

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title

Manufacturing Process Technology – Part- 2

Module- 20

Kinematics and Dynamics of ECM

by

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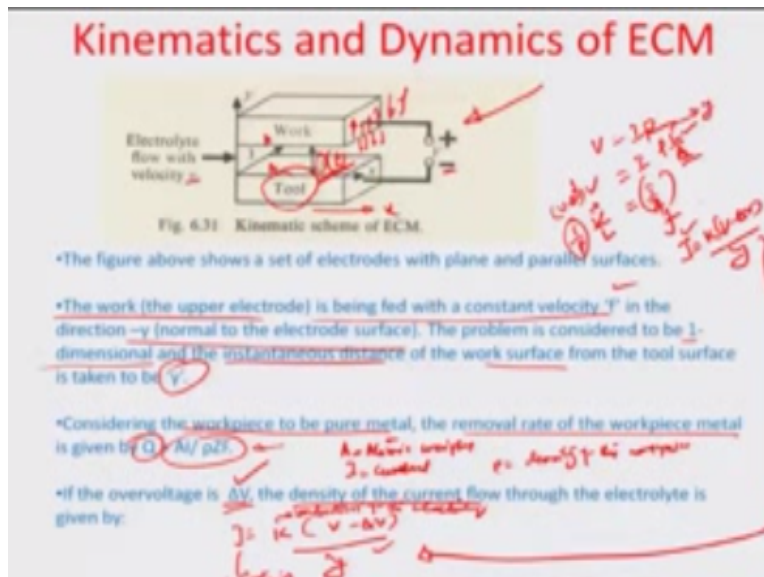
Hello and welcome to this manufacturing process technology part 2 module 20.

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We had learnt in the last module about the various over voltage potentials that are need to be crossed in order for the ions to cross over the different layers like the double layer and which essentially includes the diffuse in herborn layer and also the ion gradients which are established by verdure of the electro chemical machining process of the electro chemistry between the electrode and the solution so once we have learned about how to do this correction in order to predict the actual material removal rate the actual way that a surface behaviors you know with respect to a tooling is of critical important and for this we are going to now cover another area which is also know the kinematics and the dynamics of the ECM process.

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So let us suppose that in this particular figure we are showing that there is an ECM operation taking place where the tool is the nonnegative electrode the cathode the work piece made the positive electrode and we are further assuming that electrolyte flows with velocity V typically in the x direction, this is the x direction and it flows between the space which has which comes between the tool and work piece as the machining gap and further we assume that the work piece is having distance y as a function of time t with respect to the tool surface.

And this y keeps on increasing or decreasing based on two factors one is how the work piece is receding you know with respect to the tool because of dissolution process and the other is how the work piece is being freed into the tool so these two are the determining factors to find out how the surface receded away and how the tool gap gets created because of that okay so the work the upper electrode is being feed with constant velocity F in this particular case so let us say this is F .

Okay in the $-y$ direction normal to the electrode surface and the problem considered right now as a one dimensional simplified problem the instantaneous distance of the work from the tool surfaces taken as y so let us first estimate at what rate this surface will dissolve away which will result in some kind of a change in the distance between the tool surface and the work piece surface so if we consider the work piece to be of pure metal the material removal rate of the

work piece is give by this formulation here we have detailed the derivation of this earlier in the earlier modules $AI / \rho Z F$.

Where A is the atomic weight and I is the current the rate of change of flow rate of flow of charge within the electrolyte ρ is the density of the work piece Z is the vacancy at which the resolution is suppose to take place if is faraday constant which is 96500 coul of charge we assumed ∇V to be the over voltage in this particular case the density of the current flow therefore through the electrolyte would be given by the expression $J = k$ times of $v - \nabla v / y$ okay J is the current density current per unit area and K is the conductivity of the electrolyte.

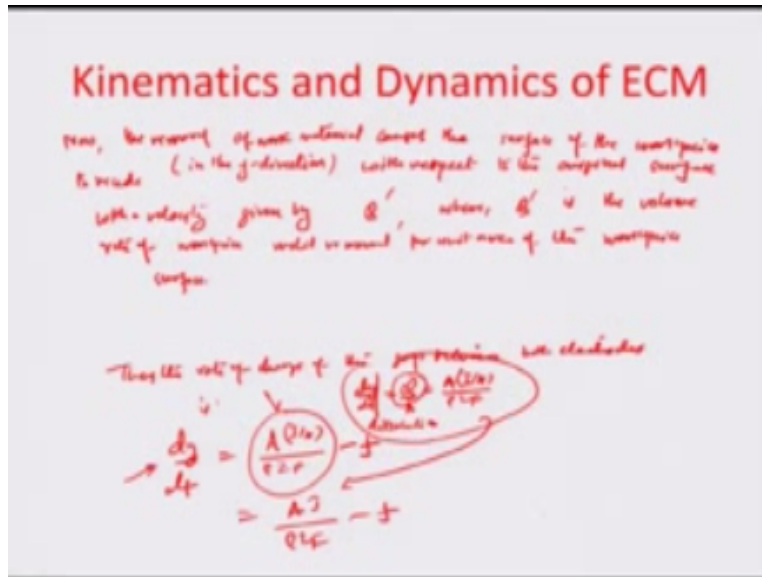
So how this arrives and this ψ is simply formulation be equal to yR so you already know that I is basically the current view voltage and the electrode does obey the Ohm's law so this because equal to the conduct the resistivity ∂ times of l over a where L is the distance across which the current flow would takes place an areas cross section perpendicular to direction of the current flow in this particular case the current is flowing as you know between the work and tool in the direction perpendicular.

To the work and tool so obviously the distance y here becomes t distance L which is the length of flow of the charges and a is the cross sectional area, assuming that the we have flat surfaces of the tool and work piece and the areas is equally equal in nature so we get the total resistance of the electro light as a ∂L over a so in other words what we are trying to find out here is that if I had the current density j to be defined as I / a or current per unit area this is the j value they should be equal to $1/\partial$ times.

Of V / L okay so in other words what we actually get is that j is nothing but 1 by resistivity is the conductivity k and this voltage available is $v - \Delta V$ because Δv is the over voltage potential and obviously, length is the distance y at the distance of time t has I have told you here, so that balanced of time the total current density comes out by this formulation which has been given here because once we have attempted this we are actually able to predict the iron migration rate just because you know the j is.

Actually indicator of how fast the irons would move and at what rate the irons would move so that the work pieces starts dissolving a way.

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Now the removal of the work material causes the surface of the of the work piece to be seen in the y direction, with respect to the original surface with velocity given by Q – whether q – is the volume rate of work piece metal removal per unit area of the work piece surface thus the rate of change of this gap between both electrodes is given by again the expression let us say $dy/dt =$ the rate at which resolution would happen which increases the gap so we write this as $A(I_a)$ obviously the rate has to be per unit distance so therefore you divide the volume rate or removal with respect to the area okay divided by the area which gives you the way at you know the amount of distance per unit time that the surfaces is going to received.

Because of the volume rate of removal Q okay, so in this case the volume rate is basically $A(I_a)/pzf$ you already know that Q was earlier derived as AI/pzf so essentially we are looking at Q/unit A or the way at which the surface is receiving because of dissolution okay, so dy/dt dissolution okay. So this becomes equal to one component and obviously I think I have mentioned earlier that the work piece also is moving by a distance F towards the tool in the other direction in the negative wide direction. So I should reduce that or minus that F per unit time with respect to the dy/dt dissolution which has been formulated okay, so this we can record as A_j/pzF which is the dy/dt dissolution okay, -f so that is how dy/dt is constituted.

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This equation becomes

$$\frac{dy}{dt} = \left(\frac{KA(V-\Delta V)}{\rho z F} \right) \frac{1}{j} - f$$

Account for some details - electrode combination operating on other voltage

$$\therefore \frac{dy}{dt} = \frac{\lambda}{j} - f$$

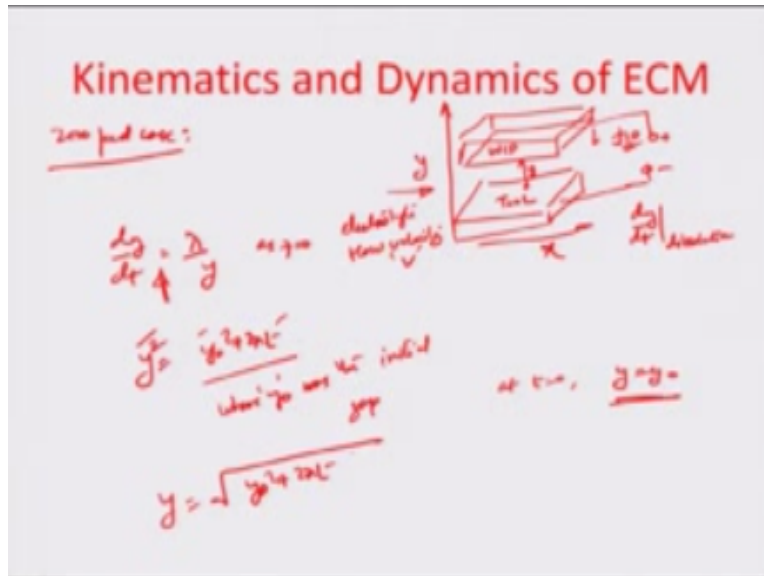
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This is the basic equation representing the dynamics of an ECM process. Let us now investigate some specific cases.

So this equation becomes equal to $dy/dt=(KA V -\Delta V/\rho zF)/y-f$ obviously because from the earlier illustration you find that you know the j current density has been earlier protected as $K(V-\Delta V/y)$ so with the substituting the value of j in this expression here to find out the net value of dy/dt . So this is how your eye want an expression for the rate of change of gap between the tool work piece in ECM on electro chemistry, therefore dy/dt can be recorded as some constant $\lambda/y-f$ obviously all these functions if you do not really change the voltage too much and not certainly not all the electrodes on the electro light combination they are more or less similar to each others.

So this is actually a factor λ constant for same electrode electro light combination operating at a given voltage, so this becomes then equal to $\lambda/y-f$ and this is the basic equation that represents the dynamics of the whole ECM process. We will now try to evaluate some specific cases based on how this would vary for different conditions of operation or parameters of operation in a ECM process.

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So let us investigate the first case which is actually to define as a 0 feed case, so this is an illustration where the tool and the work piece as I had shown you earlier let us say this is the work piece is moving with a velocity f okay, and this is the tool so these are connected respectively to the positive and the negative terminal and this is the x this is the y direction this is of course the x direction and there is some kind of a electro light flow and velocity, electro light velocity v in the x direction.

So having said that now if we assume the $f=0$ and the reason the only reason for changing of this gap y is the dy/dt dissolution or just because of dissolution the gap is changing so in that event obviously dy/dt would be equal to λ/y as f is 0 and if I would solve this equation y becomes equal to $y_0^2 + y\lambda t$ where y_0 was the initial gap. In other words you can think of it that you know a time $t=0$ y was equal to y_0 and the way that y would evolve is going to be a function of y_0 and this is going to be function of t , right. So y is basically equal to the under $\sqrt{y_0^2 + 2\lambda t}$ so if we further wanted to sort of plot this.

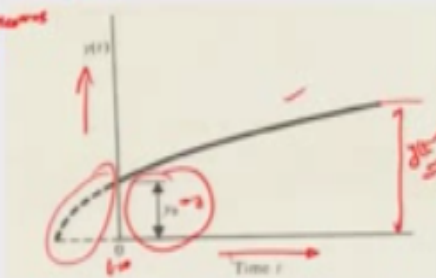
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Kinematics and Dynamics of ECM

So, the gap increases with time obeying the relation

$$y = \sqrt{y_0^2 + 2\lambda t}$$

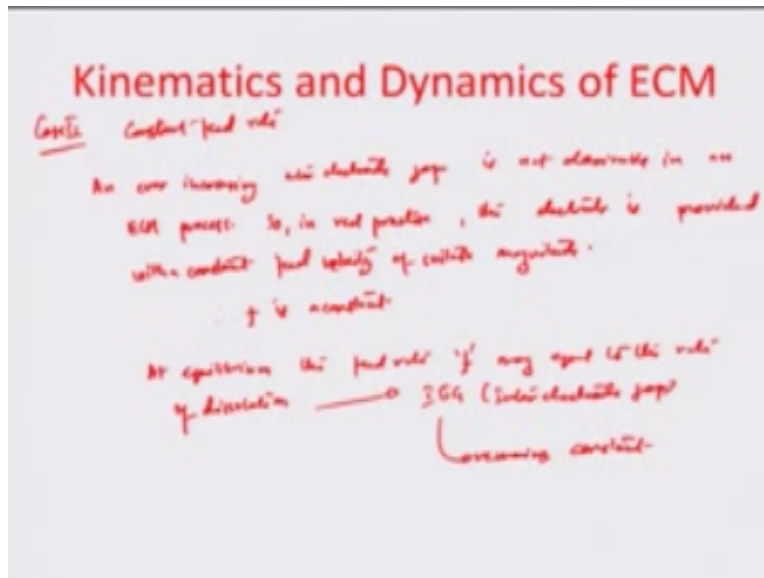
if y_0 is the initial gap
then the dissolution becomes
insignificant.



So you can say that the gap increases with time obeying the relation the zero feed cases $y = \sqrt{y_0^2 + 2\lambda t}$, so in other words this is toward the y_0 if you plot y versus t , y is on the y axis t is on the x axis obviously it is going to be a parabola about the x axis in this particular case the corresponding value of y happens to be at $t=0$ and this is a sort of imaginary projection which is not really necessary so that is how the gap keeps on changing.

In other words we can say that it is going to be an ever increasing gap up till the dissolution becomes insignificant. In other words we can say that the you know, the gap in this particular case is going to keep on increasing until you know the dissolution rate is affected by the wide distance between the tool and the work piece. So eventually this would kind of stabilize and that would be the maximum value of y which would be attainable for after a long duration of time has passed in this particular case.

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Case two which is that of the constant feed rate so for all practical limitation of the ECM process and ever increasing inter electrode gap is not desirable in an ECM process. And as you saw in the last case till and until the time is infinitely passed the gap keeps on almost getting in you know keeps on increasing. Even though where there is a small amount about at some particular value probability the field drop down and it is not enough to sort of you know start or keep dissolving the electrode and that is the probability a point of time where you can say that there is a constancy by and larger of the gap okay.

So but then that is a large gap and so typically this is not really very desirably show that the gap keeps on increasing or is very large after certain time T as past on and therefore in real practice the electrode is provided with the constant feed velocity of suitable magnitude. Therefore f is a constant, so you can say that you know at equilibrium the feed rate f may equal the rate of dissolution.

And other words as you saw before that there is a dy/dt desolation that may be equal the feed rate and in that event the IEG the inter electrode IEG the inter electrode gap remains constant. So mathematically let us examine what this condition is corresponding to.
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$$\frac{dy}{dt} = 0 = \frac{\lambda}{y_e} - f$$

$y_e \rightarrow y$
distance y at equilibrium y_e

$\text{or } y_e = \frac{\lambda}{f}$
Non dimensional

Let us use 2 non dimensional variables
 $\bar{y} = \frac{y}{y_e}$
 $\bar{t} = \frac{t}{\tau} = \frac{f y_e^2}{\lambda}$

So we are talking about k is where DY / DT overall = 0 other words λ / y_e which we also known as the distance y at equilibrium this is known as $y_e - f = 0$ or in other words the y_e becomes = λ/f , so let us now sort of change g as little bit and try to see how to make this scale independent this equation we want to make this may be non dimensional or scale independent because typically the IG the initial electro cape is very small and if you go to computationally tried to do something while predicting the various values here of the equations that I given here.

So this is the possibility that there may be a memorial term which one dominate and whole the rhythm of the process you know the estimation so therefore it is important that we get a scale factor particular minimum direction here and then sort of try to solve everything in terms of ratio so here also we will doing this so let us now use this non dimensional variables.

So which we can call as y - and t - such that we can tried y - to be ratio of the actual gap per unit the equilibrium gap magnitude y_e so we are factoring in this smallness of the y_e and somehow in this particular level f we operate this through ratios and just see that with respect to all theses into picture how the ratios varies.

You know the function of the ratios and so and then there is the time ratio which is actually equal to $t/y_e/f$ okay so this is the indication of how soon the equilibrium gas has close at the gap closes because of the filtrate of the electrode F which is being applied okay if I wanted to just use the earlier equation $y=\lambda/f$ I could actually write this further has $f^2 p/\lambda$ so these are the two variables

non dimensional variables which will try to substitute back into this equation and try to see what happens.

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$$\int \frac{d\bar{y}}{(1-\bar{y})} - \int d\bar{t} = \int k dt$$

$$[\bar{E} = -\bar{y} - \ln(1-\bar{y}) + k\bar{t}]^{\text{constant of integration}}$$

at $t=0, \bar{y}=\bar{y}_0$ ($\bar{E} = \frac{\bar{y}_0}{\lambda} = 0$ at $t=0$)

$$\bar{y}|_{t=0} = \left(\frac{\bar{y}_0}{\bar{y}_0}\right) = \bar{y}_0$$

$$0 = -\bar{y}_0 - \ln(1-\bar{y}_0) + k\bar{t}$$

$$k = \frac{\bar{y}_0 + \ln(1-\bar{y}_0)}{\bar{t}}$$

$$-\bar{E} = \frac{\bar{y}_0}{\bar{y}_0} - \bar{y} + \ln\left(\frac{1-\bar{y}}{1-\bar{y}_0}\right)$$

So we have now one side dy/dt okay which is actually you know that y is given by fy/λ and obviously is given by $f^2 t/\lambda$ just wanted to execute this in the last step here y_e again if you put would come out to be void divided by λ/f okay or yf/λ so obviously dy/dt would then be equal to f/λ times of $dy/f^2/\lambda$ times which is actually $1/\lambda dy/dt$, sorry $1/f dy/dt$. So that is how we represent $d\bar{y}/d\bar{t}$ and if I wanted to just further substitute the value dy/dt from the last step which was $\lambda/y - f$, you may recall the last equation, the equation of dynamics of these ECM process.

I would like to arrange little bit, I would write this as $1/fy\lambda - 1$ and in other words it can be written as $1/\bar{y} - 1$. So we can actually try to solve this equation $d\bar{y}/d\bar{t} = 1/\bar{y} - 1$, so as we recall this would be $1 - \bar{y} - \bar{y}$ and I wanted to just apply some partial fraction knowledge to solve this equation right here.

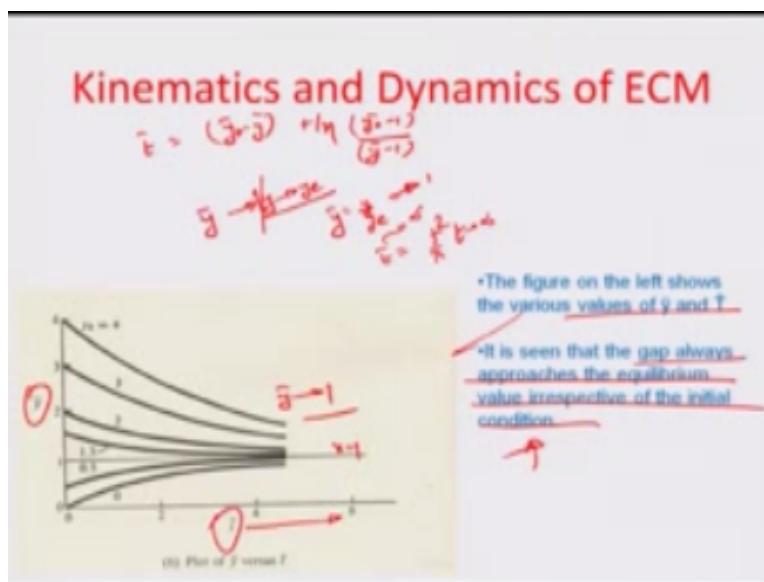
We can $\bar{y} 1 - \bar{y} = \bar{t}$ in other words let us just look at as $1 + \bar{y} - 1/1 - \bar{y}$ it is very much possible, the once cancel out with respect to each other and we left with only \bar{y} okay. So does not really matter if we lay this out as $1 + \bar{t} - 1$ in the numerator, times $d\bar{y} = \bar{t}$, so these are now enabling us to actually integrate, so we could actually try to make two integrals here $dy^- / 1 - \bar{y}$ is 1 and $-\int d\bar{y} = \int d\bar{t}$ and in other words I can easily say that $d\bar{t}$ is basically $-\bar{y}$ natural of $1 - \bar{y} + k$ is the constant of integration.

This is the infinite integral that we are trying to just solve. So let us now try putting initial conditions to check whether we can find out the value of k, so let us say at time $t=0$, the y value = y_0 so obviously \bar{t} which is actually $= f^2 t / \lambda$ should also be 0, at time $t = 0$ and we can actually write down the value \bar{y} at time $t = 0$ as the value of y at $t=0$ ye.

Let us call this factor \bar{y}_0 , so once we have let this out we will have the, if you know just substitute the value of t_0 t at 0 and you know the value of the skilled ratio \bar{y} is 0. In this expression which is obtained we have 0 on one side - $\bar{y}_0 - \ln 1 - \bar{y}_0 - \ln 1 - \bar{y}_0$, k becomes $= \bar{y} + \ln 1 - \bar{y}$ is 0. So if I substitute the value of k back into this definite integral the \bar{t} comes out be = the - $\bar{y} + y_0$, so I will just substitute $\bar{y} + \ln$ of $\bar{y}_0 - 1 / \bar{y} - 1$.

So that is how you can estimate the value of \bar{t} were these are the corresponding gap ratios at, so if I wanted to sort of plot the \bar{y} with respect to \bar{t} value, we can see that \bar{t} here.

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Is really $y = 0 - y + \text{natural log of } -1/y - 1$, okay? So if we plot this \bar{y} with respect to \bar{t} as we can see here, the values of \bar{y} actually changes to 1 okay, unity because y changes to y_e after a long point of time and so obviously \bar{y} which is y/y_e should be approaching 1. So if \bar{y} is again $f^{2/\lambda t}$ if it is very large value of t , so \bar{t} would go to infinites, so you can see \bar{t} slowly the \bar{y} approaching one the limit 1 okay. so the figure on this sides shows the various values of \bar{y} and \bar{t} .

It has seen that the gap whole is approached the equilibrium value with the respective of the initial conditions, so we are deterministically able to say that the final gap that would come out is going to be equilibrium gap through all this a mathematical analysis. So with this I would like to end this module in the interest of time, in the next module we will take up a little more different topic about tolerance in ECM and also there are other design parameters related to ECM like.

For example the velocity of the flow of electrolyte which would result in you know some kind of a design requirement of the strength of the electro materials, which are in place or things related to surface defects which would come in ECM extra but that we will look at in the next module, as if now like to close this particular module thank you very much.

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