Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology – Part- 1

Module- 39

by
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Hello and welcome to this Manufacturing process technology part 1 module 39 we were talking about how you can express F_c the cutting force in terms of a shear force and ultimate shear stress strength of the material that you are cutting and in context of that we wanted to do some optimization for the total power requirement that could be sued for cutting processes.

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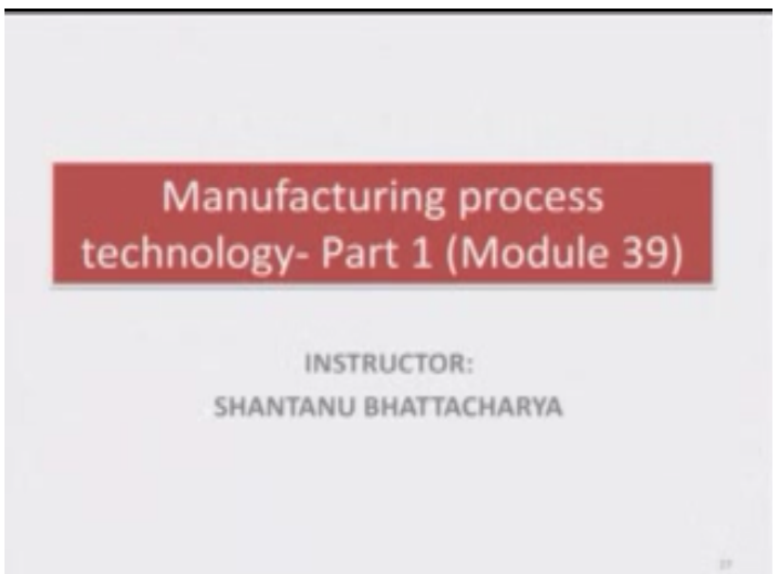

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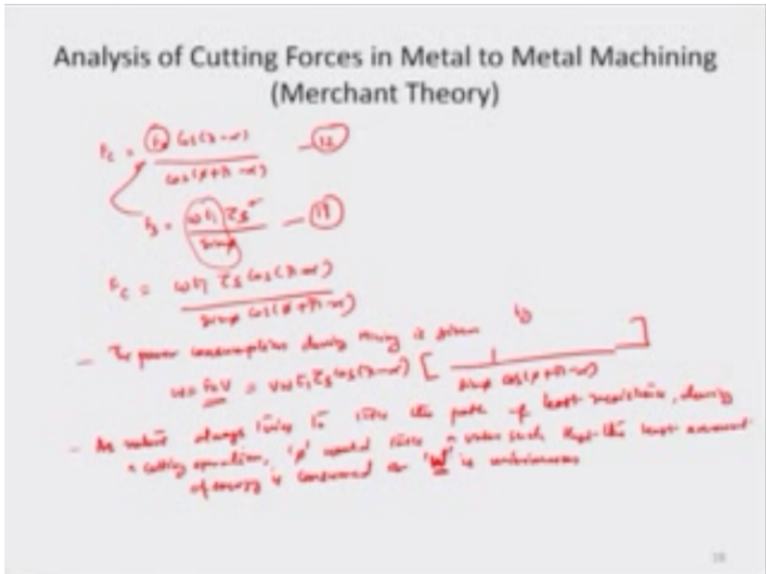

$$F_S = \frac{w t_1 \tau_s}{\sin \varnothing}$$

$$F_C = \frac{F_S \cos(\lambda - \alpha)}{\cos(\phi + \lambda - \alpha)}$$

So it basically try to map F_c the cutting force in terms of shear force F_S cos of λ- α/cos of ϕ+λ-α in the last module and we further expressed this F_S from in an earlier equation as function of the material properties and also we total cutting area by w t_1 τ_s / sin ϕ where τ_s is ultimate shear yield strength of the material and w t_1 /sin ϕ was the total interfacial area between the cutting zone and the work piece.

From which could get otherwise separated so if we substitute the value of F_S from equation let us say 11 which we are done earlier all the way to equation 12. F_C comes out to be equal to w t_1 τ_s cos of λ-α/sin of ϕ cos of ϕ+λ-α and the power consumption which is otherwise taken as the force velocity product that means the cutting force times the velocity the feed velocity or cutting velocity,.

$$F_C = w t_1 \tau_s \cos(\lambda - \alpha) \frac{1}{\sin \phi \cos(\phi + \lambda - \alpha)}$$

So that basically the power that is consumed in the separation of the chip so basically the power consumption during machining is given by W=F_C times of velocity V that is cutting force times the velocity of the cut and this comes out to be equal to V w t_1 τ_s cos of λ-α times of 1/sin ϕ cos of ϕ+λ-α. So as nature always tries to take the path of least resistance during the cutting operation ϕ would take value such that the least amount of energy.

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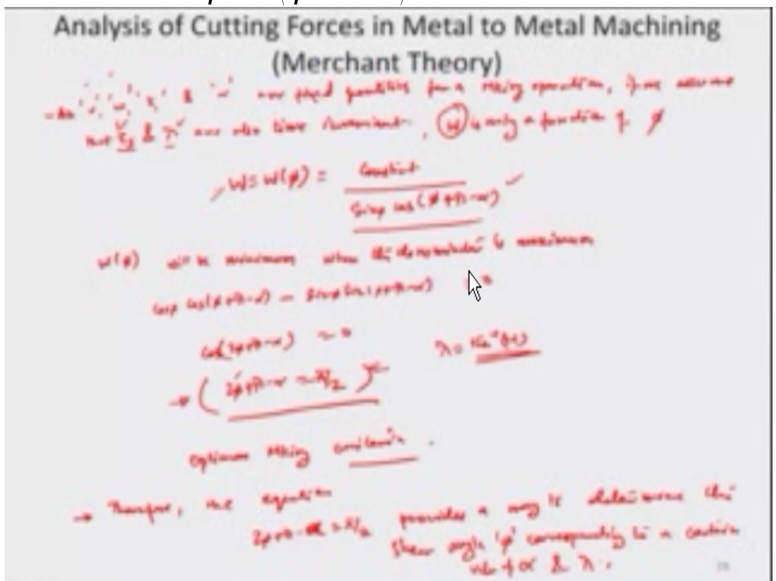
$$W = F_C v = v w t_1 \tau_s \cos(\lambda - \alpha) \frac{1}{\sin\phi \cos(\phi + \lambda - \alpha)}$$



We consumed several time assumed or W is minimum, W being the total amount of work in order to do process successfully which is minimum. so as V, w width of the cutting zone and α are fixed quantities for a certain machine given machine operation machining operation if we assume that τ_s and λ are also time in variant.

So this is something that we are just a assuming although in the real case the friction coefficient as well as the ultimate yield strength of the material shears would typically various function of the temperature rise or temperature increase. So we are not considering that here in this we are considering this to be time invariant property and as such clear to the property material.

Then you can say that W is only a function that is total work done per unit time or total model power that is consumed W is only a function of ϕ. so W actually is so therefore we can also say that this is equal to some constant here which is related to all this different parameters v, w, t_1, α you have cos λ-α even λ and ts/ the term sin of ϕ cos of ϕ+λ-α we assume that minimum power would be consumed only corresponding to a certain orientation of ϕ.

$$W(\phi) = \frac{constant}{\sin\phi \cos(\phi + \lambda - \alpha)}$$

So basically we have to minimize this number here the total amount of power or work done per unit time with respect to ϕ okay and see here what ϕ vales this power consumption would be minimum or it would again the angle would formulating way as a cutting action is happening through the least path of a resistance being offered to the tool surface which is otherwise is scrapping through the material to remove it.

So w(ϕ) will be minimum when the denominator is maximum which also can be obtain by differentiating with respect to ϕ and putting equal to 0. so we have again cos ϕ cos of (ϕ+λ- α)- sin ϕ sin of (ϕ+λ-α)=0 or cos of 2 ϕ +λ-α=0 which can only happen when 2 ϕ+λ-α is π/2. so this case this kind of condition also obviously λ is as you know tan inverse μ we assuming due to the constant in variant of time so that is the optimum machining criteria.

$$\cos\phi\cos(\phi+\lambda-\alpha)-\sin\phi\sin(\phi+\lambda-\alpha)=0$$

And definitely ϕ would take a value which would be dependent on that particular relationship corresponding to the minimum power consumption. therefore the equation 2ϕ+λ-α = π/2 provides a way to determine the shear angle corresponding to a certain value of α and friction angle λ. so we have in sort of optimization criteria here.

$$\cos(2\phi+\lambda-\alpha)=0$$

$$2\phi+\lambda-\alpha=\frac{\pi}{2}$$

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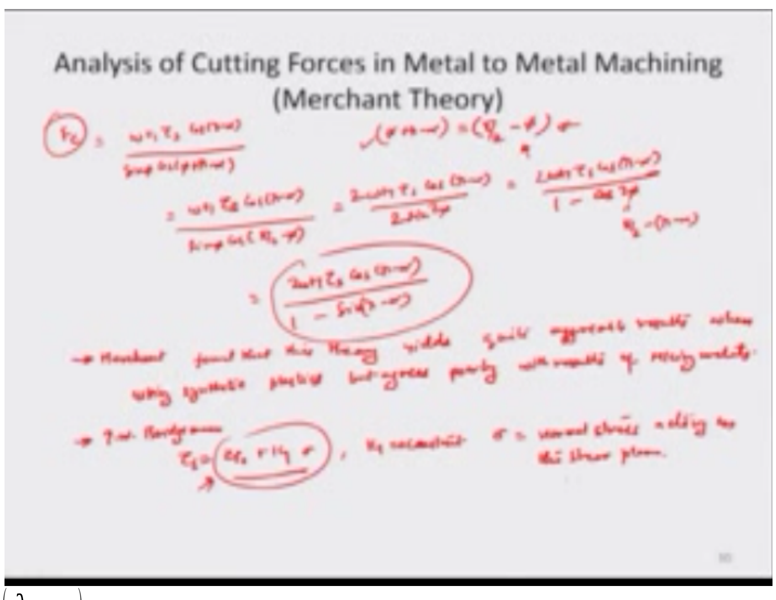

$$F_C=\frac{2wt_1\tau_s\cos(\lambda-\alpha)}{1-\sin(\lambda-\alpha)}$$

So the value of FC can be evaluated corresponding to this condition of 2 ϕ+λ-α= ϕ/2 so the FC was recorded as w t_1 τ_s cos of λ-α/sin of ϕ cos of ϕ+λ-α so if I put the value of ϕ+λ-α from the relationship obtained earlier it can be obtain as π/2- ϕ . 2 ϕ+λ-α was π/2 by the relationship. so I substitute this value here and try to see what happens to the situation so this becomes equal to w width of the cutting zone times thickness and cut thickness, τ_s shear stress cos of λ-α/sin of ϕ cos of ϕ/2- ϕ.

And in other words this can be represented as $\sin^2$ϕ and $\sin^2$ϕ if I just substitute if I just multiply numerator and denominator by 2 can also be written as 2 w t_1 τ_s cos of λ-α/1-cos of 2ϕ okay. so cos of 2 ϕ obviously is nothing but again if I look at this particular equation can be ϕ/2-(λ-α) okay. so we just substitute that again and try to see what happens in other form the equation could take this is tries w t_1 τ_s cos λ-α/1- sin λ-α.

Obviously cos of ϕ/2-λ-α would be sin of λ-α. so this is ultimately the form which comes out from for the fc value on this particular optimum criteria corresponding to the value of ϕ that is been recorded using this power minimization criteria and Merchant found out that this theory yields quite agreeable so the results when cutting synthetic plastics but agrees poorly with results of machining metals.

So this is the small modification done by PW Bridgeman who realizes that the shear stress ultimate shear stress τ_s is not completely independent of the normal stress σ which would come into the picture in fact what alternate theory he proposed was that can be relate the τ_s to basic τ_{s0} value + something proportional to the normal the normal stress σ that is coming at the shear plane okay.

$$\tau_s = \tau_{s0} + k_1 \sigma$$

Where K1 is a constant and σ is the normal stress acting on the shear plane. let take this side own k1 is a constant and σ is the normal stress at the one the shear plane . so if I just substitute this value here for the τ_s value there will be some deification which would happens to the overall you know equation and if we look at those modifications are so.

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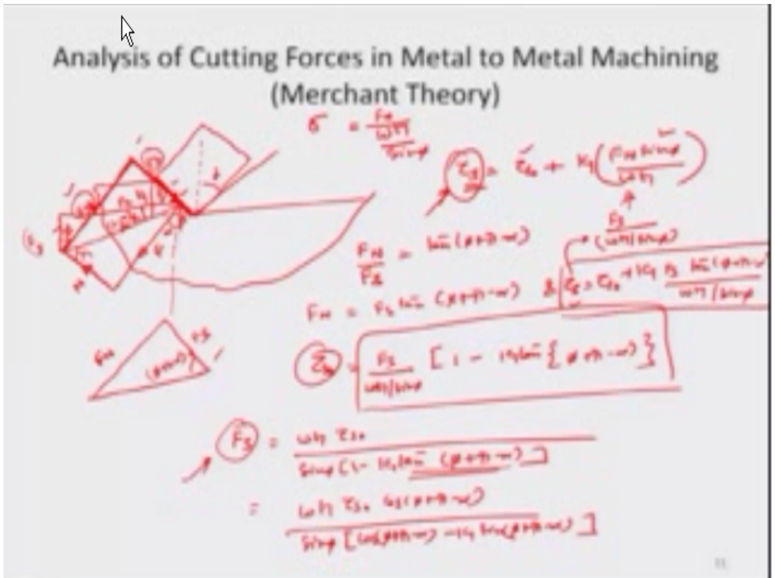

We know that the if we just sort of consider what is the normal stress. so the normal stress actually is nothing but the normal reaction which is given by the work piece on the metal per unit area and actually it is the sine component of the angle ϕ of the normal force and I just like to redraw what I did before just to explain this concept here.

$$\sigma = \frac{F_N}{\frac{w t_1}{\sin \varnothing}}$$

This is chipping power formulation of this chipping formation process which is happening and this is the tool which is at the certain rake angle α with respect to the perpendicular direction and further as I think as illustrated earlier we have shear force acting in this direction F_S there is an normal force which acts in a perpendicular direction which is given by this vector light here.

This is the normal force so this one is F_N and this results in some kind of a resulted here somewhere here and we can say this is R just wanting to sort of repeat what we did in terms of the force diagrams also we had too more forces that is the frictional force to the chip as offered by the tool rack face and the normal reaction which again was offered by the rack face we called them F_N.

And obviously there would be a third cutting and tangential force we should be offered by the work piece on to the tool surface because the tool is having a ploughing motion into the work piece okay. so having said that obviously the normal force on the shear plane it could be the normal stress would probably come up because of this F_N value right here.

And the area of the zone I think which we had done earlier was actually w t_1/ sin ϕ so that is also because the total amount of uncut chip thickness is t_1 and the angle across which this thickness needs to be translated in order to get to the shear plane is ϕ so therefore t_1/sin ϕ is what the length of the hypotaneous would be this triangle and w is the width of the cutting zone coming out of the paper or out of the board okay.

All going in sides so this w t_1/ sin ϕ would be the total area of this particular face on which force fn is acting so this is going to be σ value or the normal stress on the shear zone and further if I wanted to put this value in the suggested modification of the Bridgman, the final shear stress would come out to be into two components one is τ_{s0} and another is some constant k1 times of this σ value which again is fn sin ϕ/w t_1 so there is actually sin ϕ based variation in the total shear stress which is because there is a component of the normal stress of the shear plane acting in order to define what is the ultimate shear will stress of shear strength of the material.

$$\tau_s = \tau_{s0} + k_1 \frac{F_N}{\frac{w t_1}{\sin \varnothing}}$$

At the zone of cut or zone of chip formation so we already know that we have a correlation between F_S and F_N and so F_S/F_N was earlier recorded to be equal to tan of ϕ+λ-α if you remember the values that we talked about so the value here of if you look at F_S/F_N is really how to resolve it as component here in this particular figure.

$$\frac{F_N}{F_S} = \tan(\varnothing + \lambda - \alpha)$$

$$F_N = F_S \tan(\varnothing + \lambda - \alpha)$$

So we have this angle equated to ϕ if we realize from earlier statements this was the angle λ and the way we look at this angle was λ-α which we have done in earlier calculations this angle right here was actually α and the angle taken here in this particular area was ϕ so obviously the total amount of angle between the FT and FS in this right triangle would be ϕ+λ-α . so ϕ +λ-α so this is the total angle of the force triangle corresponding to the normal force in the shear plane and the shear force.

Let me just separate this out here this is F_S this is F_N and the angular relationship between these two therefore resulted in these two forces at an angle of ϕ+λ-α . so obviously F_N /F_S in this particular case become tan of ϕ+λ-α and having said that if we wanted to now substitute F_N value in terms of FS in this equation we would have the fn equals to FS tan of ϕ+λ-α and correspondingly τ_s as τ_{s0} +k1 times of FS tan of ϕ+λ-α/w t_1/sin of ϕ. further I can actually now the from this whole equation the value of τ_s is 0 which comes out to be F_S/w t_1 by sin ϕ times of 1-k1 tan of ϕ+λ-α.

$$\frac{F_S}{\frac{w t_1}{\sin\varnothing}} = \tau_{s0} + k_1 \frac{F_S \tan(\varnothing + \lambda - \alpha)}{\frac{w t_1}{\sin\varnothing}}$$

$$\frac{F_S}{\frac{w t_1}{\sin\varnothing}} \left[1 - k_1 \tan(\varnothing + \lambda - \alpha)\right] = \tau_{s0}$$

The reason being that you have on one side τ_s value which you can write down simply from the earlier equation as FS/ w t1/ sin ϕ so if you substitute this value in τ_s the τ_s is 0 value from this substitution comes out to be fs/wt1/sin ϕ times of 1-k1 tan ϕ+λ-α or in other words fs can be recorded as w t_1 τ_{s0} /this whole term right about here which is sin of ϕ times of 1-k_1 tan of ϕ+λ-α or n other words you can write this down as again if I just this as sin cos ratio.

$$F_S = \frac{w t_1 \tau_{s0}}{\sin\varnothing\left[1 - k_1 \tan(\varnothing + \lambda - \alpha)\right]}$$

So I should be able to write it as w t_1 τ_{s0} times of cos of ϕ+λ-α/ sin of ϕ times of cos of ϕ+λ-α-k_1 sin of ϕ+λ-α okay so having said that we have now the relationship were we can again use the same logic underline the logic of trying to find out the cutting force and trying to do the force velocity product and minimize the force velocity product to obtain a final solution which could be a slightly different.

And in the solution that was obtained earlier which we did not consider the assumption of the normal force so this brings has to the end of this module 39 and in the next module we will do this force velocity product optimization and try to find out the new angle ϕ thank you.

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