

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**

**Manufacturing Process Technology – Part- 2**

**Module -12**

**Determination of MRR of Ultrasonic Machining Process**

**by**

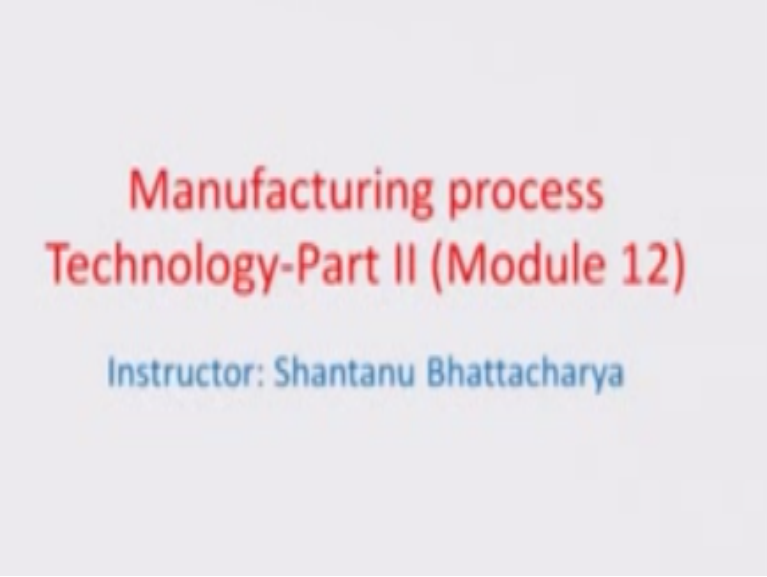
**Prof. Shantanu Bhattacharya**

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Hello and welcome to this manufacturing process technology part 2 module 12.

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**Manufacturing process  
Technology-Part II (Module 12)**

**Instructor: Shantanu Bhattacharya**

We were talking about the USM process and electronic machining process and the Shaw theory related to the material removal estimation through a geometrical model, so in this model the direct impact of the tool on the grains in contact is considered separately.

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### M.C. Shaw's model of USM mechanics

- In this model the direct impact of the tool on the grains in contact with the work piece is taken into consideration. Also, the assumptions made are:
  1. The rate of work material removal is proportional to the volume of the work material per impact.
  2. The rate of work material removal is proportional to the no. of particles making impact per cycle. ←
  3. The rate of work material removal is proportional to the frequency (no. of cycles per unit time).
  4. All impacts are identical.
  5. All abrasive grains are identical and spherical in shape. ←

And then there is another module which is constructed by assuming that there is a grain throw case where the grain comes and hits into the tool and then by virtue of the momentum transfer of the fast-moving tool the grain is impinged on to the work piece surface and it hits the surface and creates a fracture so there are two different ways one is a direct ploughing or a direct hammering the second cases where the grain makes an impact by just a free throw of the grain.

Between the gap created by the vibrating you have some tool in the work piece so and the assumptions that are made are that the rate of work material removal is proportional to the volume of the work material removed per impact the rate of the work material removal also is proportional to the number of particles making impact per cycle and the again the MRR is proportional to the frequency of the tool head.

That means the number of impacts made by the tool per unit time a number of cycles that are associated with the tool per unit time and then we assume certain other basic things like the identical nature of all impacts and the abrasive grains and by and large identified to be spherical in shape although they are not true, truly spherical abrasives are really sharp edged grains and then obviously the material removal also to a large extent depends on that sharp edge of the grain.

But in this particular case just for geometrical assumption and just for derive something more simplistically we assume all the grains to be spherical and creating brittle fracture just by penetrating perpendicular you know to the work piece surface so these are some of the

assumptions that makes in order to find out at least the order of magnitude estimation of various parameters related to the user process.

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**USM process**

- Thus, volume of work material removal rate (Q)

$Q \propto v Z \nu$

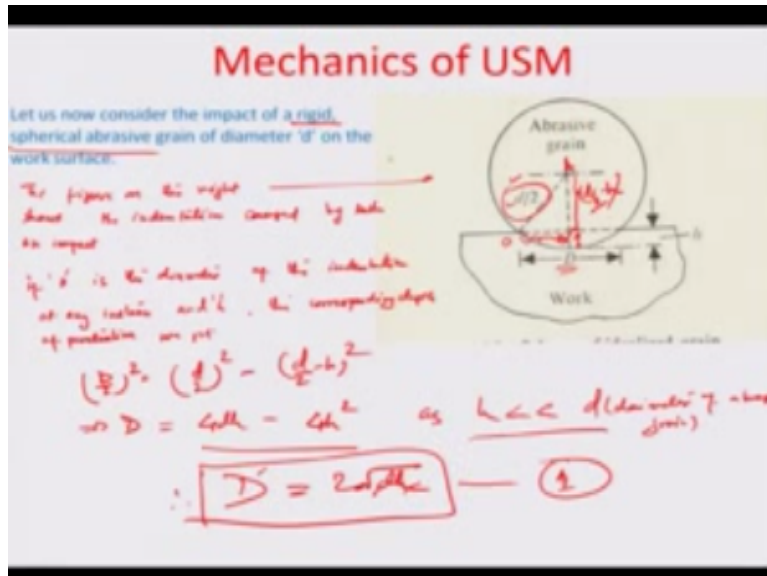
where,  $v$  = volume of the work material removal per impact

$Z$  = number of particles making impact per cycle

$\nu$  = frequency

So typically if we look at it mathematically the material removal rate  $Q$  is proportional to volume of the work material removal per impact proportional to the number of particles making impact per cycle and then the cycles per second or frequency at which the tool is making impact, so it is  $v z \nu$  which is proportional to the work material removal  $q$ .

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Let us assume that there is a case where there is an abrasive particle the spherical particle which is impinging on to a work surface as you can see in this figure right here the diameter of the abrasive grain is assumed to be small  $d$  and the total diameter of the area which gets affected because of such an impact at a certain point of time when the depth of penetration of the abrasive grain is  $H$  is given by  $D$ .

So it is basically a circular area on the surface which gets affected which is a projection of this abrasive spherical grain and the diameter of the circle on the work surface is given by capital  $D$  so having said that now let us consider this impact of this rigid a spherical abrasive grain on the work surface and what is the geometric relationship that exists between the various parameters which are mentioned here.

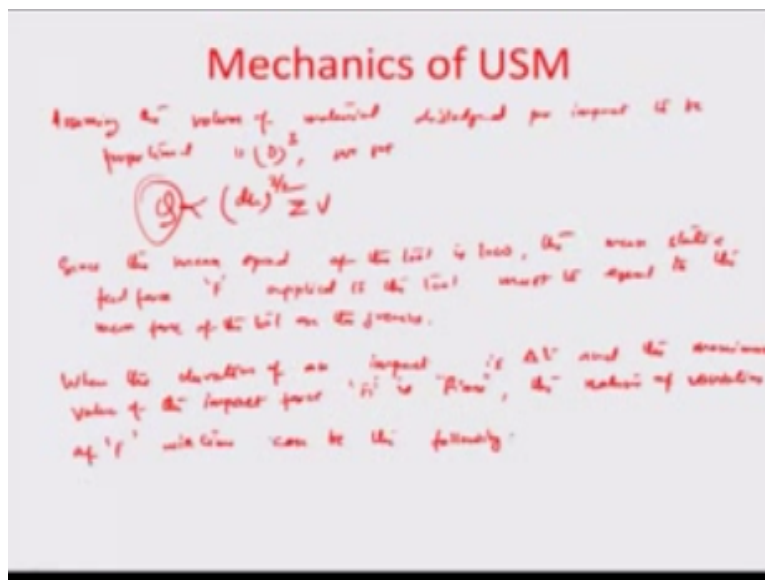
So maybe workout some active relationship with the material properties in the material removal rate, so the figure on the right shows right here shows the indentation caused by such an impact if capital  $d$  is the diameter of the indentation at any instance then and also we assume that  $H$  corresponding depth of penetration, then we get that capital  $D/2$  is really equal to the small  $d/2$ .

That is the main abrasive grain diameter here –  $(d/2 - h)^2$  that is how you know you can you can relate this length here is  $d/2 -$  length  $H$  which is happening here because of the depth of penetration and so typically this OAD is the right angle triangle, so this is square or the square of this length is actually equal to square of this length minus square of this particular length  $D/2$  minus  $n$ .

So how that is how you get the value of the indentation diameter  $D$  in other words we can get  $D$  to be equal to  $(4dh - 4^{th})^2$  and as the depth of penetration is very small in comparison to atleast the projection of the diameter or the other grain diameter or even the average grain diameter itself probably it is tenth of what the average grain diameter is which is correspondingly which is which is actually about 15 to 20 or even beyond 15 to 50 microns in range.

So this  $D$  is the average diameter of abrasive grain, so therefore capital  $D$  becomes equal to  $2\sqrt{dh}$  and that is how you get your first relationship here which talks about how the indentation diameter can be related to the diameter and the depth of penetration of the grain okay.

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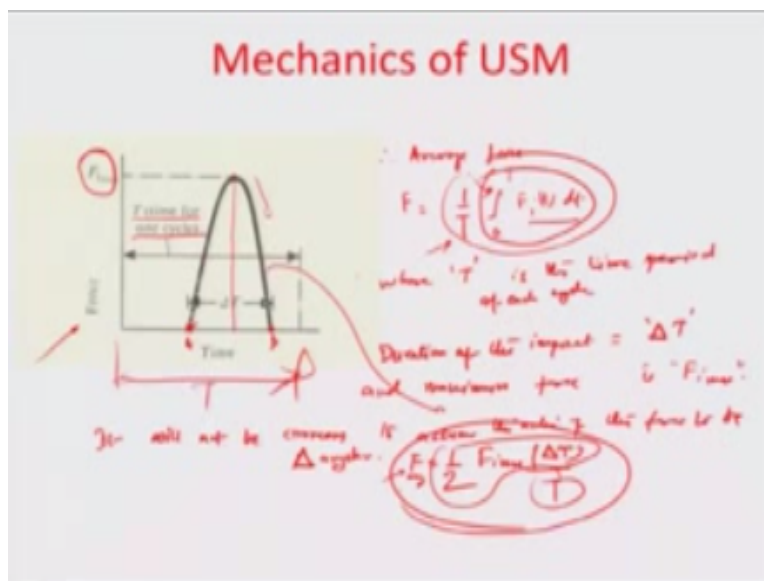


So assuming the volume of material dislodged for impact to be proportional to the  $D^3$  every easily get  $q$  from shore 0 to be equal to the volume that is involved that is  $dh^{3/2}$  remember twice  $d$   $h$  whole of the route was what the diameter  $D$  capital  $D$  was supposed to be, so obviously and the two factor is of course a constant of proportionality so we can take it off and then we have  $Z$  times of  $V$  which creates this factor  $MRRQ$ .

So since the mean speed of the tool is low in the mean static feed force  $F$  applied to the tool must be equal to the mean force of the tool on the grains many even the tool is so heavy in comparison to the grains that it is almost a direct momentum transfer of the tool and the grain mass does not really create any kind of inertial impacts on back to the tool okay, so the de Lambert's force given by the grain mass of the tool is much smaller and it is almost a direct transition of the momentum of the tool onto the work piece onto the one to the grains.

Because of the much heavier mass of the tool in comparison to the grains okay, so it is almost a direct transfer you could consider the mean static force  $F$  applied to the tool must be equal to the mean force of the tool on the grains directly, so when the duration of an impact is about let us say small  $\Delta T$  and the maximum value of the impact force if  $I$  is if  $I$  max the nature of variation of the force  $F$  with time can be the following okay. So let us look at how this variation of  $F$  would be.

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So typically the force is starting at some point along the cycle of the whole grain impingement process and if the cycle of the so the tool if you look at the tool it basically starts from a mean

position starts going towards the work piece or until it squeeze the grain between the tool surface and the work piece surface the force on the grain can be zero because the grain is moving exactly in the direction of the tool.

But with this time period on board or with this as a part of the time  $p$  total time period of oscillation of the tool there is a substantial amount of path length whether the force felt on the work piece is almost zero because the grain is not yet touched but when the grain has actually started touching and now it is giving a real reverse reaction so that is really where the force comes into picture.

And so obviously as the tool progresses and the grain kind of impinges on to the work piece the force by the grain onto the work piece keeps on increasing to an extent where it is able to cross over the ultimate yield stress of the material okay and beyond that there is a fracture so the grain and the fractured piece gets dislodged automatically in the force comes down to zero so that is how you basically representing this trend here that the force at certain point of time when the grain has started pressing.

Let us say at this particular point A is starting to increase all the way up to a maximum force of  $I_{max}$  beyond which there is a flow stress which comes into the material and there is a fracture and a failure and beyond that the force again comes back to 0 because now you can think of as if the tool is also receding a way that is because you know obviously it is coming to the extreme position and then again returning back to the mean position at the same time because of the dislodgement and everything else there is a cavity now and the fractured piece as well as the grain has moved off so there is no force active force of the work piece anymore unless the next grain comes into picture unless there is a another such interaction between the tool head and the grain which impinges.

So in one particular cycle when there is one you know near proximity of the tool to the work piece and again going back to the mean position you should have at least one if you consider only one grain you know to be squeezed between the work piece and the tool there should be one such force starting from zero all the way to  $F I_{MAX}$  and then back to again the point B were really there is no contact again of the grain to the tool to and the grain and the and the dislodgement has flown away.

You have to remember that there is a high-pressure slurry which you are injecting which has a carrier medium which would be able to dislocate every dislodged piece you know from the work piece coming off the work piece because of the brittle fracture process okay, so that is how one time cycle of force could be shown so the average force therefore the average force here given by capital F is actually  $1/t$ ,  $t$  being the time period times of the  $\int_0^t F_i(t) dt$ , okay. So is the time integral of force divided by the whole time period  $t$  is average force.

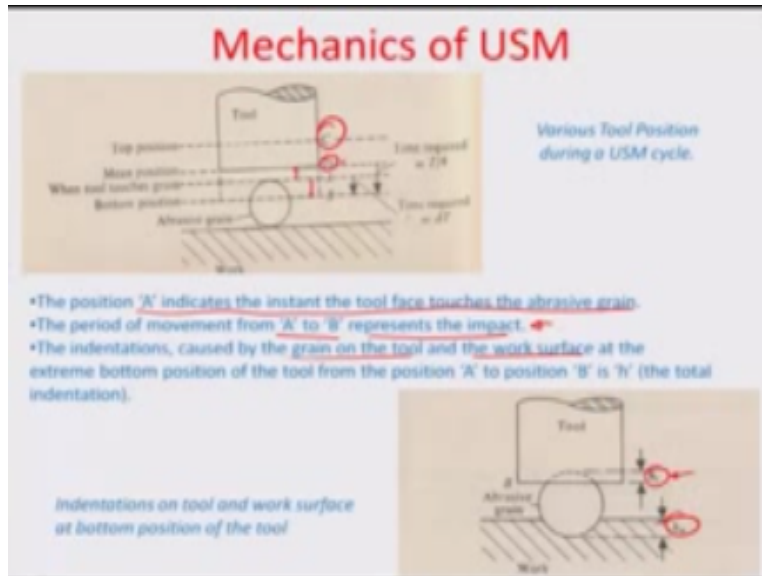
So where capital T is the time period of each cycle so you can think of this as the area under the first time curve averaged out over the whole domain of the time period  $t$  you know which we are considering for this operation, so the duration of the impact is actually  $\Delta T$  as can be seen here and the maximum force is  $F_i \text{ max}$  so it will not be erroneous to zoom this nature here of the force to be triangular, meaning thereby that you know I mean I can actually be able to estimate the area under this curve as the area of the triangle  $1/2 F_{i \text{ max}} \Delta T$  so that is how I can estimate the area under the curve given through this you know expression  $0 \text{ to } T F_i T/dt$ .

And so you can you can understand this is  $1/2$  base altitude, altitude being  $F_{i \text{ max}}$  and again base being  $\Delta T$ , and so if I divide that by the time period  $T$  I get the average force  $F$  average on the situation, so this is only an approximation however and it works very well because obviously this peak is very sharp so you know it goes all the way to an extent where the ultimate yield strength just get succeeded and then there is a fracture initiated so there is a sudden reversal of the force.

So it is quite sharp and quite high quality when we talk about the force time trend as in this particular case. So now if you look at the way that the tool really positions itself, the tool let us say is about a mean position  $O$ .

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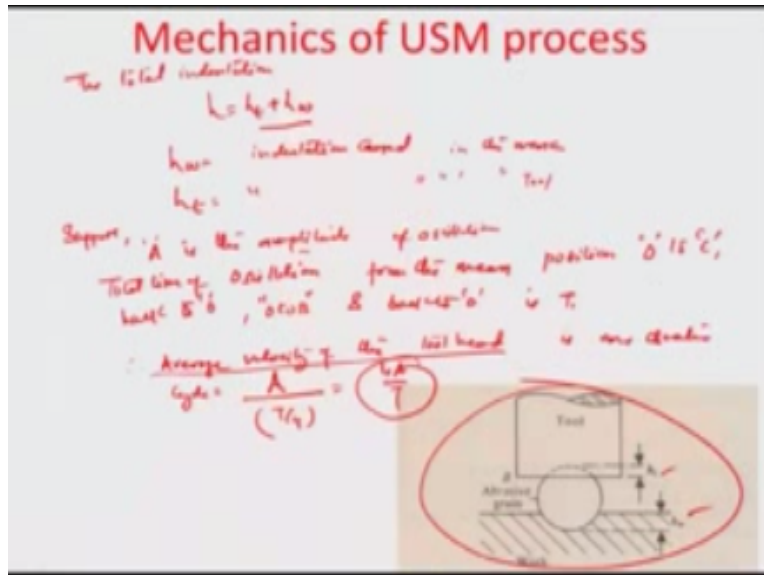


And the position A indicates the instance that the tool face touches the abrasive grain, so the grain is somewhere here lying in touch with the work piece on one side and the tool moves between O and A so that the tool starts pressing the grain the period of movement from A to B represents the impact because now it has to go all the way up to the indented value which is B here so this surface moves to be all the way meaning thereby the grain would actually now move without breaking into the work piece and A,B is really the distance that it would cover like an indentation onto the grain onto the work piece surface.

So the indentation is caused by the grain are both on the tool as well as the work surface, you can think of it here that you know instead of looking at only the work piece if I also were to find out obviously the grain is much harder than the tool as well so there is some depth of indentation on the tool side we call this  $h_t$  and some of the work piece side  $h_w$  and obviously the ratio of hardness's of the tool surface to the work piece is very, very important and is quite malleable whenever we talk about the USM process, okay.

So beyond this position be the tool receives back passes again through the mean position now go all the way up to another extreme positions C and then comes back to O, so this is how the whole tool oscillates back and forth about the mean position O, when we are talking about particularly ultrasonic frequency at which it does so, so then the total amount of indentation.

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The total indentation as can be seen right here in the figure is actually given by the total indentation on the tool side plus the total indentation on the work piece side okay, so I can always write this as  $h_w$  is indentation caused in the work and  $h_t$  is that caused in the tools side okay. Supposing that  $A$  was the amplitude of oscillation in this particular case the total time of oscillation from the mean position  $O$  to  $C$  back to  $O$  again  $O$  to  $B$  and again back to  $O$ , if that is equal to time period  $T$ .

The average velocity therefore, of the tool is exactly one quarter cycle having amplitude  $A/(T/4)$  so you have  $4A/T$  as the average velocity of the tool head which you get realized because of this fast moving you have to think of it that the amplitude  $A$  is basically from mean the extreme position and in the whole time period  $T$  you covering at least for such amplitude movements about the mean position.

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**Mechanics of USM**

∴ total time 'Δt' needed to move distance

$$h_{avg} = \frac{h}{\text{Avg. vel. } \left( \frac{v_w + v_t}{4} \right)} = \frac{h}{(v_w + v_t)}$$

$$F_{avg} \approx \frac{1}{2} F_{imax} \cdot \frac{h_{avg} \cdot T}{A} = \frac{h_{avg} \cdot T}{4A}$$

$$\therefore \frac{F_{imax}}{4} = \frac{8 F_{avg} A}{(h_w + h_t)}$$

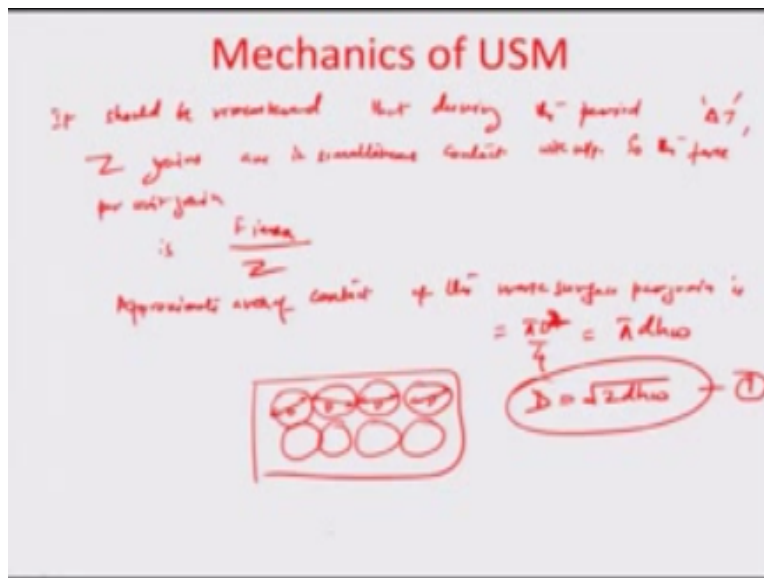
So having said that now the total time needed if I wanted to calculate to move the total indentation distance that is h should be equal to h divided by the average velocity of the tool, so in other words it is actually  $h/(4A/T)$  when h can already be supplemented with the tool work piece indentations times of  $T/4$  so that is how the average velocity that is how the total time  $\Delta T$  that is needed to move the distance  $\Delta H$  or move the distance h is represented, okay.

So mind you again this h is composed of the indentation across the tool as well as the work piece. So the average force on the other hand as we had calculated earlier is actually equal to  $1/2 F_{imax}$  times of you know the time  $\Delta T$  which I had just calculated time times  $\Delta T$  is obviously the time in which the grain gets indented, so just substitute this value  $h_t + h_w / A \cdot T / 4 \cdot 1/T$  that is how the average force was represented.

If you may look at the average force here was  $1/2 \cdot F_{imax} \cdot \Delta T$  which is actually the total time period needed for the four cycle to go to  $F_{imax}$  and back in the graph earlier, so having said that now the  $F_{imax}$  therefore can be represented as eight times the average force times the amplitude divided by  $h_w + h_t$  the whole goal behind what we are trying to do here is this  $F_{imax}$  can be very easily correlated if we knew the area of the indentation to the ultimate in strength of the material which is actually a property.

So with that on board we would like to do something so that we have now a very important estimate of what is going to be the average force, what is going to be amplitude obviously the depth of indentation can also be studied by looking at the various features on the surface which get formulated and so that is how I would like to bring in something related to the property and something related to the geometry to sort of predict what is going to be at this order of magnitude wise the material removal rate.

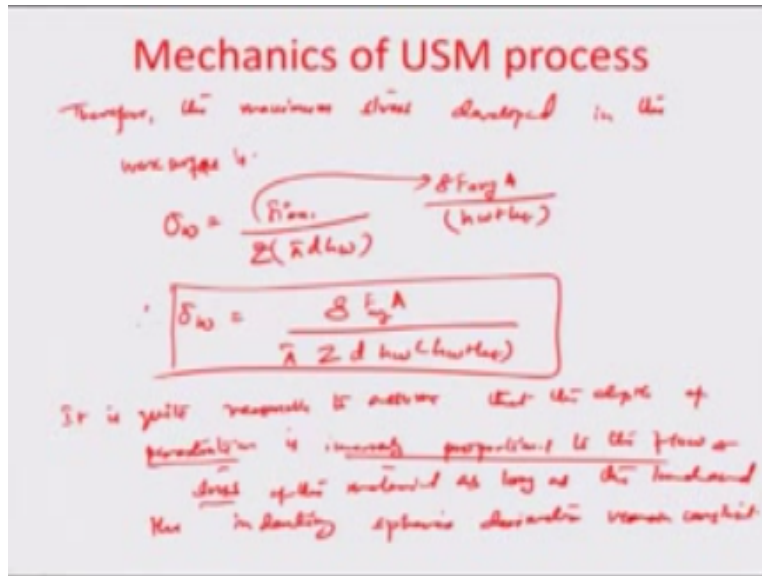
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So it should be remembered that during the period  $\Delta T$  around Z grains are in simultaneous contact with work piece and they are all being simultaneously plowed into the work piece by the tool head and so the force per unit grain is represented by the  $F_{max}$  per unit Z okay, so the approximate area of contact of the work surface per grain you can think of a case that the grains are all spherically indented into the material, right.

So if i look at the top view they are all half diameter indentation diameter D to the work piece, so the total area of contact of the work surface per grain that is available I will actually be equal to  $\pi D^2/4$  okay, and so this results in  $\pi dh_w$  because D is  $\sqrt{2dh_w}$  has had been calculated earlier in equation 1.

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So therefore the maximum stress developed in the work surface is  $\sigma_w$  equals force of  $F_{\max}/Z$  that is force per unit grain divided by the area of contact of the grain which is  $\pi d h_w$  or in other words  $\sigma_w$  of the total stress exerted by the grain at the indentation point is given by  $AZA$  this is the average force okay, this is the  $F_{\max}$  so  $8$  average force  $A/\pi Z d h_w \cdot h_w + h_t$ . I am just merely substituting the value of  $F_{\max}$  in terms of the average force as borrowed from the earlier illustration just made few lines back.

So that is how you can actually record the total amount of stress pollen grain and so if it is quite reasonable to assume that the depth of penetration is inversely proportional to the flow stress of the material and this is so as long as the load and the indenting spheres diameter remain constant okay, the load varies or the diameter of the indenting sphere varies which happens because there is a grain to grain variation in terms of the diameter obviously changes this approximation that penetration being inversely proportional to the flow stress.

So I am going to now use this probably in the next module to plug the material properties into this equation, so that now we have everything in terms of material properties and some of the geometric parameters which are involved and so that is how you find MMR in case of USM, so we will do that in the next module there until then thank you so much for patiently listening, thank you.

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