

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title

Manufacturing Process Technology – Part- 2

Module- 24

Theoretical determination of Tool shape

by

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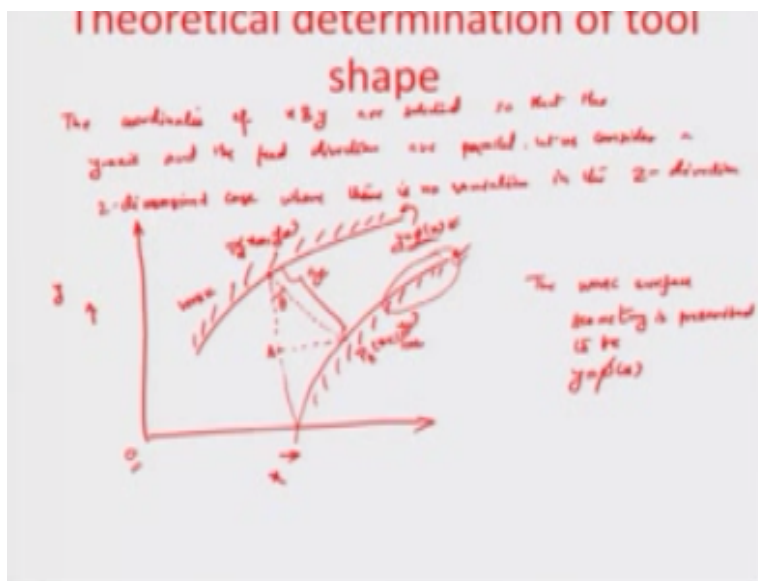
Hello and welcome to this manufacturing process technology part 2 module 24.

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We were talking about the estimation of the tool shape given a particular work shape and you know if you have a functional representation on the work piece I'd can be really mapped what is going to be the inverted tool surface so in respect of that decoy let us say the coordinates of both X as well as why are selected.

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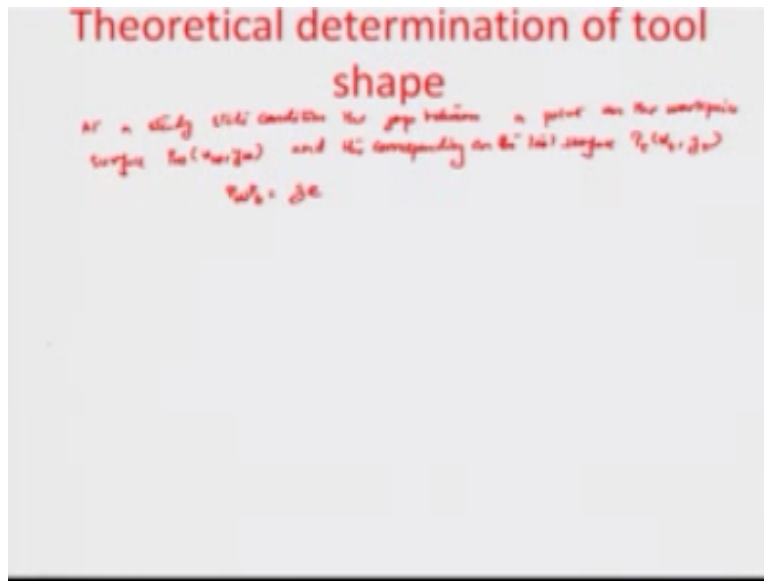


So that the y axis and the feed direction are parallel so let us consider only two-dimensional case where there is no variation in the Z direction so let us suppose this is the XY coordinate system which we are talking about this being the Y direction that is being the X direction and further we have been given a particular work surface profile which follows a functional relationship so the work surface geometry here is prescribed to be y equal to X and let us suppose there is a point here p which is determined by the coordinates XW and YW and further.

The work surface follows a functional relationship y equal to X so the goal DD here is to determine what is going to be the tool surface so this let us say is the predicted tool surface and let us say corresponding to this point PW there is a point which exists exactly you know opposite to the point PW of the work piece we call this point PT so this has coordinates XT and YT so given this relationship on the work piece I did not we have to see what is the kind of functional relationship.

Which happens on the tool side so further this is a case where the work is actually moving at an inclination angle theta with respect to the tool surface so let us say if the tool surface goes like this is how the work piece is going to move and if I were to just connect the surface PT with respect to the perpendicular there is a point here which it makes we call this the point a okay.

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Whereas this origin is 0 further the gap between the two points opposite to each other in a perpendicular direction this gap is given by equilibrium gap GE so having said that now let us try to map the function that exists on this side to side so which we have to obtain from the parameters given like x w YW and the functional relationship on the x I so here at a steady state condition the gap between a point on the work piece surface pw given by x w YW.

And the corresponding point on the tool surfaces given by p txt whitey would be given by p WPT equal to the equilibrium gap go obviously from the illustration that has been drawn earlier here what we can find out is that the XT which is actually all the way here from the origin x w which is here would be given by this distance right here which is nothing but the equilibrium gap g times of sine of theta so.

Let this be one of the equations let us say equation2 and the other one is in the Direction so let us assume the coordinate here as YW and the second coordinate all the way here as YT so y YT should be equal to G theta let us assume this to be equation 1 we already know that so let me

just write this down here so we have equation 1 say YT okay G an equation to as x w cause G sine theta too so we already know the value of the equilibrium gap particularly from the process parameters.

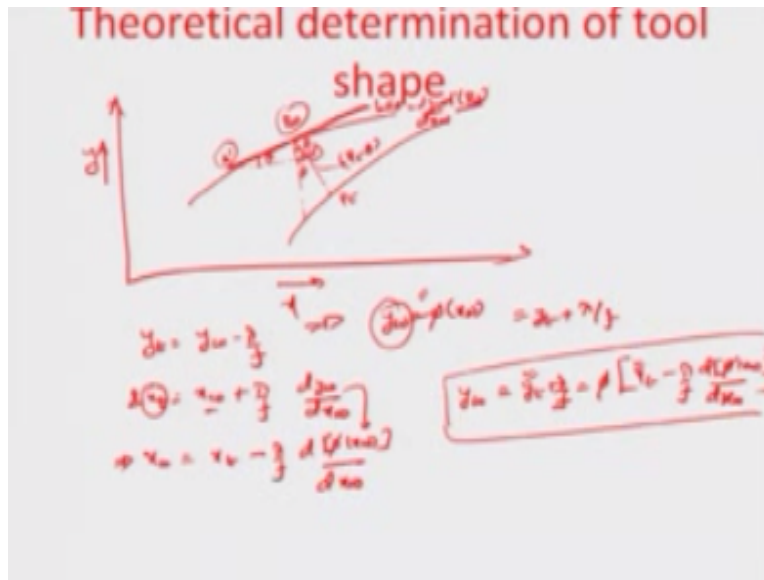
it is given by conductivity times of the atomic weight/ or times of V divided by Z the faraday constant capital F times of feed cost theta so that is how you estimate the equilibrium gap as if have shown earlier several times in other occasions while calculating the me are so having said all this there is a possibility of mapping whitey into given that there is a functional relationship which already exists between the YW and XW .

So let us try to explore that point here so we can easily say therefore $Y T$ becomes equal to $Y W$ of obviously $k AV$ by $ZF f \cos \theta$ times of \cos of theta cancels out we are left with $y w$ _ some λ by f where λ is this whole expression $k a V$ divided by $\rho Z F$ capitals this is how we had also earlier defined the equilibrium gap particularly when investigating the dynamics of the tool with respect to the work piece the second equation obviously results in equals $x w + k a V d /$ by $Z F$ small F times of $\sin x \cos t$.

What $\tan \theta$ okay and this can again further be written down as $X W + \lambda x f \tan$ of thetas this is the modified one and two equations as has been shown earlier sultan of theta further can be estimated as the slope $t YW \times DX w$ at the point $P WX w YW$ let us go back to the figure and have a look at how it is slope so \tan of the is basically really the tangent at this particular point.

So it is about how $d x w YW$ varies with respect to $X Watt$ this point $x w YW$ so having said that now the $\tan \theta$ can be substituted in the above equation and I can write XT to be equal to $x w + \lambda x f d YW DX w$ also since the work piece geometry is given by y equal to of Therefore $d YW \times DX w$ can also be written as d of $x w \times DX w$ so I am going to further put this back here to find out what it results in and probably.

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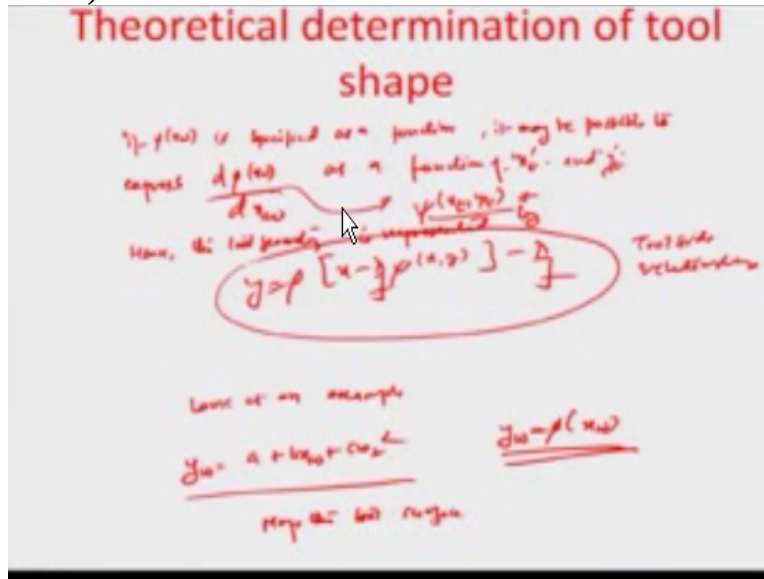


I can you know show you a little more detail just to understand this slope concept which may be you know from the geometrical idea it can come out so this is the work piece surface in the tool surface these are the two opposite points pw and PT which are connected and they are perpendicular to each other and I am talking about the feed direction the in which this particular work piece moves towards the tool surface.

So if I looked at the tangent at the point P which is represented by again to of theta okay d YW x t x w at the point P Who I can actually try to draw another right angle here let us say a dash pw 0 okay in which this angle is already theta okay so this obviously in the right triangle given by this area here becomes by 2 and so in this tribe triangle a pw 0 this becomes again that so really it is Stan of the same that that is the inclination direction with respect to the equilibrium gap okay.

So that is how we assume tan theta so therefore the YT can be simplified or just written as Y W _ lambda by F and the xt can be written as X W by F times of d YW x DX w in other words from the YW what we can interpret is that YW which is otherwise function of X W II X W can be written down as YT + l by F and as we already know that now XT has been included in x w somehow so x w can be further written as x t _ x f of d x w by t xw we are just subtly representing the YW value here in this expression.

So obviously then the YW ok which is a very well a function of the xw can be written as YT + x equals of the new value for the x w which is l by f d x w x DW DX w that is how you write the final form of the expression which is a map of the into the x t given the map on the to the work piece's of Y into the X on the W or work I.
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So if X W is specified as a function it may be possible to express d x W X DX w what we did in the last step as a function of X T completely which is our goal really so we want to find everything in terms of YT and XT and Y T so therefore the tool geometry is represented by y equal to π , $X_{\lambda} x f x$ of this function here $5\pi x t YX YT$ which is actually the you know how you represent the slope of the function $d \pi x w x DX w$ as some function π of x and $y_{_V} x$ that is how you represent the tool side relationship so let us actually look at an example to get a little more feelabout what we want to say here let us suppose we have a relationship on the work piece YW given by $a + xw + CXw^2$ so this is the $y = 5xw$ so now we want to map the tool surface from the logic that has been represented above in the few slides.

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Theoretical determination of tool shape

$$y_t = f \left[x_t - \frac{\lambda}{f} \Psi(x_t, y_t - \lambda/f) \right]$$

$$y_t = x_t - \frac{\lambda}{f}$$

$$x_t = x_w + \frac{\lambda}{f} \frac{dy_w}{dx_w} \quad \frac{dy_w}{dx_w} = b + 2cx_w$$

$$x_t = x_w + \frac{\lambda}{f} (b + 2cx_w)$$

$$= x_w + \frac{\lambda b}{f} + 2c \frac{\lambda}{f} x_w \Rightarrow x_t = x_w \left(1 + \frac{2c\lambda}{f} \right) + \frac{\lambda b}{f}$$

$$x_w = \frac{x_t - \frac{\lambda b}{f}}{1 + \frac{2c\lambda}{f}}$$

$$y_t = a + b x_w + c x_w^2 = a + b \left[\frac{x_t - \frac{\lambda b}{f}}{1 + \frac{2c\lambda}{f}} \right] + c \left[\frac{x_t - \frac{\lambda b}{f}}{1 + \frac{2c\lambda}{f}} \right]^2$$

$$y_t = \frac{a + b x_t + c x_t^2 - \frac{\lambda}{f} \left[\frac{(b + 2c x_t)^2}{1 + \frac{2c\lambda}{f}} \right]}{1 + \frac{2c\lambda}{f}}$$

So in this particular case what I would like to express is that already we know the relationship y_t equals $\phi(x_t - \lambda/f, \Psi(x_t, y_t - \lambda/f))$ okay so this is all of the tool surface relationship that we are obtaining for the coordinates which are there on the tool surface so we do the same thing so we already know that y_t equals $y_w - \lambda x/f + x_t$ equals $x_w + \lambda x/f$ times of $\Delta y_w/x_w$ so obviously as y_w is given as $a + b x_w + c x_w^2$ the dy_w/dx_w now is given by $b + 2cx_w$ okay.

And what I can probably say here that if I wanted to put back in terms of x_t what is x_w so basically you can write it as $x_w = \lambda x/f + b + 2cx_w$ again taking this off we have x_t equals x_w times of $1 + 2c\lambda x/f + \lambda b/f$ in other words x_w can be $x_t - \lambda b/f$ divided by $1 + 2c\lambda x/f$ so if I were to put this back into the formulation so you already know that y_w is related to x_w in the way $a + b$ now the value of x_w is now done here which is $x_t - \lambda b/f$ divided by $1 + 2c\lambda x/f$ okay + c times of the square of x .

So x_t again $-\lambda V/F$ divided by $(1 + c\lambda/f)^2$ okay value of x_w and this y_t is again written as $y_t + \lambda/f$ so that is how you express this thing so if I simplify this term together I would actually have y_t equals $a + b x_t$ okay and if I wanted to just play around this term and then I do not want to simplify this in great details I mean algebraic detail but it is just about if you probably just add these terms together and one obviously has $1 + 2$ some $\lambda c \lambda/f^2$.

So in this particular case then I can get $a + b x_t + c x_t^2 - \lambda/f - \lambda/f$ times of the term that I am expressing here as $b + 2c x_t^2$ divided by $1 + 2c\lambda/f$ so this is actually more like

an approximate solution of this value of yt equal to this whole expression which is here so I can actually now predict the relationship between the wise and the excess on the tool surface given the values or given the functional relationship on the work piece surface.

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Numerical Problem

The geometry of a work-piece surface with single curvature is given by the equation $y = 10 + 0.3x - 0.05x^2$ where x and y are in cm. The process data are:

Applied potential = 15V, Overvoltage = 0.67V, Feed velocity = 0.75 mm/min (given to the work in the $-y$ direction), work material = copper, electrolyte conductivity = $0.2 \Omega^{-1} \text{ cm}^{-1}$. Determine the equation of the required tool surface geometry.

$z = 1$, $A = 13.57 \mu\text{m}$ and $\rho = 8.96 \text{ g/cm}^3$

$F = 94500 \text{ Coulomb}$

feed velocity $f = 0.00125 \text{ cm/sec}$

$$\lambda = \frac{11A(V - 0V)}{\rho F z} = \frac{0.2 \times 13.57 \times (15 - 0.67)}{8.96 \times 1 \times 94500} \text{ cm}^2/\text{m}$$

$$= 2.14 \times 10^{-4} \text{ cm}^2/\text{m}$$

$$\frac{\lambda}{f} = \frac{2.14 \times 10^{-4}}{0.00125} = \underline{\underline{0.143 \text{ cm}}}$$

Let us look at the overall problem for you know just for the sake of theoretical clarity there is a geometry of the work piece surface or the single curvature which is given by this equation now y is equal to $10 + 0.3X - 0.05 X^2$ x and y are both in centimeters and the process data are that the applied potential is about 15 volts over voltage in this case is 0.67 volts the feed velocity is 0.75 millimeter per minute given to the work in the $-y$ direction work material is equal to copper.

And the electrolyte conductivity is also given as 0.2 centimeter inverse Ω inverse and we have to determine the equation of the required tool surface geometry when the work piece geometry has been given by this y equal to this functional relationship between y and x here given here in this part of the question so we find that for copper the values that we have to assume is that it dissolves let us say at a +1 valency.

So the minimum resolution limit is like +1 for this copper and the atomic weight is about 63.57 grams and the ρ that is the material density is about 8.96 grams per centimeter³ we have to find out what is the value of F so what is so basically we already know that the value of F is 96500 Coulomb and the feed velocity in this particular case comes out to be equal to the feed velocity in this case comes out to be equal to F 0.00125 centimeter per second.

We find out that as you know λ equal to ka divided by $V - \Delta V$ by ρ times of z times of f and we can actually calculate this λ value as 0.2 times of 63.57 the value of the atomic weight times of 15 minus the overvoltage potential which is in this case 0.67 okay divided by the total material density 8.96 times of the dissolution valiancy which is a univalent in this particular case assuming it to be in valentines of the faraday constant.

So this many centimeter² per second of λ is recorded so this is 2.11×10^{-4} centimeter² per second and so λ by F typically in this case becomes 2.11×10^{-4} divided by centimeter² per second divided by 12.5×10^{-4} centimeter per second which is actually calculated or converted from this 0.75 millimeters per minute feed velocity and this in centimeter is reported to be 0.169 centimeter.

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Theoretical determination of tool shape

Now $\phi(x) = 10 + 0.3x - 0.05x^2$
 Find equation for the tool surface geometry

$$y = 10 + 0.3 \left[x - \frac{0.159(0.2 - 0.14)}{1 - 0.1 \times 0.14} \right] - 0.05 \left[x - \frac{0.113(0.2 - 0.14)}{1 - 0.1 \times 0.14} \right]^2 - 0.169$$

$y = 9.8134 + 0.3157x - 0.0517x^2$

Tool surface equation
x by numerical

So having said that now the ϕ or the functional relationship between the work piece x and y coordinates are explored as given in this equation ϕX equal to $10 + 0.3 X - 0.05 X^2$ and the final

equation for the tool surface geometry from the derivation that has been done in the earlier step so final equation for the tool surface geometry is found by equation $y = 10 + 0.3$ times of $X - 0.169$ that is the λ / f x of $0.3 - 0.1 x$ divided by again $1 - 0.1$ times of 0.169 .

And this is the converted value of the x coordinate the tool side given the functional relationship on the work piece side – 0.05 times of again $x - 0.169$ times of $0.3 - 0.1x^2$ divided by $1 - 0.1$ times of 0.169 okay so this actually whole square here and $- 0.169$ the λ / f term so we simplify this equation after solution and we arrived at formulation $9.8154 + 0.3157 X - 0.0517x^2$ as the final tool surface equation.

So given the work piece relationship in the question I can now find out what is the relationship between the position coordinates Y and X on the tool surface where both x and y are in centimeters so this is typically how you estimate you know some of the theoretically the tool surface of the tool shape given a certain functional relationship which exists on the work piece shape so it is all about mapping.

Because ECM is really a dicing process it has to really comply to that the tools are is exactly the negative of the work free surface and for this such a formulation is actually kind of valid so if supposing the tool geometry were to change and supposing.

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Theoretical determination of tool shape

$$y = a + \frac{\lambda}{f} x^2 + \frac{\lambda^2}{2f^2} x^4 + \frac{\lambda^3}{6f^3} x^6 + \dots$$

$$y = a + \frac{\lambda}{f} x^2 + \frac{\lambda^2}{2f^2} x^4 + \frac{\lambda^3}{6f^3} x^6 - \dots$$

$$y = \frac{a + \frac{\lambda}{f} x^2 + \frac{\lambda^2}{2f^2} x^4 + \frac{\lambda^3}{6f^3} x^6}{1 + 2(\lambda x^2)/f}$$

Now if I were to say that you know the tool geometry is a three-dimensional surface where y is represented as $a + bx + cx^2 + dz + z^2$ now it depends on both x as well as z so in that event the final formulation of the transformation into the two side would actually have a $y = a + bx + cx^2 + dz + z^2 + e + gxz - \lambda / f - \lambda x f$ times of a whole term here $b + \text{twice } CX + gz \text{ square} + cd + \text{twice } e z + gx$ this again so sorry bracket $d + \text{twice } e z + gx$ whole square divided by $1 + z$ times of $c + e$ times of λ by f .

So this is again using the same transformation as I had earlier done it may be food for thought or maybe you can try to solve you know by looking at this function relationship and then substituting the way that the tool coordinates Express into the work piece coordinates are try to may make a map of the function of the work piece on the tool surface to arrive at this particular formulation.

So in any event I will probably give a complete solution to this particular how it is formulated in a later on module okay of our work so far you have in ECM covered various topics like how to design for the velocity of the electric light flow what are the different criteria is which will result in different surface roughness a-- is what is the basis of the tolerance in the ECM process and then finally you are able to estimate the overall shape and size of the particular tool surface given a certain shape requirement that we have on the work piece surface.

So I would like to now sort of end this module in the interest of time but in the next module we will start with another version of our issues related to the you know the non-uniformity in the mr are or the material removal rate which is based on the improper flow of the electrolyte and the way that the flow separates over a surface particularly surfaces which may have disruption in terms of sporadic you know dissociation and other mechanisms as have been described before.

So that we can see that how Eddie and vortices and flow separations along the path of the flow may result in completely sort of a gradient mr are or a difference in the mr are at different zones of the machining so with this I would like to end this particular lecture thank you for being with me thank you.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
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