

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**

**Manufacturing Process Technology – Part- 1**

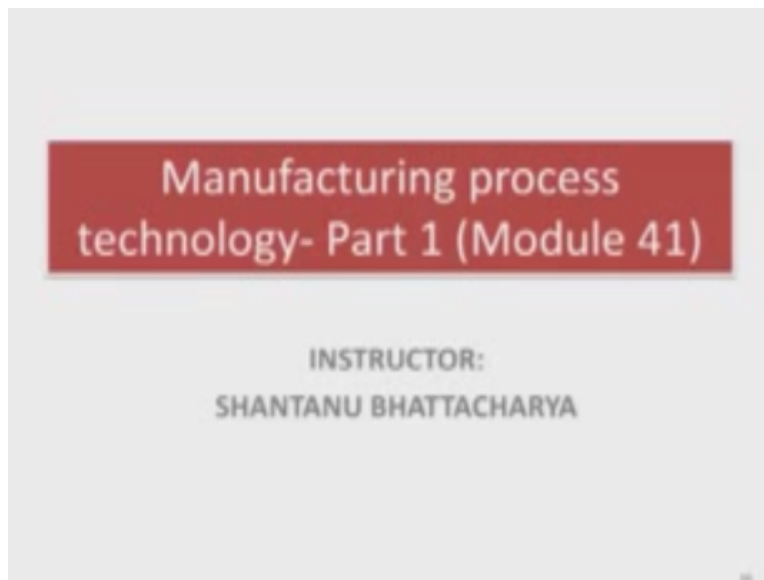
**Module- 41**

**by**

**Prof. Shantanu Bhattacharya**

Hello and welcome to this manufacturing process technology part 1, Module 41.

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In the last lecture we had talked about numerically deigning a case where we had non value of the measured forces as well as the geometrical parameters based on that we estimated what is going to be the ultimate shear streets of material in the cutting zone. let us actually reverse that cycle and try to estimate from material properties and geometric parameters given what is going to be the cutting forces okay in the following example.

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## Analysis of Cutting Forces in Metal to Metal Machining (Merchant Theory)

Mild steel is being machined at a cutting speed of 200m/min, with a tool rake angle of 10deg. The width of cut and the uncut thickness are 2mm and 0.2mm resp. If the average value of the coefficient of friction  $\mu=0.5$  and the shear stress  $\tau_s$  of workpiece = 400 N/mm<sup>2</sup>. Determine (1) Shear angle (2) The cutting and thrust component of the machining forces

$\tau_s = 400 \text{ N/mm}^2$   
 $\mu = 0.5$   
 $\alpha = 10^\circ$   
 $\phi = ?$   
 $W = 2 \text{ mm}$   
 $t = 0.2 \text{ mm}$

Merchant's first solution  
 $2\phi + \lambda - \alpha = 90^\circ$   
 $2\phi + \lambda = 100^\circ$

(1)  $\tan \phi = \frac{t}{W} = \frac{0.2}{2} = 0.1$   
 $\phi = \tan^{-1}(0.1) = 5.71^\circ$

(2)  $R = \frac{\tau_s W t}{\sin \phi} = \frac{400 \times 2 \times 0.2}{\sin(5.71^\circ)} = 478.6 \text{ N}$

$F_c = R \cos(\phi - \alpha) = 478.6 \cos(5.71^\circ - 10^\circ) = 462 \text{ N}$   
 $F_t = R \sin(\phi - \alpha) = 478.6 \sin(5.71^\circ - 10^\circ) = 134 \text{ N}$

So we have one example here which talks about a mild steel piece being machined at a cutting speed of about 200m/min and this is being done with the tool rake angle of 10° that means respect to the vertical the tool rake face is at 10° the width of cut and the uncut thickness are given to 2mm and 0.2mm respectively and the average value of the coefficient of friction and now we are just treating this to be taking this to be average value although the presumption of taking an average value is a not completely true.

Because due to the unequal distribution of pressure of the chip on the rack face that may change in the normal force that it receives and from the tool face there by resulting in differential friction And also the coefficient friction changes with respect to temperature as the cutting process happens so but we are right now assuming this to be uniform and a average value reported of 0.5 and shear stress  $\tau_s$  of the work piece is now is material property again we want to consider it to be time invariant although it is not completely true because shear stress also sometimes depends on the temperature of the cut zone so here in this case we treat that to be temperature independent and as 400 N/m<sup>2</sup>.

So we need to determine what is  $\phi$  the shear angle is this particular case and we also need to determine what is the cutting and thrust component of the machining forces so using the merchants first solution let us actually assume that we are using the first solution for our calculations here, we already know that  $2\phi + \lambda - \alpha = 90^\circ$ ,  $\alpha$  here is given as 10° the friction

angle  $\lambda$  can be calculated as tan inverse of average  $\mu$  which is actually = tan inverse 0.5 over 26.57°.

$$\text{Ernst \& Merchant: } 2\phi + \lambda - \alpha = \frac{\pi}{2}$$

$$\lambda = \tan^{-1} \mu = 26.57^\circ$$

Having said that we have now the  $\phi$  value recorded as  $90 + 10 - 26.57 / 2 = 36.7^\circ$  and the fs value the shear force value here they call it as  $Wt_1 \tau_s / \sin \phi$  as I had illustrated earlier this being the total shear plane area and this actually = to ultimate yield strength of the material in shear so it is already given to be a constant value 400 N/mm<sup>2</sup> and the problem example so we want to put this value of the width which is 2 mm times of the uncut chip thickness within this case 0.2 mm.

$$\phi = \frac{\pi - \lambda + \alpha}{2} = 36.7^\circ$$

So 0.2 times of  $\tau_s$  which is 400N/mm<sup>2</sup> /  $\sin \phi$  so sin of 36.7° so in this case the FS would come out to be here the shear force would come to be close to about 262.3 N and the way we calculate from the shear force the various components  $F_C$  and  $F_T$  so you already know that the resultant force times of cos of  $\phi + \lambda - \alpha$  from the merchants circle diagram represents a shear force so in this case obviously.

$$F_s = \frac{wt_1 \tau_s}{\sin \phi} = \frac{2 * 0.2 * 400}{\sin 36.7} = 262.3 \text{ N}$$

Then the resultant of the shear and the normal forces in the shear zone or you know the similar to the resultant of the frictional and the normal reaction of the tool face on the chip so that is going to be equal to 262.3 i.e. FS value divided by cos of  $\phi$  which is 36.67 +  $\lambda$  again 26.57 -  $\alpha$  which is 10° okay so this whole thing corresponds to about 438.6 N which is reaction force or resultant force R and you can calculate  $F_C$  and  $F_T$  by putting the R component and  $F_C$  is defined as  $R \cos \lambda - \alpha$ .

$$R = \frac{F_s}{\cos(\phi + \lambda - \alpha)} = 438.6 \text{ N}$$

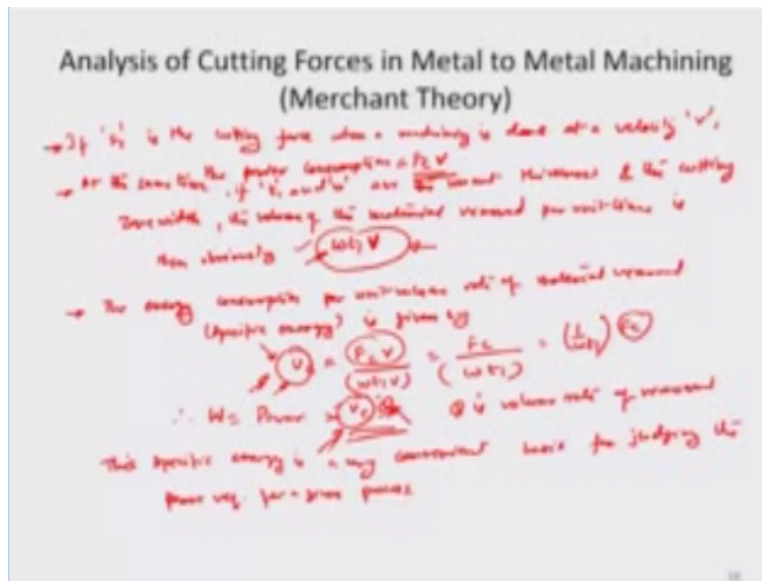
I think we have illustrated this earlier in the merchant circle so this becomes equal to 438.6 times of cos of 26.57 - 10° 420 N and  $F_T$  becomes =  $R \sin$  of  $\lambda - \alpha$  which is 438.6 sin of 26.57° - 10° which is = 125 N, so that is how you calculate the cutting force as well as the tangential force to the direction of cut so in a nut shell you can now estimate from the material property and the geometrical property the normal forces and the cutting forces in the tooling zone or the cutting zone.

$$F_C = R \cos(\lambda - \alpha) = 420 \text{ N}$$

$$F_T = R \sin(\lambda - \alpha) = 125 \text{ N}$$

And in the earlier example we had done the reverse way by looking at the measured cutting and tendencies forces what is going to be the you know the final shear ultimate shear strength, so having said that I think we have covered more or less everything that is need to understand Merchant theory properly.

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We now need to go to a little bit different aspect about calculating the specific cutting energy of then material and for that let us assume that if  $F_C$  is the cutting force when a machining is done. let us say at a velocity  $v$  the total power consumption I think we had earlier discussed comes out to be the force velocity product right so it is basically  $F_C * v$  that is used to defined the power at the same time if  $t_1$  and  $w$  are the uncut thickness and the cutting zone width, the volume of the material removed per unit time is then obviously  $w t_1 v$  right. so this is basically the total volume or total sectional area of the material and  $v$  is the velocity and so that is going to be the total amount of volume per unit time that is being removed okay.

Now what happens to the deformation process or how the chip gets deformed and wise or rights on the rake face is a different question but the amount of volume that you are flying out as chip is typically the volume that you are saving of the material right so in this case this is the if I assume  $v$  to be in a direction perpendicular you know in the cutting direction, so cutting velocity so obviously cutting velocity times  $w t_1$  should be able to give the sufficient volume per unit time removal.

Which would result in again deforming and riding on the tool face and formulating into a chip, so having said that the energy consumption per unit volume rate of material removal okay which we commonly also known as the specific cutting energy most specific energy is then given by  $U_c = F_c v$  which is the amount of energy consumed per unit time / by the volume rate of removal per unit time which is  $w t_1 v$  by another words it is  $F_c / w t_1$ .

$$W = F_c v$$

$$U_c = \frac{F_c}{w t_1}$$

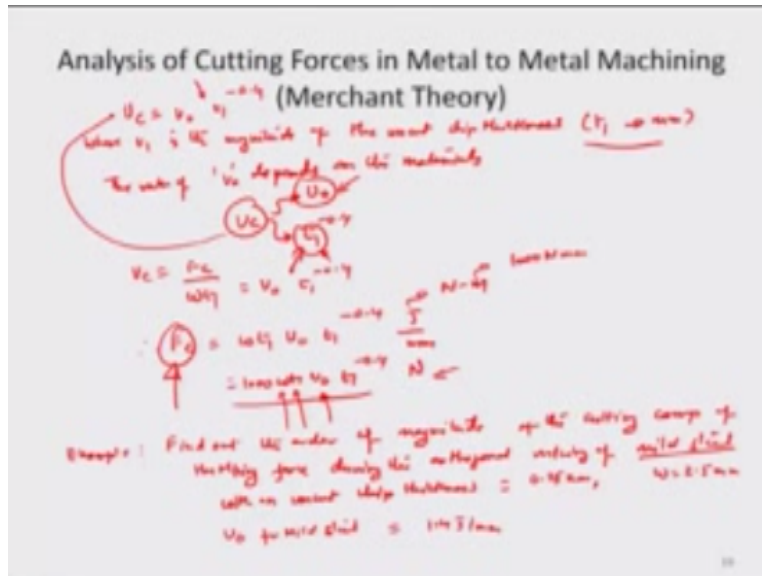
So this  $1 / w t_1$  factor is really pretty important and it give you a relationship between the cutting force and the specific energy that is being spent in order to remove a certain material at a certain volume rate of removal. therefore now I can probably say that the amount of energy that is need per unit time or the amount of cutting power that is needed represented by  $w$  can be represented as  $U_c Q$  where  $U_c$  is the specific energy of the material and  $Q$  is the volume rate of removal that is how many  $\text{mm}^3 / \text{min}$  or  $\text{s}$  is being removed of the materials of  $Q$  is the volume rate of removal.

$$W = U_c Q$$

And this  $U_c$  is basically again the power spent per unit volume rate of removal so multiplying that with the volume rate of removal would actually give you the total power that is need for cutting the system or cutting performing the whole cutting process. so this specific energy is a very convenient cases for judging the power required for a given process. if I say the specific energy related to a volume certain volume rate of removal is higher or for a certain process is higher it means that you can basically you would like to consume more amount of power to remove the material in that particular system.

If the specific energy is lower of a certain system then you have to use less power in order to cut more material of the system so an examination of the various.

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$$U_c = U_0 t_1^{-0.4}$$

A literature data suggest that you know you can relate this  $U_c$  to the uncut chip thickness  $t_1$  through a relation  $U_0$  to the power of  $-0.4$  where and this is again completely a sort of a core relational formula so where this  $t_1$  is the magnitude of the uncut chip thickness of the cutting process okay and this is measured because this is kind of regression formula this  $t_1$  actually is measured in mm.

So in this particular relationship the  $t_1$  is an mm and the value of  $U_0$  really depends on the materials so obviously we are splitting down the  $U_c$  into two components: one is material related property and another which is actually related to the thickness of the cut that you are excising so at one particular you know round cutting how much material or how much depth you are giving in terms of you know cut chip thickness is also critical to determination of the rate of consumption of energy which would be need to remove that material.

So therefore we are trying to split it up into material property  $U_0$  and thickness  $t_1$  to power of  $-0.4$  so having said the value of  $U_c$  can be substituted in the relationship which we developed here right here and we can say that you know  $U_c$  which is actually  $= F_c / w t_1$  and found out before is nothing  $U_0 t_1$  to the power of  $-0.4$  and basically  $F_c$  therefore become equal to  $w t_1$  times of  $U_0 t_1^{-0.4}$  J/ mm this J can be corresponding to Nm you could actually represent this you know by changing this to 1000 N mm and I can write this down as  $1000 w t_1 U_0 t_1^{-0.4}$  N okay.

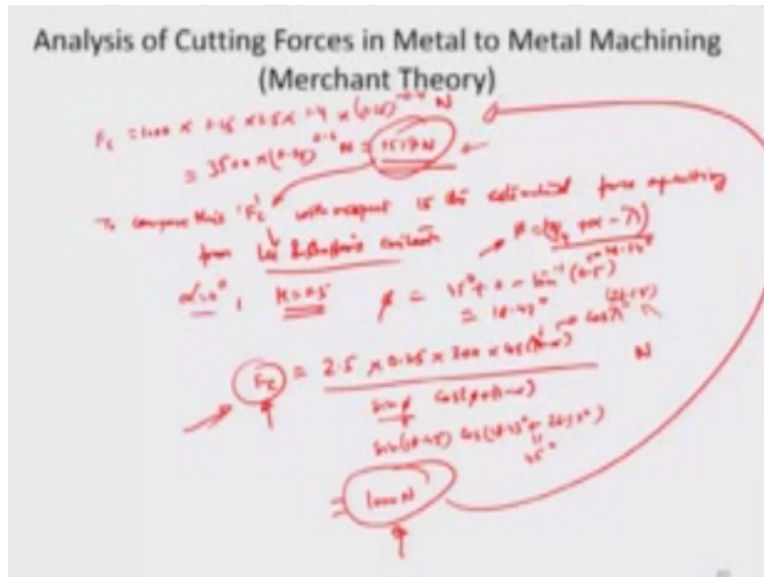
$$F_c = 1000 * w t_1 U_0 t_1^{-0.4} N$$

So that is how you can actually find out the cutting force requirements given material property  $U_0$  of particular material that is what is the specific energy of the material and also the width and

the depth of the cutting zone so this is very important to determine how much force would be necessary And sufficient to start removing the material so let us do a numerical problem example to find out how we apply this specific energy concept in the cutting.

So let us say we have a case where we need to find out the order of magnitude of the cutting component of the machining force and during the orthogonal machining of mild steel with an uncut chip thickness of 0.25 mm and width of a cut w being equal to 2.5mm so we already know that the  $U_0$  which is a material specific property assuming this to be a mild steel as I already told you here component  $U_0$  for mile steel is recorded as  $1.4 \text{ J} / \text{mm}^3$ .

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So having said that the amount of cutting force  $F_c = 1000$  times of the value of the uncut chip thickness which is 0.25 mm times of the width of the cutting zone  $w$  times of 1.4 the  $U_0$  value from mile steels specific cutting energy of mile steel times of  $t_1$  the uncut chip thickness to the power of -0.4 which is again represented here  $0.25^{-0.4} \text{ N}$  so this becomes equal to 3500 times of  $0.25^{0.6} \text{ N}$  which is 1517 N.

$$F_c = 1000 * w * t_1 * U_0 * t_1^{-0.4} = 1000 * 0.25 * 2.5 * 1.4 * (0.25)^{-0.4} = 1517 \text{ N}$$

So this is the kind of force requirement which is there for starting to cut on this mile steel sample to compare this  $F_c$  value which is predicted in the or which is calculated in to the last step here with respect to let us say something to the estimated force of cutting from using one of the

criteria which we illustrated earlier let us say we use the Lee & Shaffer criteria which talks about  $\phi = \pi/4 + \alpha - \lambda$  okay  $\tan^{-1} \mu$ .

So we can probably find out whether we are still in the range of you know what ever is estimated by this specific energy method at least order of magnitude y is are the force value same so in this case let us assume that  $\alpha = 0^\circ$  rack angle of tool which is almost vertical  $\mu$  some of the parameters we have assumed earlier as 0.5 and the  $\phi$  calculated from which the Lee & Shaffer condition which is  $45^\circ + 0 - \tan^{-1}$  of 0.5 which is actually  $26.54^\circ$ .

$$\text{Lee \& Shaffer} :: \phi + \lambda - \alpha = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{4} - \lambda + \alpha = 18.43^\circ$$

$$F_C = \frac{w t_1 \tau_{s0} \cos(\lambda - \alpha)}{\sin \phi [\cos(\phi + \lambda - \alpha) - k_1 \sin(\phi + \lambda - \alpha)]}$$

$$F_C = \frac{2.5 * 0.25 * 300 \cos 26.57}{\sin 18.43 [\cos 45]} = 1000 \text{ N}$$

This becomes equal to 18 so the FC values here would be equal to obviously the width of cut times of the uncut chip thickness  $t_1$  times of  $t$  ultimate shear incase of mild steel which is  $300 \text{ N/mm}^2$  is assumed to be that value times of  $\cos$  of  $\lambda - \alpha$  or  $\cos \lambda$  okay divided by  $\sin \phi$  times of  $\cos \phi + \lambda - \alpha$ , so this value is basically  $\cos$  of  $\lambda$  and because  $\alpha$  is 0 in this particular case as you have seen in the assumption itself so  $\cos$  of  $\lambda$  which actually becomes =  $\cos$  of  $26.57^\circ$  okay and right here is basically  $\sin$  of  $\phi$  which is  $\sin$  of  $18.43^\circ$  and  $\cos$  of  $\phi + \lambda - \alpha$  which is actually =  $\cos$  of  $18.43^\circ + 26.57^\circ - 0$  this becomes equals to roughly about  $45^\circ$  So that much N would be the force FC from normal Lee & Shaffer's criteria for optimization of power. so which find out that the values of pretty specially very close FC in this case comes out to around 1000 N if I just calculate by substituting the value of this  $\cos 26.57$  and all the other  $\sin$  and  $\cos$  parameters in the denominator the total force of cut happens to be about 1000N which is at least order of magnitude wise write close to this 1517 N okay.

So at least with both methods we can predict similar orders of magnitude of the cutting force as illustrated here now the other issue which I would probably like to take after in the next module would be related to heat transfer process and temperature rise in the cutting zone because of the cutting action obviously there is a shear plane across which the chip is being deformed and that all energy is stored in terms of increase in temperature or heat energy.



And so there is going to be a you know toll temperature zone which comes up because of all this energy exchange so apart of the energy probably goes to the work piece a part is flying away with the chip and the fly is retain by the part is retained the tool so in that manner we will try to model to see what is the heat flow and what is the temperature rise of all these 3 zones while doing a machining operation as if now in this particular module I would like to end it here. thank you so much.

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