Nature and Properties of Materials
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Lecture 29
Cantilever Design (High Stiffness and Light weight)

Good morning, our today's focus is material selection for the design of a cantilever beam. So we will apply our knowledge of materials while actually working for a design problem. And we will also use the knowledge of Ashby chart. So the problem statement is that you have to select a cost-effective material for a circular cross-section cantilever beam loaded at its end having high stiffness and lightweight.

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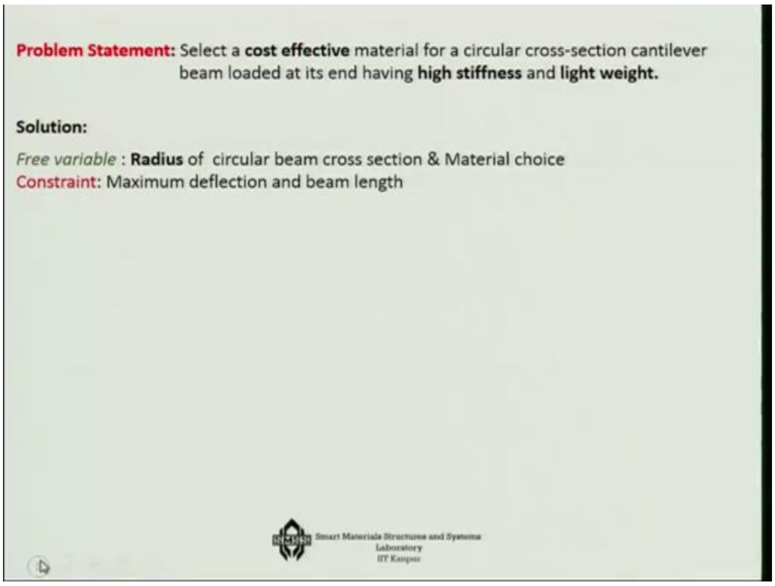

And since the length or the span of the cantilever beam is always fixed, so what you can play with in the geometric is with the radius of the circular beam cross-section and also the choice of the material, so these are the 3 variables for us and the constraints will be the maximum deflection and the beam length. So let us look into it that how we can solve such a problem and then how we can use our knowledge of materials after solving such a problem.

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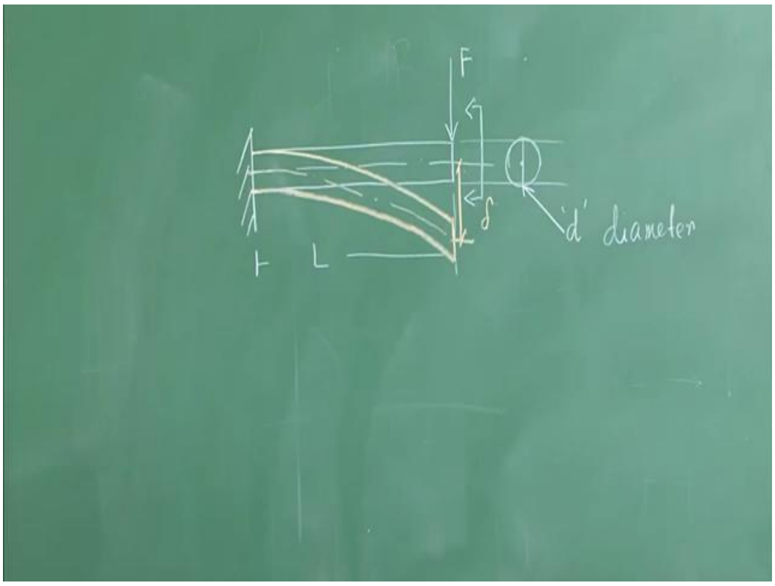

To begin with that what is important for us is a circular cantilever beam that means if I look at it from this direction, then it is a circular cross-section. And the diameter d of this circular cross-section is a free variable, okay. This is a cantilever beam and the span of the beam is let us say, the length is L, what is the loading condition that is carrying a end load, so it has a end load here the end load is F okay. Now we have to have a little bit of knowledge on the deflection or the strength of materials basically, the deflection of cantilever beams how it looks like.

So if you try to exaggerate the deflection let us say the load is quite high then this is the way it is going to deflect, which means that the central point of the beam and it is here the same centerline, this distance is Delta, so we can keep in our mind that what are what is the free variable that is d and material choice. What is the constraint, what we cannot play with? Length of the cantilever L and tip deflection Delta, we cannot play with these 2, if the deflection is too high, then the cantilever beam will be kind of unusable. And if the length changes then also functionally become unusable for the force F it has to carry.

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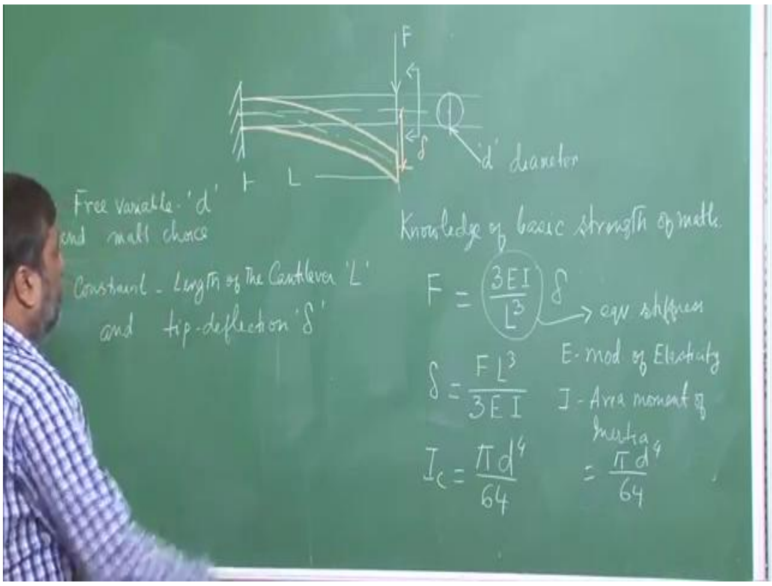

Now from our knowledge of strength of materials, apply that knowledge, knowledge of basic strength of materials. We know that the end force and the tip deflection, they are related in such a manner that we can say that F equals to 3 E I by L cube times Delta, where 3 E I by L cube, this is the equivalent stiffness. In other words, I can also write that delta equals to F L cube over 3 E I. Now in this relationship, we have to keep in our mind that you cannot touch F that is the force, L you cannot, E you can change and I you can change, where E is the modulus of elasticity and I is the area moment of inertia.

Now for circular cross-section that right, so I in our case is actually pie d to the power 4 over 64 okay, so I c is pie d to the power 4 over 64. Now let us try to use all these facts together and let us try to carry out the analyses further to find out something called the material index or the performance index. So we already know that delta equals to F L cube by 3 E I, so the other point is that our objective is to get a light, so objective is to get a light and stiff beam, this should be light enough so that generally if it is light you are actually the total weight is less and hence the cost will be less.

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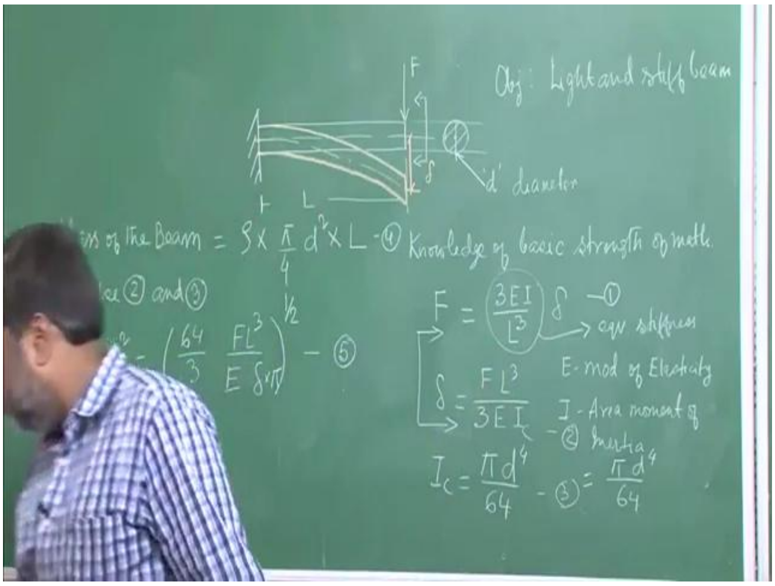

And it should be stiff enough so that the deflection does not go beyond a particular level. So what is the mass of the beam? Mass of the beam is actually density of the beam times the volume of the beam itself, so we can write the mass as Rho times area of the beam that is pie by 4 d square times L that is the mass of the beam itself considering that it has a uniform circular cross-section that is the mass of the beam. So then, and we have to make it as a lightweight right okay.

What is d in terms of our 1st equation? Let us look into that, in this equation that means the further developed from this one okay. So in this equation if we try to use this equation, so let us give these equations some numbers, 1, 2 and 3 and this to be our 4th equation. So let us use 2 and 3, so what we can get is that I is here and that is nothing but I c, which is pie d to the power 4 by 64, so I can write d square equals to little bit of algebra, 64 by 3 F L cube over E delta whole to the power half that is square root of the whole thing that is what is my expression of the d square.

And substitute this, so let us say this is my equation 5. Yes, we also have E and the pie here, E delta pie right. Now we substitute this whole thing and we try to get an expression, so let us try to erase this part now because we have used these equations so I think we can very easily erase this part, so let us try to erase it and let us try to use this expression of d square that is equation 5 let us try to use it in 4, so use equation 5 in equation 4. If I do that, I can now write the mass of the beam let us say the mass of the beam I denote it as m.

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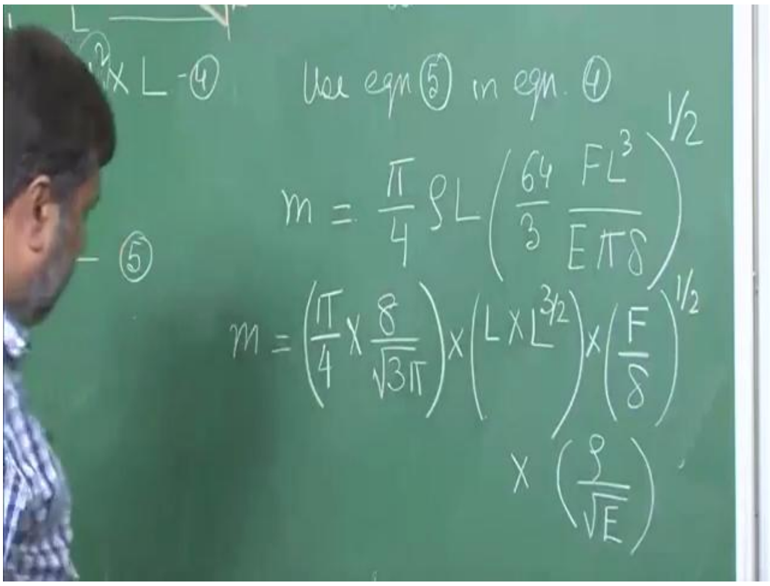

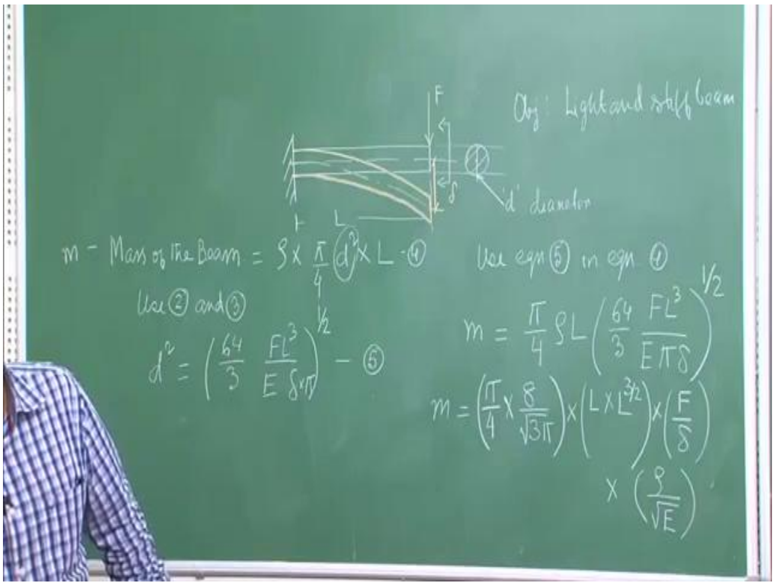

So I can write m equals to what we have, pie by 4 Rho L 64 by 3 F L cube by E pie delta square root right that is the mass of the system that is what we are getting once we are substituting d square term here by this expression of the equation 5, so that is the mass of the system that we are getting form it. So let us try to now write down this mass in terms of some different parts okay. So what we can do is that we have let us say the constant part, so that is pie by 4 that is one constant times we have square root of 8 over square root of 3 pie, so this is like all the constants together, so let us keep all the constants together.

Then let us see, we have some geometric parameters with us, what are these geometric parameters? L times L to the power 3 by 2, right. So that is the geometric parameter that we have so let us keep that also separately. We have some functional parameters, what is it? That is F over delta okay function the force is given to us and the deflection is also the maximum value is given to us. Now, if I take these things apart what else is remaining, what is remaining is the material property of the system. And what is it, Rho over square root of E okay.

So once again let us look into the term by term situation that we have and F by delta we have and we have a half here right okay, so let us look into the whole thing now. The 1st part is the constant right, the 2nd part here this is a geometric parameter, the 3rd part here is a functional parameter, where as the last part here our focus for this course is on the last part that is why we call it to be the material property index because that is the material property related parameter.

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In fact, we can define a performance index PI which is the reciprocal of this MPI, 1 over material property index. So if you do that, then it will be square root of E over Rho, correct. So what should be our objective? Our objective is to get a light beam as well as stiff and the stiffness is such that the deflection delta is satisfied that is why the E term is already here. Now, in order to from all the solutions if I want to get the minimum mass, that means I have to have minimum of Rho over square root of E and that means I have to maximise this term, right.

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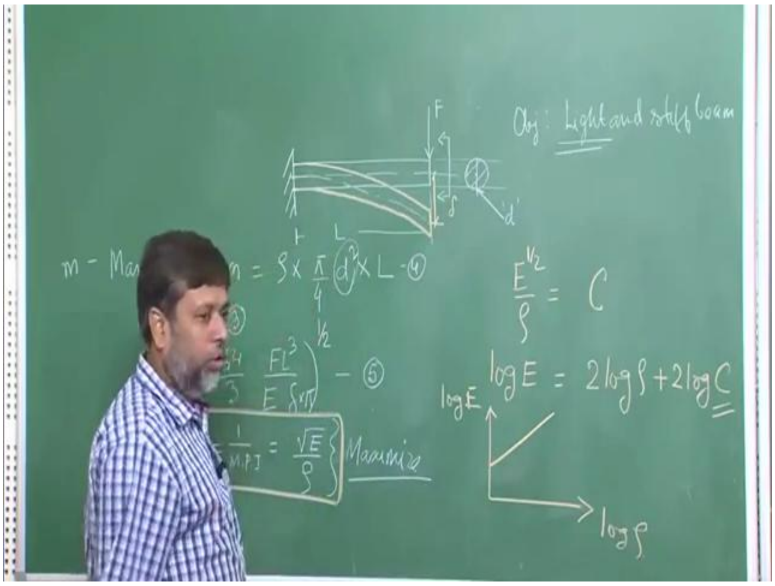

So maximization of this performance index square root over E by Rho is what can give us the solution that is one thing that is clear to us. Now sometimes on top of this we may also say that there is some kind of a cut-off value in our particular analysis. So if I do that, that if I also say that there is a cut-off value, then I can develop an equation. Suppose, I say that E to the power half over Rho has a cut-off value C, so that means you have to maximise it fine, but accretes should be beyond a certain point.

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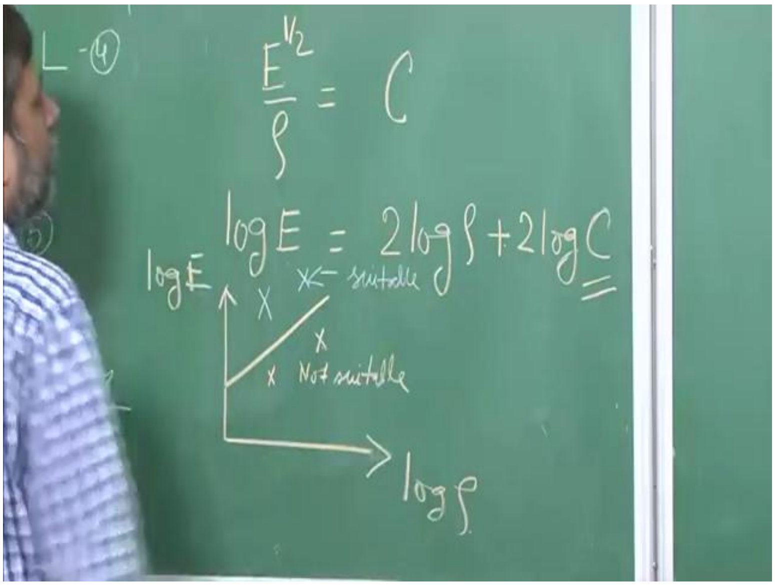

Then that cut-off value if it is C, by taking a logarithm in both the sides I can get it as 2 log Rho + 2 log C. In other words, if we now get a plot of a log-log plot of the system that means log E over log Rho, then depending on this cut-off value C, we actually have a straight line in the log-log plot. And the materials which are below this point, they are not suitable and the materials which are above this point, those materials will become suitable for our application.

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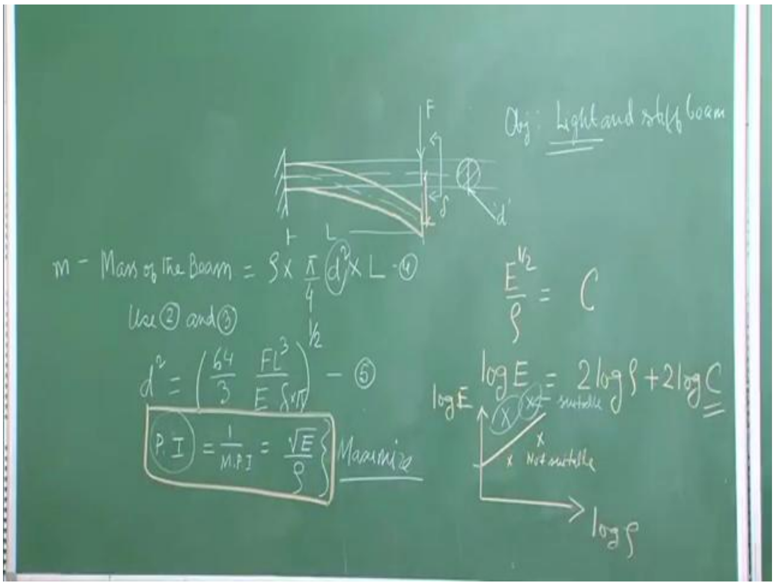

So thus if we have modulus of elasticity versus Rho graph, which is there in the Ashby chart corresponding to various materials and then on that I am looking for a high value of square root over E by Rho because I know the higher this is the lighter will be the beam. To do that also if I know that what is my the cut-off point, then I can actually search materials and find out what will be the what will be a very good material for our applications, so this is what precisely is done using the Ashby chart and let us go back to the Ashby chart to do this part of the work.

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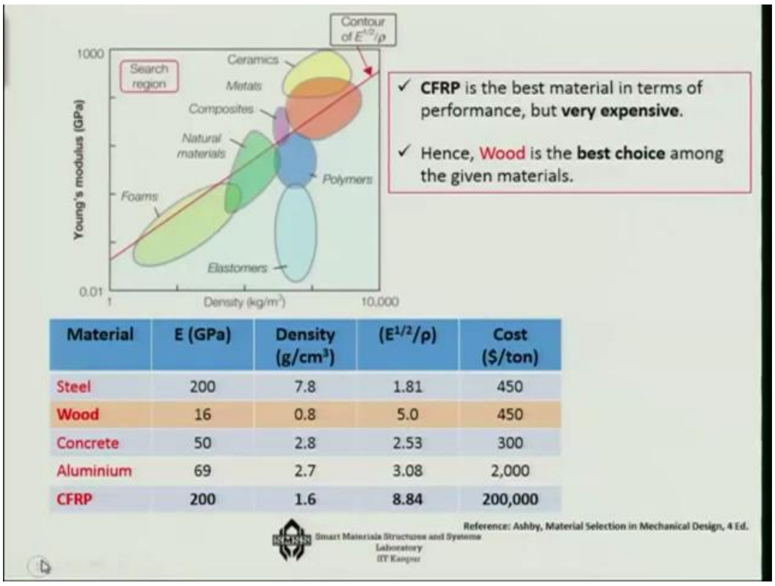

If we now look at the Ashby chart, we can see here that there are various possibilities of density as you can see that you have very lightweight elastomers here okay and you have foams here, which are even lighter. But corresponding to our fixed cut-off value many materials are not actually suitable, so this is not suitable the elastomers are not suitable, group of foams are not suitable, group of natural materials, group of polymers are not suitable.

So material about this that is some natural materials, some composites, some metals, and some ceramics are suitable for us because they are above this region. And what is this line; this is precisely the line that we have just now obtained the line for E to the power half over Rho. Now, on the top part we have options of ceramics, metals, composites, et cetera and some natural materials, so let us try to take some of the materials.

1st we started with the cheapest material that is the steel, modulus of elasticity is about 200 gigapascal, density is about 7.8 g per cc, square root over E by Rho will come something like 1.81, cost is about 450 dollar per ton, so it is one of the cheapest, 1.81 we are getting. Let us look at wood, a kind of a reasonable quality of wood, modulus of elasticity is about 16, density is 0.8 that is less than water it is very light, so I get a high vector, square root over E by Rho is 5, so it is much higher than the steel and this is about 450.

No wonder I can tell you for many of the traditional applications, even today if you look at the Chariot of Jagannath for example, it is made of wood, so wood no wonder is a very good choice it is better than steel because it is light you see that is why for this application it is good. Let us look at concrete it has modulus of elasticity of 50 gigapascal, density of 2.8 gram per cm cube so the factor will come out to be 2.53. Cost wise it is cheap but this vector is showing that it is not as good as the wood is.

Let us look at aluminium, aluminium is one of the lighter metals although it is more expensive, 69 gigapascal, 2.7 density gram per cc, the vector will come 3.08, which means it is better than steel but look at the cost, you are spending thing like 3 to 3 and half times more than steel. Well if pocket is not a problem, why do not we try with CFRP? What is CFRP that is carbon I have talked about it right, Carbon Fibre Reinforced Plastic that is one of the very advanced material that is available today.

It has modulus of elasticity of about 200 gigapascal just like steel but the density is 1.6 much less than the 7.8 density of steel, so what is the vector square root over E by Rho, 8.84, this is the topmost material as you can see even better than wood, but what is the cost? Something like very high cost per ton as you can see here. So, I would say that CFRP is the very good choice but it is very expensive, so if there is some application where you want still a lightweight, but you cannot afford CFRP then you should go for the next best which is steel or otherwise you should think of or maybe the cost wise if you tell me then from CFRP you should go to concrete.

And if concrete is not okay, then you can go for also wood you can try and then your choice will be between steel and aluminium, but though steel is cheaper so steel is better in that way because aluminium is more expensive even though this point is slightly higher, so thus from this simple overall this kind of a problem, what we are trying to show you is that in many practical design applications, it is not just one property but a combination of the properties in a certain way because here it is not simply E and Rho, but the square root of E over Rho, so it is non-linear with respect to the modulus of elasticity.

So in a certain way it actually affects our choice of the material and depending upon the objective, this change. In the next class I will show you some of the examples of similar type. So this is what we are going to close and more problems are coming in the next section, thank you.