind of increase so obviously this value right here the that is the 2 β or 2/ √ π this integral value of 0 to ζ e – ζ2 d ζ is going to be bigger okay so this going to increase with the increase in ζ and so is this going to increase so you know you can say that there is condition where because of the increase value of this negative coefficient here and sometimes this value is going to reduce because it is obviously it is one / e ζ2.

So this would be much small than 1/ √π okay so this is a case which can heat upon when the power density is too high particularly when the time tm becomes smaller because of increase power density. Let us just look at the reverse case.

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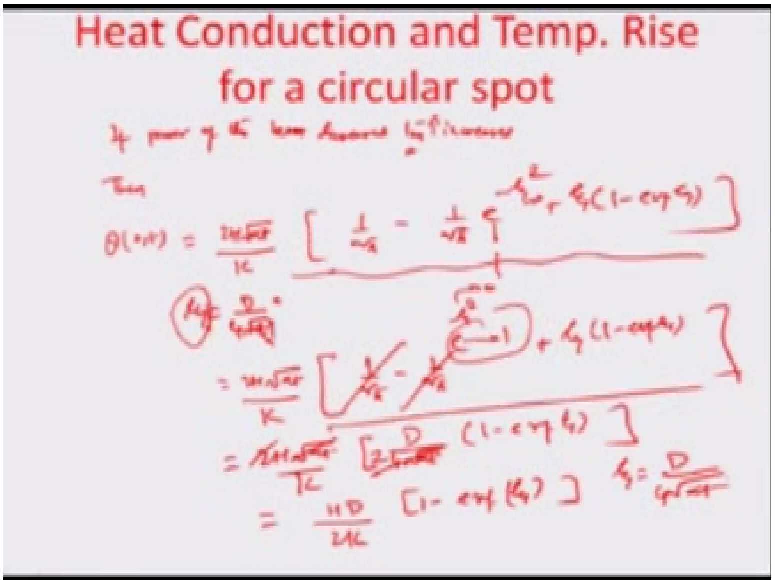

Let us say for example there is a case where the power of beam decrease and rather than increasing the tm increases now because you are operating at a lower power obviously the time to wait before the melting temperature is set up on is more so there in that even the θ 0 t you know which is again let us just write the expression down fully 2 h √ α t / k times of 1 / √ π – 1 / √ of π to the power of – ζ^2 + ζ times of 1- the error function ζ okay.

So let us see how this behaves obviously we know that ζ is actually a ratio between the beam diameter okay which is d / 4 √α / α times of t and as I have already increase or decrease the beam power therefore the tm is increased oaky. So this value tm write here is increase because of width ζ should come down okay so this much smaller. So having settle that now let us see that if tm is more and the ζ comes down how this equation is going to behave so if the ζ value is smaller let us say the ζ approaches a 0 then this increasing the approaches one okay.

And so we are left with 2 h √ α t / k 1/ √ π – something which approaches 1/ √ tie obviously if ζ equals or approaches a 0 this guy approaches one plus you know the ζ time of 1- d are a function of ζ of value which is normally there okay. so if this approaches one if this condition is obeyed then these guys cancel out and we are left with a condition here where we just talk about 2h √ α t / k times of the ζ value which is again d/ or let us just write down the diameters capital d as we have been following earlier so capital D/ 4 √ α time of t times of 1 – the error function of the ζ value okay.

And we can easily cancel some of the terms here and we can just try to convert this in to h d / 2 k 1 – the error function of ζ value with ζ is actually ratio d / 4 √ α t, so having set that now you know this is a kind of condition which corresponds to low beam powers you know because the tm value is reasonability high in this case.

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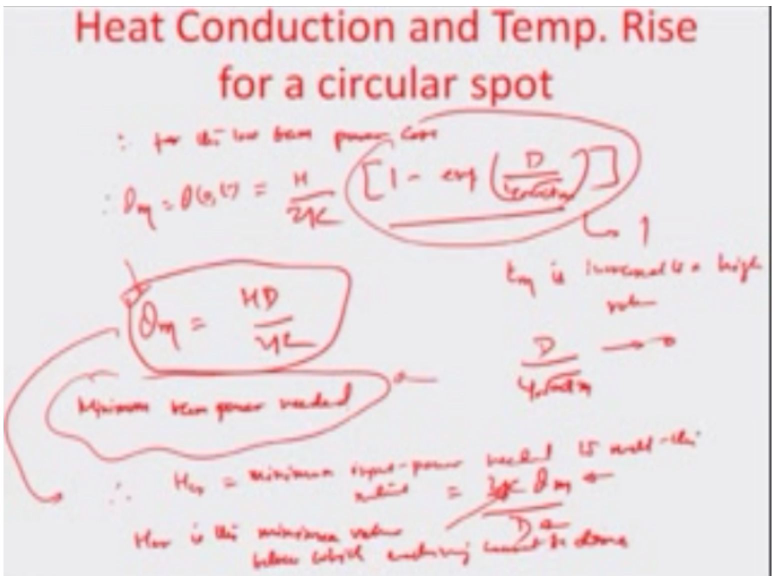

And therefore for the low beam power case the θm = θ0 t as such a power is set upon lower beam power is set upon is actually equal to hd / 2k time of 1- error function of d/ 4 √ α t m so this is the more simplistic form of the expression and you know obviously the maximum value of this particular expression is one when the you know tm value approaches to a very, very high value so tm is increased to a high value so that d/d root of αtm the ζ value approaches 0.

Then you know it is the case where I can say that the maximum you know for the lowest beam power case okay or we can talk about it has the minimum beam power which is needed okay the minimum beam power needed would be corresponding to when the θm becomes Hd/ 2k or this si sorry to represented it has small d this capital D that we are talking about here okay.

So Hd/2k that is what the θm value is going to be and this is how you can estimate what is going to be the criteria corresponding to which minimum beam power can be used laser beam power can be used for the temperature of the surface to hit upon the maximum temperature of melting okay so I can probably now write a little bit about thus low power cases or low beam power circular beam cases.

As that the critical power that is needed fcr which is the minimum input power needed to melt the material is actually given as k θm/d d is the beam diameter θm is the melting temperature of the material and K is the thermal conductivity and so below this critical value the melting temperature will never be reached you can say that Hcr is the minimum value below which machining cannot be done.

So by now I think it is clear that how we have expressed the one dimensional heat conduction equation and the circular beam case and how we have tried to actually estimate the minimum power requirement that could initiated machining process to happen so I think in the interest of time we will close this module but in the next module we will try to once again review this issue and do some empirical design problem were we see what is the minimum power which is needed.

And then also try to look at what is the conduction aspect on the work piece side that would happen you know in the melting zone where they would be a heat transfer which would happen from the center to the sides okay and then in that event we will try to again see what is going to be effective power that is coupled into the system from the laser with this I would like to end this module thank you for being with me thank you.

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