

**Indian Institute of Technology Kanpur**

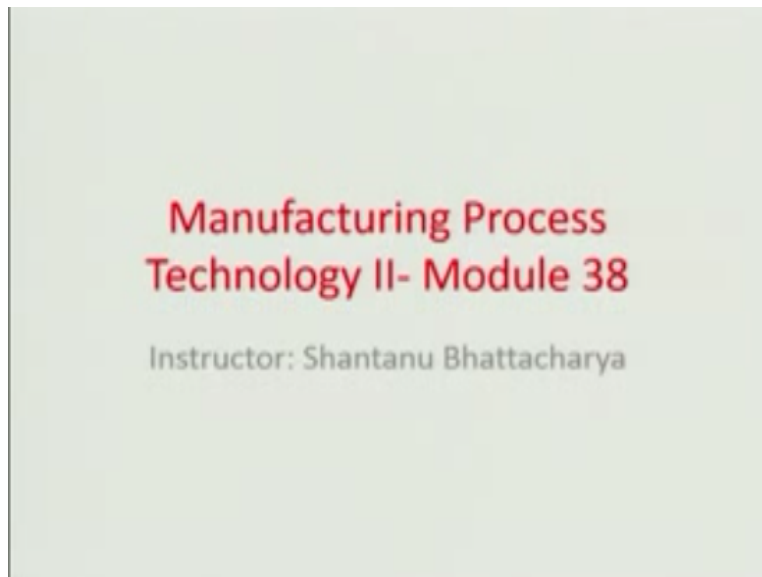
**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title  
Manufacturing Process Technology-Part-2**

**Module-38  
Material removal rate of LBM**

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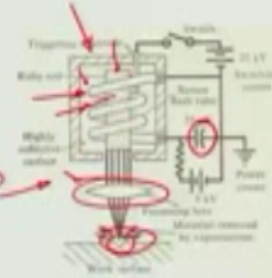
Hello and welcome to this manufacturing process technology part 2 module 38. We were talking about laser machining in the last lecture and we discussed from basic physical principles behind the light amplification of stability machine of relations.

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## Mechanics of Material Removal

- The figure below shows a typical pulsed Ruby laser.
- A coiled Xenon flash tube is placed around the lasing material and the internal surface of the container walls containing the lasing system is well polished and is made highly reflecting so that maximum light falls on the ruby rod for pumping operation.
- The capacitor is charged and a very high voltage is applied to the triggering electrode for the initiation of the flash.
- The emitted laser beam is focused by a lens system and the focused beam meets the work surface, removing a small portion of the material by vaporization and high speed ablation.

- A very small fraction of molten material is vaporized so quickly that a substantial mechanical impulse is generated, throwing out a large portion of the liquid material.
- Since, the energy released by the flash tube is much more than the energy generated at the lasing head therefore the system needs to be continuously cooled.



Context of that we were looking at a pulsed ruby system where we talked about, you know the xenon flash tube and a capacitance circuit which would actually discharge all of a sudden and a packet of energy, so that the flash would can be operated. And there is a shine material of the surface of the flash tube, because of which all the pump or the energy the light energy is pumping to this ruby crystals center right here.

And gender rates a laser signal the signal is basically focused again with a lens into a small spot size, so that there is a hotspot which is created. And there is always small portion of the material kind of surface which is being removed by vaporization, and very, very extremely high speed ablation, because the power density is a very high in this case almost of the level of about  $10^7$  watt per millimeter square.

So a very small fraction of the molten material is vaporized at such a high energy built up at a small spot. And so, there would be a substantial mechanical impulse because of such an energy being incorporated in a very small region. So there is a splitter, you know there is a large portion of the liquid material around which kind of comes out and it is thrown out because of such a high beam power.

Since, the energy released by the flash tube is much more than the energy generated by the lasing head, therefore the system has to be continuously cooled, so that the ruby laser does not get so hot it gets start melted or it starts to melt.

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## Mechanics of LBM

Machining by laser beam is achieved through the following phases:

- Interaction of laser beam with the work material.
- Heat conduction and temperature rise.
- Melting, vaporization and ablation.

Interaction of laser beam with work material:

- The interaction is a thermo-optic interaction between the beam and the workpiece.
- It is obvious that the work surface should not reflect back too much of incident energy.

The figure on the left shows a laser beam falling on a solid surface.

The absorbed light propagates into a medium and its energy is gradually transferred to the lattice atoms in form of heat.

The absorption is described by Lambert's law as:

$$I(z) = I(0) e^{-\mu z}$$

where  $I(z)$  denotes the light intensity at a depth  $z$  and  $\mu$  is the absorption coefficient.

So that is how you produce a solid state laser and you can actually now in detail study the basic physical mechanism behind the laser beam machining process. So it works on the famous VS law and the absorption of light okay. So machining is normally achieved by the following phrases one is the interaction of the laser beam with the work material. Now there of course, is because of such an interaction there is a, you could say there is a photon to phonon convergent okay.

So photon to phonon, so this is one vibration aspect which comes in on the surface. And so, therefore, there is a rise in temperature and there is obviously thermi layer portion rises in its temperature, that is going to be heat conduction across the surface. And this would lead to various phenomenon like melting, vaporization, ablation on the material surface. The interaction of the laser beam with the work material has renew the thermo-optic kind as I told you here photon to phonon conversion.

And obviously when we are talking about a beam of light there is the question of reflection and surface should be designed in the manner, so that there is not much reflectance of the laser light, because if this happen is then we lose quite a bit of laser power. Already laser power overall in reading look at its solid state lasers particularly are not very well to do. So therefore, and also that these two okay.

So reflective power is a major, major issue in all the laser machine processes and it is obvious the

work surface should not reflect back to much of the incident energy so if supposing  $I_0$  intensity falls on the surface of a solid by the Beer Lambert law as a function of depth you have a changing intensity of the material which is dependent on the molar absorptivity coefficient of the material here  $\mu$  this is highly dependent of course on the wave length that we are using typically.


The wave length does not change much when we are talking about a single source laser it is more or less at the same wave length that we are operating so we do not really consider this aspect of the variation of  $\mu$  with respective wave length okay bur definitely there is substantial portion of drop an intensity because of the absorption by the material and it goes by the expression  $I_0$  the surface intensity time of  $e^{-\mu z}$ .

Okay  $I_z$  denotes the light intensity red depth and  $\mu$  is the absorption coefficient in this particular area so you have a incident beam on a work piece surface you have a beam intensity  $I_0$  and then there is an  $I_z$  intensity which is at a certain depth  $Z$  from the surface work surface and because of this absorption the material gets heated up and obviously.

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**Heat conduction and temperature rise**

We will perform a one dimensional analysis by assuming that the beam spot diameter is larger than the depth of penetration.



If we assume that the thermal properties (the conductivity, specific heat etc.) remain unaffected by the temperature change and also assume an uniform heat flux at the surface of a semi-infinite body then we can write the heat conduction equation as follows:

Eventually melted and vaporized so let us now try to model this whole thing by looking at one dimensional analysis and looking at heat source at the laser would typically formulate of the surface so we will perform a one dimensional analysis by assuming that the beam spot diameter is larger than the depth of a penetration in other word we are talking about shadow hole okay the diameter of the hole is very high and this depth here is not as high as the diameter so if we assume that the thermal properties like conductivity specific heat etc...

Remain unaffected by the temperature change and also assume a uniform heat flux at the surface of the semi infinite body, so then we can write the quantum machine conduction heat conduction as the following.

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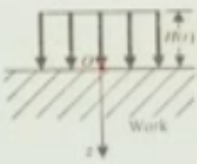
### Heat Conduction and Temp. Rise

$$\frac{\partial}{\partial z} \left( \frac{\partial \theta}{\partial z} \right) - \frac{1}{\alpha} \frac{\partial \theta}{\partial t} = 0$$

$\alpha$  is the thermal diffusivity  
 $\theta$  is the temperature

The boundary conditions applied are  
 At  $z=0$ , when the heat flux has just started  
 the temp  $\theta = 0$

At  $z \rightarrow \infty$ ,  $\frac{\partial \theta}{\partial z} = -\frac{1}{k} H(t)$



The diagram shows a horizontal surface of a semi-infinite body. A vertical z-axis points downwards from the surface. A series of downward-pointing arrows above the surface represent a uniform heat flux  $q(t)$ . The word "Work" is written below the z-axis.

So we can write this  $\partial^2 \theta$  this  $\theta$  is temperature and function of the depth  $z$  and  $T$  by there is a  $2 - 1 / \alpha$  by  $\theta z T / \partial T = 0$  obviously  $\alpha$  is a thermal diffusivity where should be in the conductivity in the volume specific heat,  $\theta$  is the temperature and we applied the following boundary conditions so the boundary conditions applied  $R$  or  $T = 0$  when the sheet flux has just started the

temperature at the body  $\theta = 0$  similarly at  $z = 0$ , has need that surface here wrote about in this particular think here.

And you have the temperature gradient  $d\theta / dz$  really depended on  $1 / k$  the heat flux or a heat per unit area, okay so with this boundaries and this PB quantum machine should be the for solution of the temperature refuse to investigate the solutions it would come out.

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**Heat Conduction and Temp. Rise**

The solution is the PDF above

$$\theta(z,t) = \frac{2H}{k} \left[ \sqrt{\frac{\alpha t}{\pi}} \exp\left(-\frac{z^2}{4\alpha t}\right) - \frac{z}{2} \exp\left(-\frac{z^2}{4\alpha t}\right) \right]$$

At  $z=0$ ,

$$\theta(0,t) = \frac{2H}{k} \sqrt{\frac{\alpha t}{\pi}}$$

If  $\theta_m$  is the melting temp. for the material, the time require for the surface to reach the melting point. Let us assume this to be  $T_m$  so obviously.

As  $\theta(z,t) = \psi h / k \sqrt{\alpha T / \pi} \exp(-z^2 / 4\alpha t) - z / 2 \exp(-z^2 / 4\alpha t)$  where functioning of  $z / 2$  twice root of  $\alpha T$  that is how would categorize a the solution for temperature and function of  $z$  and so obviously at  $z = 0$  find outlet  $z = 0$  of here and we have the maximum temperature so  $\theta(0,t)$  which obviously has to be  $\theta_m$  melting for the material to be starting to melt, okay so this = twice  $H / k \sqrt{\alpha T / \pi}$  obviously this is going to be 0 and  $z = 0$  and so is this going to be 0 that because there is factor here so if  $\theta_m$  is the melting temperature of the material the time require for the surface to reach the melting point okay. Let us assume this to be  $T_m$  so obviously.

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**Heat Conduction and Temp. Rise**

$$t_m = \frac{1}{2} \left( \frac{\theta_m k}{H} \right)^2$$

$$\theta_m^2 = \left( \frac{2H}{k} \right)^2 \frac{1}{2} t_m \propto \frac{1}{k}$$

$$\theta_m = \theta_m = \frac{2H}{k} \sqrt{\frac{1}{2} t_m}$$

**Numerical Problem**

A laser beam with a power intensity of  $10^5 \text{ W/mm}^2$  falls on a tungsten sheet. Find out the time required for the surface to reach the melting temperature =  $3400 \text{ deg. C}$ . Thermal conductivity =  $2.15 \text{ W/cm. deg. C}$ , Volume specific heat =  $2.71 \text{ J/cm}^3 \text{ deg. C}$ . Assume that 10% beam is absorbed.

$$\theta = \text{thermal depth} = \frac{k}{\rho C} = \frac{2.15}{2.71} = 0.79 \text{ cm}^2/\text{sec}$$

$t_m$  then would be indicated by  $\frac{1}{2} \frac{\theta_m^2}{\alpha} \text{ time } K/2x^2$  this is actually borrowed from the earlier illustration  $\theta$  at  $0 T = \theta_m = \frac{2H}{k} \sqrt{\frac{1}{2} t_m}$  time for a temperature to reach melting point on the surface so  $\theta_m^2$  becomes  $\left(\frac{2H}{k}\right)^2 \frac{1}{2} t_m$  and so therefore the terminology here right here so tis an estimate of what is going to be the time for the beam to dual on a surface so that the  $\theta$  which is  $\theta_m$  or the melting point or the material.

Again for sake of brevity I am not going into the solutions of the PD because this is a very standard equations that has been solved in most of the you know mathematical or mathematics text books and probably that are questions regarding how this PD has been right at or what is the solution of the PDE we can discuss about the portal more so layer on. So here we are concerned with what is going to be the time of machining and things like what is important for a machines to know for a laser beam to work on a surface.

So let us have a numerical design problem here we have a laser beam with a power intensity of about  $10^5 \text{ volt/mm}^2$  and it form a tungsten sheet okay. Now I need to find out what is minimum

time which is needed for the surface to reach the melting temperature, melting temperature would be consider is about 3400°C we have the other values related to the surface including thermal conductivity 2.15 W/cm degree Celsius the volume specific heat  $\rho c$  which is 2.71 joule/cm<sup>3</sup> degree Celsius.

We also assume that about 10% the total beam power is absorbed so having said that let us now calculate first the  $\alpha$  value the thermal diffusivity value is a ratios I mentioned earlier between or the thermal conductivity K and the volume specific heat  $\rho c$  K being 2.15 W/cm degree Celsius volume specific heat being 2.71 joule/ cm<sup>3</sup> degree Celsius so we have the thermal diffusivity here as 0.79 cm<sup>2</sup>/ second.

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**Heat Conduction and Temp. Rise**

$$t_m = \frac{3}{2} \left( \frac{\rho_m k}{m} \right)^2$$

$$t_m^2 = \left( \frac{\rho_m k}{m} \right)^2 \Rightarrow t_m = \frac{\rho_m k}{m}$$

**Numerical Problem**

A laser beam with a power intensity of  $10^8 \text{ W/mm}^2$  falls on a tungsten sheet. Find out the time required for the surface to reach the melting temperature = 3400 deg. C. thermal conductivity = 2.15 W/cm. deg. C, Volume specific heat = 2.71 J/cm<sup>3</sup>. deg. C. Assume that 10% beam is absorbed.

$$\alpha = \frac{K}{\rho c} = \frac{2.15}{2.71} = 0.79 \text{ cm}^2/\text{sec}$$

Let us assume that the beam power in this case is uniform with time obviously the heat flux which is also the you can say the energy per unit time or the beam power okay, the heat flux which is the energy per unit time per unit area or in this case you know it can be the coupled power density you know power density being the power you know per unit area, so as we all have I mean as the question says that the power density in this particular case of the laser beam is about  $10^7 \text{ W/m}^2$ ,  $105 \text{ W/mm}^2$  just converting the units here okay.

So and the coupling is you have to an extend if about only 10% because has mentioned here that only 10% beam is absorbed remaining is reflected. So the power the heat flux H which would really comes about  $0.1 \times 10^7 \text{ W/cm}^2$   $10^6 \text{ W/m}^2$  which still is a very large value very large number



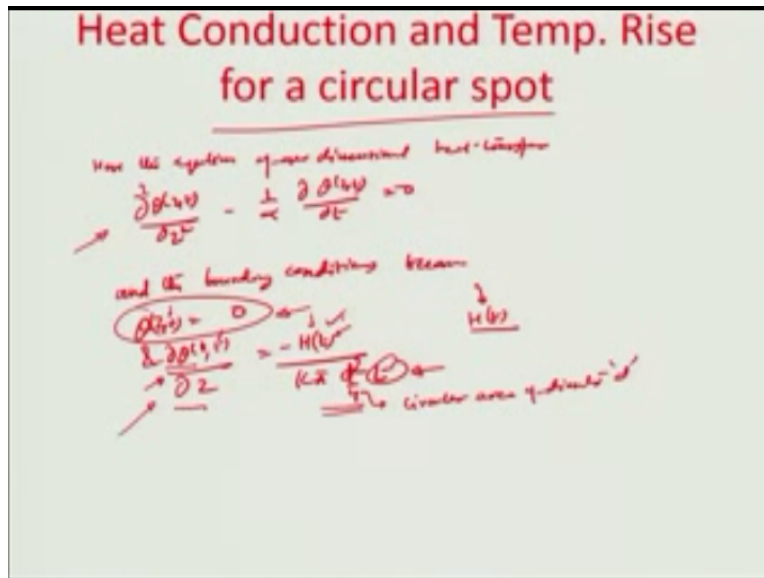
but you can see that most of the power almost 90% is loss because of deflection in this case, so if that where the H value then the time is machine  $t_m$  would really be  $t_m$  and the machine is considered to be a one sort process at the whole material would be either vaporized or melted.

And there would also be a mechanical impulse which will creates patches of the material and some re-deposition would happen etcetera. So the time that you really wanting machining to happen is really what we are meaning by the time of start of machine okay, and the starter machining would only admit the  $t_i = \theta_m$  by melting point of the material which is given by the earlier formulation here  $t_n = \pi/\alpha$  times  $\theta_m$  times of thermal conductivity  $k/2H^2$  so in this case the  $\alpha$  being equal to  $0.79 \text{ cm}^2/\text{s}$  and  $\theta_m$  obviously is  $34^\circ$  k values  $2.15 \text{ W}/\text{cm}^\circ\text{C}$  and also the extend of the power here which is  $2 \times 10^6 \text{ W}/\text{cm}^2$ .

So many seconds of the total time that it would take for the surface temperature to reach the melting point  $\theta_m$  is needed and this time is really very small as you can see it is  $0.000053$  seconds okay. So it is very small time that it would take for such a high power density to the goal surface and create a spark which would need necessity at the  $\theta$  to go to  $\theta_m$  or  $\theta$  to go to the  $\theta$  melting. So let us this is only a case when we are talking about a semi infinites surface I would actually these a beam the circular in nature and you would really cannot to demote proudly surface to be a semi infinite or the surface to be two big for the hat flux to be almost infinite.

So these are all the assumptions and realistic level it is really the beams which is having a certain circularity which is to question most of the laser beams are focused on circular spot sizes. And so let us look at how this equation would change or how the formulation would change for a case where we have a circular spot in to picture.

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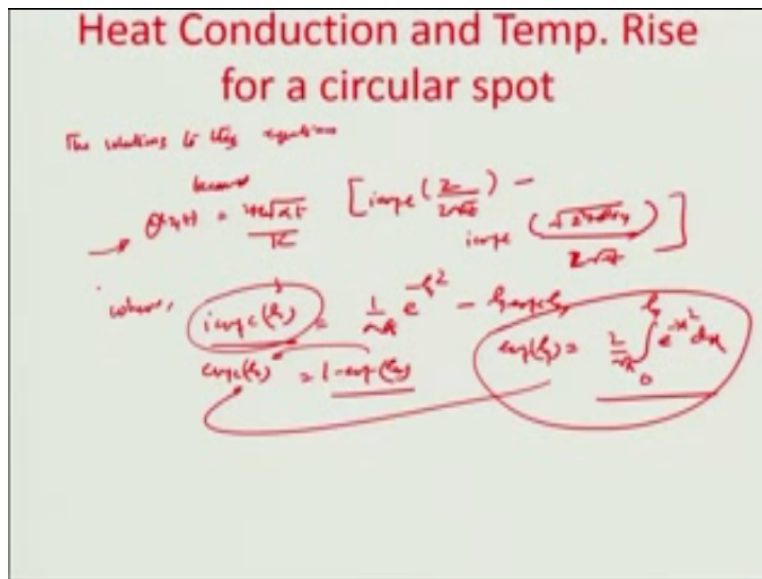


And the surface no longer with semi infinite equation obviously would be equation of one dimension heat transfer would be  $\frac{\partial^2 \theta(z,t)}{\partial z^2} - \frac{1}{\alpha} \frac{\partial \theta(z,t)}{\partial t} = 0$  and the boundary conditions become  $\theta(z=0) = 0$  and  $\frac{\partial \theta(z,t)}{\partial z} = -\frac{ht}{k \pi d^2 / 4}$  times  $t$ . So  $ht$  is really the you could say it is the power or the quantity of the heat so heat energy as a function of time coming on the surface by the circular spot and this could be treated as a constant which we see later when we try to evaluate the solutions.

And you can basically say that the thermal gradient  $d\theta / dz$  at point on the surface when  $z = 0$  as a function of time is really dependent on how much energy the surface is getting per unit area okay. So this  $\pi d^2 / 4$  is basically represented if of a circular area of diameter  $d$  and time  $t$  basically the quantum of time it takes for the during which the heat energy is being supplied to the circular spot by the lasing source or the laser power. The laser source so in a way it is a four years equations or one dimensionally heat conduction which talks about what is going to happen on surface because of a cost and heat energy addition to the surface or may be an energy which varies to the function of time on the surface.

So this is energy per unit area per unit conductivity per in time so this is equal to the temperature gradient so having set that the solutions under these boundary conditions again I am not going to be really solve the whole one dimensional condition heat transfer here systematic solution with the boundary conditions that have been indicated.

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So the solutions so this equation becomes  $\zeta Z t$  equals to thrice  $X$  root over all for  $t/k$  ierfc  $Z/\text{twice}$  root  $\alpha t$  – of gradient ierfc root of  $Z^2 + V^2/4$  divided by twice root over all for  $t$  where this ierfc is only representational of  $v$  error function in general represented by  $1/\text{root of } \pi e^{-\zeta^2} - \zeta \text{erfc } \zeta$  where erfc  $\zeta$  is again represented as  $1 - \text{erfc } \zeta$  so it is only a representation you know that their function is in numerical integral of anywhere  $\zeta Z$  is given by twice the root of  $\pi$  divided by root of  $\pi 0$  to  $\zeta^* e^{-x^2} dx$  okay.

So how will said that we are trying to find out a certain representation here ierfc  $\zeta$  through which you can represent this so this is really how the particular distribution happens on the surface of work piece if there is a circular spot okay because of the lasing on the surface so with this I would like to close today is module but we will investigate this more on the more on the later module.

Where we talked about the circular spot and actually couple energy and try to see if we can calculate estimate the depth of melting temperature. And what is going to be zonal extend up to this  $\theta$  is equal to  $\theta_m$  so that we would construct the zone of the machining for laser spot so with this I would like to end to this module thank you again for being with me and I will see you soon in the module 39 thanks bye, bye.

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