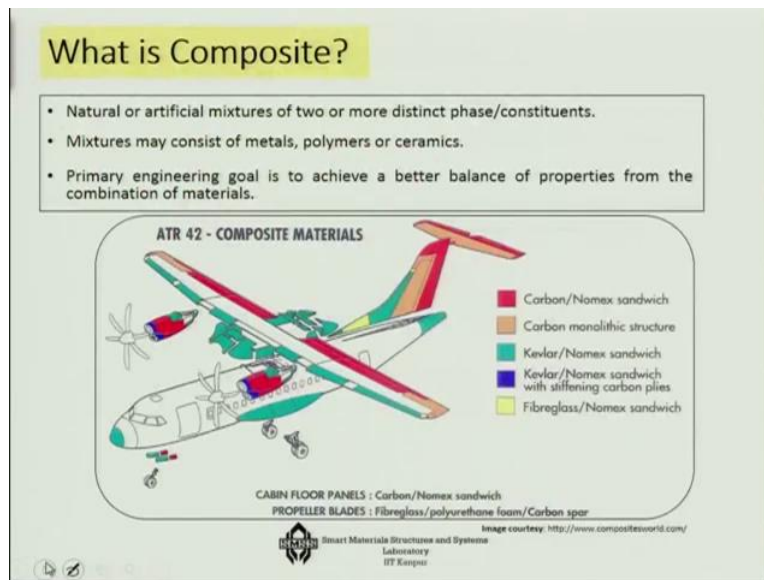


Nature and Properties of Materials
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Lecture 20
Composite Materials 1

Today we are going to talk about composite materials. So we now have a fair knowledge of metals, ceramics and polymers, so with that background now we can develop composite materials, which is actually a mixing of all these various types of systems. So what we are planning to cover today are composite, definition of composite material and a bit of history, a bit of evolution from where this composite materials have actually this whole journey have started and where are we today and also we will talk about the composite classification today.

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So first about that what is a composite material? Now, it is basically as the name suggests that it is indeed a mixture of 2 or more materials, it can be natural materials or it can be artificial materials. But the point is that when these 2 materials you are mixing them together, they have to be mixed such that they remain, there phases are distinct phases or constituents. That means, they should not chemically react with each other and form a third compound.

Then for example, that is how you can say that alloys are not composite because in the alloys, there are definitely a new material compound that gets formed out of it, a new chemical formulation comes because there is a reaction that takes place at the atomic levels, but in composites that does not happen. The 2 materials get mixed with each other very intimately and a mechanical bonding takes place between the 2.

And that is good enough to create certain favorable properties in the material. So the mixture may consist of metals, polymers or ceramics, it can be between metals and ceramics, sometimes of composites, polymers and ceramics or metals and polymers is of course very rare, but that is also possible or polymers and polymers and ceramics and ceramics, so various types of such compositions are possible.

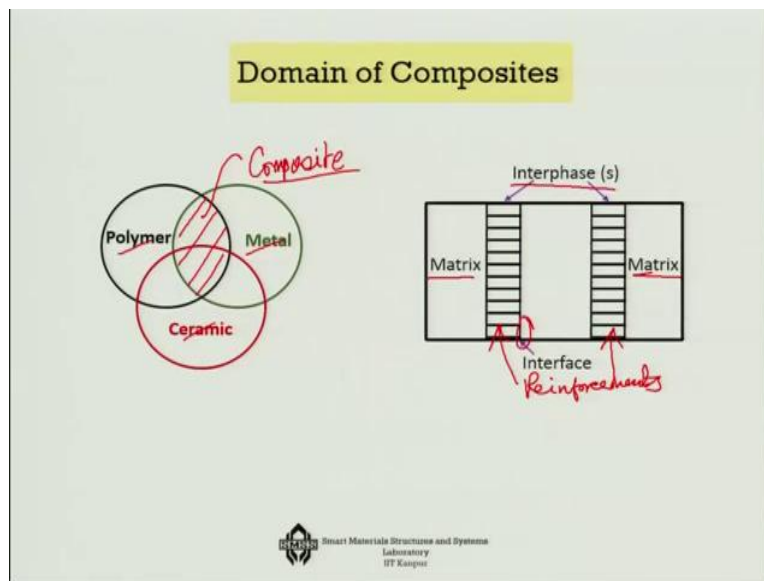
Now, what is the primary engineering goal? Why are we mixing? We are mixing to achieve a better balance of properties from the combination of these materials okay a better improvement in terms of property. What improvements you can think of when you can think of mechanical applications?

Suppose, the whole system must have better strength or better stiffness or better thermal conductivity or better against moisture and the expansion would be lower, so better kind of resistance to moisture, things like that, so various this type of properties you can actually achieve by making a composite. Here is an example for example this is an aircraft.

And you look at it than how many different places at the aircraft today composites are used. Say for example, as you can see that the left part, carbon nomex sandwich that is used here in the radar part that is also here in the backside of the wheels you can see their use, they are also used in the engine portion, so there are many portions, where the carbon nomex sandwich is used okay.

And if you look at it the Kevlar nomex sandwich that is the green part, is used in the (())(4:09), it is used in the tip, it is used in the tip of that the wings and the front part of the radar and the backside. Similarly, if you look at the fibre glass nomex sandwich, the yellow part that is also used in some of the sections. In fact, it is also that is of course a low performance one hence, it is not used much, but that is also getting increasingly used. So thus, you can see all these composites at various places, they are using today's aircraft as much as in some of the aircrafts as much as 60% of the aircraft structure is replaced by composite materials. What other domain of these composites?

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So I told you that it is a domain of mixing of polymers, metals and ceramics, so that means all these domains interaction domains, they are all the composites okay. And in the composites, generally you will find that there are 2 different phases. One phase which is more in terms of volume, we call it the matrix phase, which actually holds the entire system, system integrity and the other phases are the inter phases which are like reinforcement okay.

So the other phase; let us call it more name, the formal technical name is reinforcement. So this reinforcement have interfaces with the matrices and that inter phase is to be designed in such a manner that there is no discontinuation or no de-bonding between the reinforcement and the matrix, if that happens then the purpose of the composite will be defeated.

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The slide, titled "History of Composites", lists four key historical milestones in composite materials:

- **Indus Civilization** (~ 3000 B.C.) - Straw reinforced bricks.
- **Hittites and the Samurai** (~ 100 B.C.) - Steel composites (formed by the repeated folding of a steel bar back on itself).
- **Industrial revolution** (~ 1800 A.D.) - concrete and cast iron.
- **Modern composites** (~1950-present) - fiber-reinforced composites.

Accompanying images include: "Straw + Mud brick" (top right), "Samurai Sword" (middle right, credited to the Metropolitan Museum of Art, Japan), and "Concrete" (bottom right).

If you look at the history of the composites, we can claim that it is our country who are the drawer development of composite materials because something like 3000 to 4000 BC Indus Valley civilisation, you will see the first application of reinforced composites, which is in the form of straw + mud brick that means the clay, which is like ceramics would act as the matrix and the straws, which is like the natural fibre acted as the reinforcement.

So this straw reinforced bricks, they were so much better in comparison to if you look at the bricks at that time in the Mesopotamian civilisation and other civilisation that even today you would see Mohan Jodaro and Harappa and all such places, the evidences of beautiful structures, which are made of these straw reinforced bricks.

And their secret behind this strength is that the straw as a reinforcement, it works beautifully, it adds strength and it adds other properties like thermal insulations okay, moisture prevention, et cetera, so it is the brick system and hence, this is now the best example of the composites that was developed in the you can say in the very beginning of the civilisation.

Another interesting example of the composites this the Samurai Steel composites, which is of course used for the famous samurai swords, so there the steel composites which is you may say that it is the beginning of something like cast iron kind of things, but there also they have 2 different types of metals or metallic systems, which are actually bonded together through a heating and beating procedure.

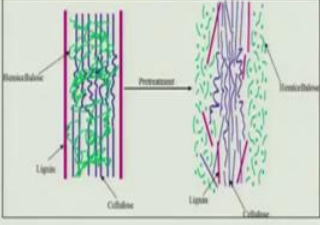
At a later stage, we have as in Industrial Revolution, two great products of composites we got one is the concrete and another is the cast iron, both of them began actually revolutionized post Industrial Revolution. Now then there are many modern composites okay. So concrete and cast iron are composites, but if we look at the reinforcements they are not like a continuous reinforcements, they are more like extensions of particular reinforcements.

On the other hand, the modern composites which are like fibre reinforced composites, they are the truly speaking the composites, which have all the very good properties of a composite for example, the Tailor ability is one property that you may not find in the concrete or cast iron, but you get a very high degree of tailor ability in this modern fibre reinforced composites. So what other examples of natural composites around us?


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Natural Composites

- Exist throughout nature - almost all natural materials
- Some examples:
 - **Wood** = lignin matrix + hemi-cellulose wound in a spiral form
 - **Bone** = organic fibers + inorganic crystals, water and fats
 - 35% of bone consists of organic collagen protein fibers with small rod-like hydroxyapatite crystals



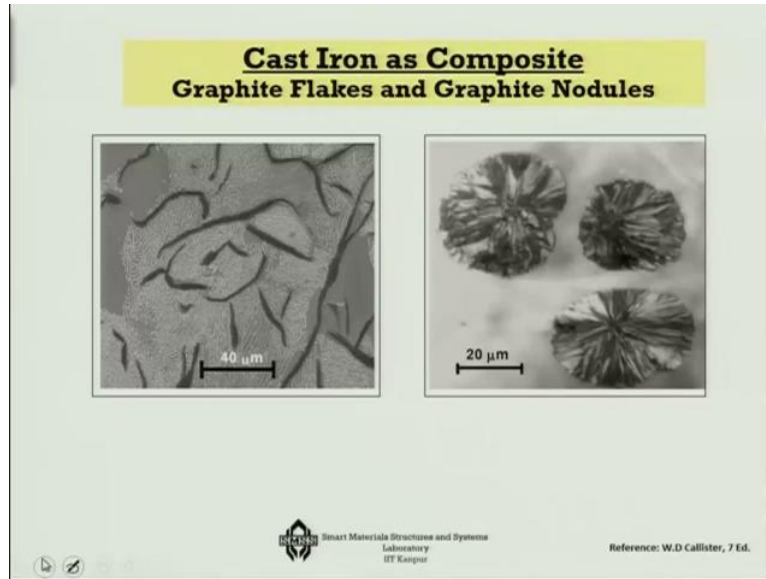
Reference: <http://genomicsgtl.energy.gov/biofuel/>

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There are at least 2 very good examples, one you carry within yourself that is the bone okay, so the bone has the organic fibres and inorganic crystals, waters and fats. In fact if you look at a bone structure, you will find that you have the collagen fibre okay, so collagen fibre is very important because once again the structure of the collagen fibre was actually discovered in India okay by Professor Ramachandran.

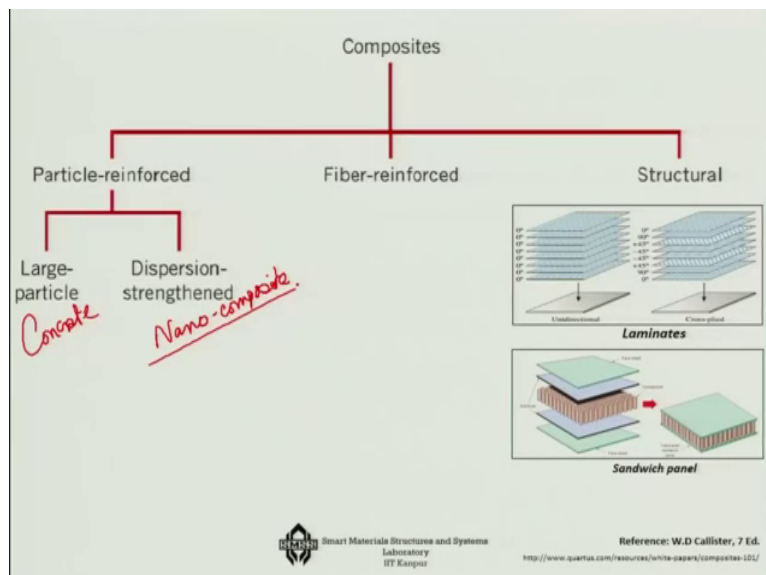
So the collagen fibre and the matrix is hydroxyapatite matrix, this together has formed the bone structure okay, so that is a composite that you carry with you. The other one is the Wood itself, if you look at any wood you would see that it has a lignin matrix okay and it has cellulose or heavy cellulose, which are wound in a spiral form, so making once again it as a candidate composite material.

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So you see nature is very important intelligent to begin with itself because most of the structural elements like wood for the tree or bone for the animals are the composite materials. I told you that cast iron is a composite material, why is it so? Because if you look at cast iron as you can see from this SCM images, you would see that there are actually graphite flakes or graphite modules in the cast iron and that is what makes it a composite because it is like a particle reinforced composite material.

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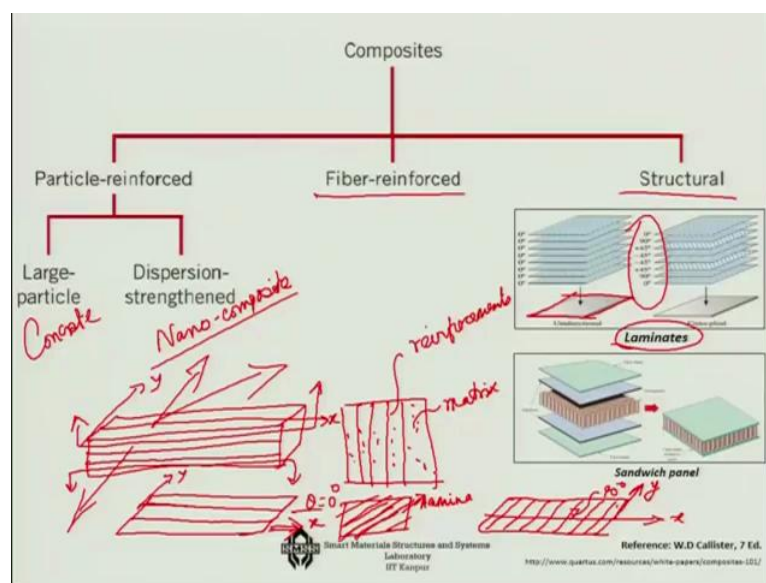
So let us divide the composite then into 3 parts, one part is the particle reinforced part and this particle reinforced could be large particle something like concrete okay or could be discussions strengthened, something like nano composites where the particles are so small

that you need special measures to actually disperse them inside the material, inside the matrix and you get wonderful results out of it.

The other types of composites are fibre reinforced composites, various fibres; you put them in matrix and to obtain a composite. And of course there are structural composites, now structural composites there are certain things that are there in it. The 1st point is that structural composites are developed layer by layer okay. So there is it is like each single page or paper and you are stacking all of them together.

Now each one of this paper, we will call it as a lamina. So if you look at each one of this lamina suppose, in the lamina there are going to be at least 2 phases, so this is a lamina. What are these 2 phases? You are going to see the reinforcement inside the lamina, so these are actually reinforcements and you are going to see the rest part, which is nothing but the matrix.

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So there are matrix with actually holds the reinforcement and then between the 2 combination you are getting the lamina. Now when you are making the lamina, the direction or so to say the orientation of the reinforcement actually makes a lot in terms of changing the properties of the system. For example, you consider a beam okay, so this is a beam and suppose this beam consists of 3 different lamina, so I have 3 different lamina.

Now, you might cut the 1st layer and if I look at it okay, so this is the X direction longitudinal direction, this is the Y direction. So, I am cutting the 1st layer and looking at it, and we find that so this is my X direction and this is my Y direction. I will find that all the reinforcements

are parallel to the X direction, so this is one type of an orientation of the fibre. If I look at the 2nd part on the 3rd part suppose I cut it here and I look at it, I will find that all the fibres are actually perpendicular.

So, here the angle of the ply so that means here the angle $\theta = 0$ degree because with respect to the X axis or the longitudinal axis, the angle of the reinforcement orientation is 0 degree, whereas here with respect to the X axis this is 90 degree and hence this is like a 90 degree ply and I can have thus 0 degree, 90 degree or anything in between, 30 degree plies or 45 degree plies, et cetera, et cetera.

I can have various types of such plies and that is what I have shown here in terms of the ply angles. And all these plies you stack them together and you are going to get something from the lamina, which we will call as the laminate that is what the structural composite is. Why are we doing like that okay? Why are we keeping 1 at 0 degree, 1 at 90 degree, et cetera, there must be a reason for it.

The reason is you see that the lamina is highly anisotropic in nature. Now generally reinforcements provide the high elastic modulus because they are having higher modulus of elasticity. So if you have a 0 degree ply, you are going to get high modulus of elasticity in the X direction. If you make one 90 degree ply, you are going to get high modulus of elasticity in the Y direction.

If you take any angle ply, you are going to get high modulus of elasticity in that direction. So what will happen that when a load comes on the structure, if the load is in the X direction, the X direction ply takes care of it, if the load is transverse this manner then the Y direction ply takes care of it, if the load is angular then this angular ply takes care of it, so that means you have provision to take care of the loading from any direction.

In fact, if your application has certain preferable direction of loading, you can keep all the plies preferably oriented towards that direction. That (())(16:57) exist in comparison that is the beauty on the composite structures. Now let us talk about the particle reinforced composites.

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Particle-reinforced

Large-particleDispersion-strengthened


LARGE-PARTICLE COMPOSITES

Example: Concrete, which is composed of cement (the matrix), and sand & gravel (the particulates)

- The **particulate phase is harder and stiffer** than the matrix.
- Reinforcing particles tend to **restrain movement** of the matrix phase in the vicinity of each particle.
- Matrix transfers fraction of applied load to particles.
- Improvement of mechanical behavior depends on strong bonding at the matrix-particle interface.

+

Sand, stones & water

=

Cement Concrete

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So I told you that there are 2 types of particle reinforced composites, the large particle composites which are like concrete for example okay, which is composed of the cement as the matrix and sand and gravel the particulate phase is harder and stiffer than the matrix. And the reinforcing particles tend to restrain movement of the matrix phase in the vicinity of each particle.

Matrix transfer fraction of the applied load to these particles, So thus you get an overall strength in the system then the stones that you use they give the improves strength, whereas the cement works like a binder in the system, so that is the large particle. How about the dispersion strengthened?



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Particle-reinforced

Large-particleDispersion-strengthened

DISPERSION-STRENGTHENED COMPOSITES

- Particles are normally much smaller, with diameters between 10 – 100 nm.
- Strengthening occur on the **atomic or molecular level**.
- Matrix bears the **major portion** of an applied load, while the **small dispersed particles hinder the motion of dislocations**.
- Thus, **plastic deformation is restricted** such that yield and tensile strengths, as well as hardness improves.

→

Strong Particle <100 nm Dislocation stopped

Example: Sintered aluminum powder - flakes of Al coated with Al_2O_3 , which are dispersed within an aluminum metal matrix.

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These are like I told you something like nano composites. Here, particles are very small, something like 10 to 100 nanometre. And here the strengthening occurs on the atomic or molecular level, the matrix bears the major portion of the applied load, while the small dispersed particles hinder the motion of the dislocations. So thus plastic deformation is restricted because the dislocation movement is hindered and good improvement in the tensile strength as well as the hardness improves.

So one of the examples is the sintered Aluminium powder, flakes of Aluminium coated with Al_2O_3 , these are dispersed within an Aluminium metal matrix to get ceramics metal composites or CMCs, so this is one of the examples of Dispersion strength composite. Now, how about the fibre reinforced composites? Here, as the dispersed phases are the fibres and then you have the matrix phase in it.

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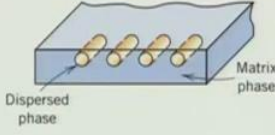
The Fiber Phase

Role of fibers in composites includes:


- To enhance stiffness
- To enhance strength
- To provide crack-bridging
- To enhance thermal resistance

On the basis of diameter and character, fiber phase can be grouped into

- **Whiskers** : They are very thin **single crystals** that have extremely large l/d ratio but has the form of fiber.
 - ✓ Flaw free and thus extremely high strength – but expensive.
 - ✓ Include graphite, silicon carbide, silicon nitride, and aluminum oxide, etc.
- **Fibers** : A material that has at least l/d ratio equal to 10 : 1
 - ✓ Either polycrystalline or amorphous.
 - ✓ Generally polymer and ceramics
- **Wires** : Relatively large diameters
 - ✓ Typical materials include steel, molybdenum, and tungsten



Dispersed phase Matrix phase



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Reference: W.D Callister, 7 Ed.

So, what are the roles of these fibres? You are getting better stiffness, it is stronger, you are getting better strength and it also works like a crack bridging, so if a crack wants to propagate then it will even if it propagates these is one of these fibres, so we have applied bridges because they will not give up so easily, so they will work like a crack bridge and it enhances the thermal resistance of the system.

Now, there are 3 types of fibres; some fibres are actually little bit of extension of particles, we will call them Whiskers. They are actually very thin single crystals that have L by D ratio and have the form of the fibre. And they are flaw free and thus they are of extremely high

strength, but expensive, this includes graphite, silicon carbide, silicon nitride and Aluminium oxide, et cetera.

And then you get fibres which has L by D ratio equal to about 10 is to 1 and either they are polycrystalline or amorphous. Generally they are polymers and sometimes they are ceramics. Then there are relatively large diameters, which are wires. Typically like steel, molybdenum and tungsten. Tungsten particularly is used with boron a coating layer and that is used in the space shuttles. Now what is the role of the matrix phase?

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The Matrix Phase

Role of matrix in composites includes

- Binding fibers together
- Protect the individual fibers from damage by external environment.
- Act as a medium to transmit and distribute externally applied stress to fibers.
- Serves as a barrier to crack propagation.

It should be noted that :

- ✓ The matrix material should be ductile.
- ✓ Elastic modulus of the fiber should be much higher than that of the matrix.
- ✓ There should be adequate bonding between matrix and fibers.

The slide also features a hand-drawn diagram of fibers and a stress-strain graph. The graph shows a linear elastic region followed by a yield point and a large plastic strain region. A vertical line marks the yield point, and the word 'debonding' is written next to it. The x-axis is labeled with the Greek letter epsilon (ε) and the y-axis with the Greek letter sigma (σ). The logo for the Smart Materials Structures and Systems Laboratory at IIT Kanpur is at the bottom.

1st thing, with the fibre you cannot develop a structure, so you need something to develop the structure on integrating, so matrix works for binding the fibres together. Also, it protects the individual fibres from damage by external environment and it acts as a medium to transmit and distribute eternally applied stresses to fibres and it serves as a barrier to crack propagation, so it has a lot of role matrix not only it helps the fibres, but also it itself works as a barrier to crack propagation.

In many cases, it is the matrix that gives actually because the fibres fail of a sudden. If you look at the fibres and the matrix, the fibres, this stress strain diagram, the fibres fail all of a sudden. It is like a brittle material, whereas the matrix has an typical large plastic strains it can take, it is very ductile and hence the matrix holds the system for some time, it gives you some warning, so that is the good part of it.

So matrixes materials are generally very ductile, elastic modulus of the fibre are generally much higher than that of the matrix. And they should adequate bonding between the matrix

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Volume Fractions

If V_f, V_m, V_v and V_c are the volumes of fiber, matrix, void and composite, then

$$v_f = \frac{V_f}{V_c} = \text{fiber volume fraction}$$

$$v_m = \frac{V_m}{V_c} = \text{matrix volume fraction}$$


$$v_v = \frac{V_v}{V_c} = \text{void volume fraction}$$

Where,

$$v_f + v_m + v_v = 1$$

$$V_c = V_f + V_m + V_v = \text{Composite Volume}$$

$v_f + v_m + v_v = \frac{V_f + V_m + V_v}{V_c}$
 $= \frac{V_c}{V_c}$
 $= 1$
sp. case
void free case.



Now the most important things then are 3 important things in fibre matrix composites that is the fibre volume fraction which is the ratio of the volume of the fibre to the volume of the composite V_f , so this is fibre volume fraction. Then v_m , which is V_m over V_c and V_v voids, which is V_v over V_c . So naturally, if I add all of them together so what I am going to get? $V_f + V_m + V_v = V_f + V_m + V_v$ over V_c .

Now $V_f + V_m + V_v$ is nothing but V_c , so that is why V_c over V_c is unity, so we can say that all the 3 things together is actually unity. If suppose there is no void, suppose if we assume such systems are there, where it is perfectly void less system, then it will be $V_f + V_m = \text{unity}$, this is a special case, this is void free composite okay.

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Weight Fractions

$$w_f = \frac{W_f}{W_c} = \text{fiber weight fraction}$$

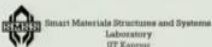
$$w_m = \frac{W_m}{W_c} = \text{matrix weight fraction}$$

Where,

$$w_f + w_m = 1$$

$$W_c = W_f + W_m = \text{Composite Weight}$$

Note: Weight of voids neglected



In some of the cases we will assume it to be almost like void free, so we will derive best of the things based on that.

Now, if you look at the weight fraction than that is actually w_f over W_c , W_f . And w_m is W_m over W . Void has the weight, so which means that the weight fraction $W_f + W_m = \text{unity}$ unlike the weight of the volume of the voids, which you have to consider.

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Densities

Density, $\rho = \frac{W}{V}$

Composite weight, $W_c = W_f + W_m$

Therefore, $\rho_c V_c = \rho_f V_f + \rho_m V_m$

Hence, Composite density, $\rho_c = \rho_f v_f + \rho_m v_m$

"Rule of Mixtures" for density

$d_c = \sum_{i=1}^n d_i v_i$ $\rho_c = \sum_{i=1}^n \rho_i v_i$ *Transport Properties*

$k_c = \sum_{i=1}^n k_i v_i$

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Now if I look at the density, density is what, W over V . And this W , W_c for the composite is $W_f + W_m$. So what is W_c , it is ρ_c times V_c , when ρ_c is the density of the composite and V_c is the volume of the composite and that = $\rho_f V_f + \rho_m V_m$, so the composite density ρ_c is actually $\rho_f V_f$ over $V_c + \rho_m V_m$ over V_c , we can write it shortly as $\rho_f v_f + \rho_m v_m$, this v_f is the volume fraction v_f and volume fraction v_m okay.

This is also this basic rule is also known as the "Rule of mixtures". That means I can write a ρ of a composite is actually $\sum_{i=1}^n \rho_i v_i$ okay suppose there are n different phases there, then ρ_i and v_i okay. And this is not only valid for density, but for many transfer properties of a composite. It is the moisture absorption capability, you call it β_c okay that also is the same rule that $\sum_{i=1}^n \beta_i v_i$.

And similarly the thermal conductivity, α_c exactly the same rule. $\alpha_c = \sum_{i=1}^n \alpha_i v_i$ so does this is general name of Rule of mixtures is there for the density and such other transport properties. Some properties are not transport properties okay, so like strength and stiffness, for them there are special formulas.

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Longitudinal Modulus

The tensile load is acting along fiber direction.

Assuming perfect bonding between fibers and matrix,

$$\epsilon_f = \epsilon_m = \epsilon_c \quad (1)$$

where ϵ_f , ϵ_m , ϵ_c are the longitudinal strains in fibers, matrix and composite respectively.

Since both fibres and matrix are elastic, the longitudinal stresses are

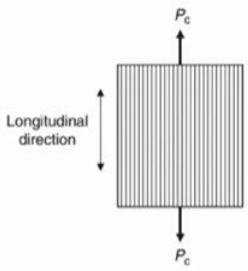
$$\sigma_f = E_f \epsilon_f = E_f \epsilon_c \quad (2)$$
$$\sigma_m = E_m \epsilon_m = E_m \epsilon_c \quad (3)$$


Since $E_f > E_m$ Hence $\sigma_f > \sigma_m$.

The tensile load P_c applied on the composite lamina is shared by fiber and matrix. So,

$$P_c = P_f + P_m$$

Since load = stress x area

$$\text{Therefore, } \sigma_c A_c = \sigma_f A_f + \sigma_m A_m$$


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For example, if I consider the longitudinally modulus, then if you assume perfect bonding between fibre and the matrix, then the strain in the fibre equals to the strain in the matrix that is strain in the composite. In this case longitudinally modulus, you are having fibres and matrix as if there are two different springs which are in parallel. So something like I can show it to you through this particular example.

That like you consider that there are this is this one this spring is for the fibre and this spring is for the composite, so if I apply the load, what I am going to see that the entire load is actually shared between the 2 springs okay, so the 2 springs are sharing the load, one is the fibre and another is the matrix. If all you get the similar analogy, you can consider that as if you have a fibre as a spring and a matrix as another spring okay.

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And this is a parallel model for longitudinal modulus. And if I am applying a load, so I am applying a load on the system and the load is shared by the 2. But in terms of the strain, the same strain you are going to see in the matrix as well as the fibre okay, so that is what the assumption is in this case okay. So we can say here that the $\epsilon_f = \epsilon_m = \epsilon_c$ and the load is shared between the 2 that is so $P_c, P_f + P_{matrix}$ okay, so this is how the load is shared between the 2.

Now in terms of the stress if I want to define each one of these loads, how do I do it? P_f is nothing but you can call it to be $\sigma_f A_f$ okay and σ_f is actually $E_f \epsilon_f$. And since ϵ_f is equal to ϵ_c , so you can also write it as $E_f \epsilon_c$. Similarly, E_m is so this is $P_f, \sigma_f A_f$ and that is σ_f is denoted by this one right, $E_f \epsilon_c$.

Similarly, $2 m$ is $\sigma_m A_m$ and where $\sigma_m = E_m \epsilon_m$ and that is nothing but $E_m \epsilon_c$. So then and the thing is that P_c is nothing but $\sigma_c \epsilon_c$. So if I work on this whole thing, so what we will reach?

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Now, $\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$ (4)

Or

$$\sigma_c = \sigma_f \frac{A_f}{A_c} + \sigma_m \frac{A_m}{A_c}$$
 (5)

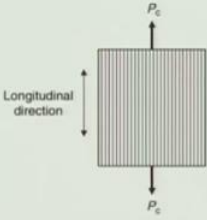
where
 σ_c = average tensile stress in the composite
 A_f = net cross-sectional area for the fibres
 A_m = net cross-sectional area for the matrix


$$A_c = A_f + A_m$$

Since, $\vartheta_f = \frac{A_f}{A_c}$ and $\vartheta_m = \frac{A_m}{A_c}$

Thus, $\sigma_c = \sigma_f \vartheta_f + \sigma_m \vartheta_m$

Dividing both sides by E_c , and using (2), (3) we get

$$(E_c)_{\text{longitudinal}} = E_f \vartheta_f + E_m \vartheta_m, \text{ "Rule of Mixtures"}$$




We see that $\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$, which means $\sigma_c = \sigma_f A_f / A_c + \sigma_m A_m / A_c$. And since you are considering the cross-sectional area and you consider the unique thickness of the system, so in this direction okay, so consider a unit thickness, so this is really thick. Then A_c you can say it to be $A_f + A_m$ and that is nothing what V_f and V_m , so hence $\sigma_c = \sigma_f V_f + \sigma_m V_m$.

So interestingly this is also following the Rule of mixtures, but because of the difference, because of the load share okay. And similarly, if you actually put this E_c in the terms of the strain, you will see the E_c longitudinally also follows the Rule of mixtures as $E_f V_f + E_m V_m$, so that is in the longitudinal direction. What happens in the transverse modulus?

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Transverse Modulus

- The tensile load is acting normal to the fibre direction.
- The total deformation (strain) in the transverse direction is the sum total fibre and matrix deformation.

$$\delta_c = \delta_f + \delta_m$$
 (1)

- Tensile stress in fibre, matrix and composite are equal.

$$\sigma_f = \sigma_m = \sigma_c$$
 (2)

From definition of normal strain

$$\delta_f = \epsilon_f L_f$$

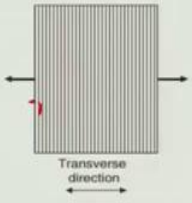
$$\delta_m = \epsilon_m L_m$$


$$\delta_c = \epsilon_c L_c$$

Substituting in Equation (1) and dividing by L_c both sides, we get

$$\epsilon_c = \epsilon_f \frac{L_f}{L_c} + \epsilon_m \frac{L_m}{L_c}$$
 (3)

Since $\vartheta_f = L_f / L_c$ and $\vartheta_m = L_m / L_c$





Transverse modulus, the load is now coming first the fibre suppose, then to the in between matrixes. So that means as if there are 2 springs, one is the fibre spring and another is the matrix spring and the load is getting transmitted. So unlike the other case where they are sharing here the same load in actually getting past, something like this okay. So something like you have one as a fibre, one as a matrix, as you are applying the load the same load is applied, if passing through both the fibre and the matrix.

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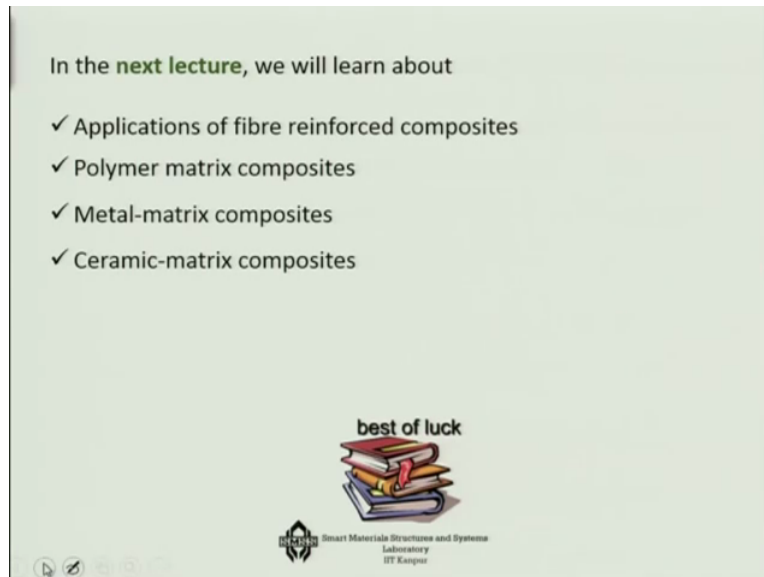
This is not shared, but the same load is going and you can see that this system is more flexible, right. So no wonder that the transverse modulus is actually much smaller in comparison to this system, which is the same type of spring depressed, but it is much stiffer. So in this particular case as you can see that the tensile load is acting normal to the fibre direction.

The total deformation is $\Delta C = \Delta f + \Delta m$, deformation in the fibre and deformation in the matrix. And since the same load is passing, so the stress is same now. $\sigma_f = \sigma_m = \sigma_c$. And your $\Delta f = \epsilon_f L_f$, Δm is this Δc is this, so instead of this you will substitute it by ϵ_c , so you get ϵ_c as $\epsilon_f L_f$ over $L_c + \epsilon_m L_m$ over L_c .

Now the volume fraction in this case is L_f over L_c and L_m over L_c , which means in this particular case you can write that σ_c over $E_c = \sigma_f$ over $E_f V_f + \sigma_m$ over E_m . And since $\sigma_f = \sigma_m = \sigma_c$, so I can make I can cancel all of them, so what I

get is $\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$. In other words, E_c here is $\frac{E_f E_m}{V_m + E_m V_f}$ okay.

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Or you can also write it in this particular relationship okay, so this is the composite modulus in the transverse direction. So this is where we will complete our lecture, in the next lecture we will talk about applications of fibre reinforced composites, we will talk about some of the polymer matrix, some of the metal matrix and some of the ceramic matrix composites, thank you.