Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title Manufacturing Process Technology – Part- 1

Module- 30

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Hello and welcome to this manufacturing process technology part I module 30.

(Refer Slide Time: 00:16)



We were talking about two different cases where in one case we had a predominant thermal resistance given by the solidified metal and in case previous to that there was a problem related to the wetting of the surface because of which there was the thin film which was the interfacial thermal resistance which is of predominant nature. So in the case there was.

(Refer Slide Time: 00:40)



Predominance of the solidified metal or this portion was predominated, with a constant casting surface temperature meaning there by that this mould was a thin mould and we would like to water cool or something, so that it is like gate heat sink and so this surface temperature would become more or less constant because of that process. So in this particular case.

(Refer Slide Time: 01:04)

Solidification with constant casting surface temperature 0,-0,74

We already calculate that the terms ζ is the error function of ζ was made = $\theta f - \theta s / \sqrt{\pi} Cs/L$ from the previous derivation that we have made and this value ζe^{ζ^2} had to be calculated from you know various values of ζ where the right and left equal to each other. So basically corresponding to different ζ there would be standards tables called error function tables something like what you can see here.

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2	Conta	29(1)	6000		ets	Si(1)	41(A)
29	0.0000	0.0000	-	2.0	0.9953	1.6054	-0.4230
1.2	0.2227	0.1996	1.0422	2.2	0.9981	1.6576	-0.1751
1.4	0.4284	0.3965	0.3788	2.4	0.9993	1.7525	-0.3173
1.6	0.6039	0.5881	0.0223	2.6	0.9998	1.80334	-0.2513
1.8	0.7421/	0.7721	-0.1983	2.8	0.9999	1.8321	-0.1865
1.0	0.8427/	0.9461	-0.3374	3.0	1.0000	1.8487	-0.1196
2	0.910.9	1.1080	-0.4205	3.2	1.0000	1.8514	-0.0551
4	0.9523/	1.2562	-0.4620	3.4	1.0000	1.8419	0.0045
.6	0.9763/	1.3892	-0.4717	3.6	1.0000	1.8719	0.0580
8	0.9891/	1.5058	-0.4568	3.8	1.0000	1.7934	0.10116
0	0.9953/	1.6054	-0.4230	4.0	1.0000	1.7562	0.5410

So you have the x value here and you have the error function of x here, so corresponding to certain x value the error function is defined in this particular entry here on the column corresponding error function of x. so the idea is that you know this ζ e ⁽²⁾ whatever be that we have to plot with the respect to ζ value okay in a graph, various values could be obtained of the ζ corresponding to which the error functions are defined here and from this graph we would like to see where along this graph exactly.

The right hand side of this particular equation $\theta f - \theta s / \sqrt{\pi} Cs/L$ would be matching and that would be corresponding ζ value and from that you can easily find out what is the time of solidification by the formulation $f^2/16 \zeta^{2\alpha s}$ and this was in the case the predominately solidifying material or metal to be the most important component defining the thermal resistances.

(Refer Slide Time: 02:51)

Solidification with predominant resistance in mold and solidified metal ald-metal intertace Geions 2 and 4 Fig. 2.25 Temper

The other case we would like to illustrate here with predominant resistance in mould and solidified metal, would be the case where obviously we assume the copper mould which is a highly conducting mould with respect to the metal where that it is basically solidifying, it is quiet thick. It is the thin mould that we had illustrated earlier and this is not water cool. So therefore there is going to be a gradient of temperature from the mould surface, all the way to outer surface of the mould and let us consider this temperature to be θ_s and θ_0 .

So this is the only difference in this particular case where there constant casting surface temperature was established because of that, so in the mould- metal interface the temperature θ_s can no longer be assumed to remain at its initial value θ_0 . So obviously θ_s will change and θ_s still assume to be constant is the thermal properties of the mould and the solidified metal. And moreover after the initial stage of the solidification interface resistance also becomes very very negligible.

So we do not consider because it is metal to metal weld the hardly any interface that would happen. So in this particular case only significant thermal resistance is offered by region 2 and the region 4 okay, so the 2 is this region and 4 is particularly this region and the resulting distribution temperature any instances shown in the figure. So assuming the mould to be semi infinite medium in the negative x direction. So this mould starts from x = 0 at the interface of the phase of the mould and all the way to the other end of the atmosphere end, so it is the negative x direction.

The temperature distribution can be again written down as $\theta - \theta_s / \theta_0 - \theta_s =$ the error function of (x/2 $\sqrt{a}t$). So this is completely on the basis of distribution, you know the solution of the one dimensional equation, that we did at first instance when we consider the temperature distribution of the sand moulding only thing here changing is the boundary condition and nothing else. So this is for all the x < 0 because obviously corresponding to x more than 0 the temperature would be higher because the solidified metal at least freezing point is higher.

So for x< 0 this would be the relationship that holds and in the solidified metal that means the corresponding x > 0 the equation would change $\theta -\theta_s / \theta \infty - \theta_s$ again this is the integration constant just as what talked out in the earlier cases and this actually is due to the error function of $x/2\sqrt{\alpha}$ st where the α s is the thermal diffusivity of solidified metal. This α corresponding to the α p mould okay, so please be very careful about what this metals.

$$\frac{\theta - \theta_s}{\theta_0 - \theta_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) \text{for } x < 0$$
$$\frac{\theta - \theta_s}{\theta_\infty - \theta_s} = erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right) \text{for } x > 0$$

So I am not going to prove the distribution function as such because I think I have already illustrated corresponding to the sand boundaries how these functions behaves. So θ_0 obviously is the initial temperature of the mould and it has mould metal interface temperature, θ_{∞} integration constant as been defined here earlier.

(Refer Slide Time: 07:15)



The other conditions need to be satisfied one of them is the balance of heat flux at the mould metal interface that is $K_s \partial \theta / \partial x$ at x = either 0+, K_s basically is that the thermal conductivity of the solidified metal sight okay and that of the mould sights, so that we called as K $\partial \theta / \partial x$ at x = 0-oaky. So the amount of heat of the solidified metal side and that of the mould side there should be equal to each other in this particular illustration.

$$x = 0 - k = 0 + i k \frac{\partial \theta}{\partial x_{\Box}}$$
$$k_s \frac{\partial \theta}{\partial x_{\Box}}$$

Number 2 is that the rate of energy flow balance at the solid, liquid interface that is $K_s \partial \theta / \partial x$ at x = Δ , which is basically at instant t okay. If I just look back in this figure the metal as probably gone all the way upto Δ at the certain time is t at it is still expanding at the rate $\partial \theta / \partial \partial / dt$. So thi should be = to the amount of heat that is liberated by virtue of the solidification process and I do not need to illustrate how we obtain in the earlier. So it is ρ_m times of L $\partial \theta / dt$ assuming again the poring and freezing temperatures to be same to each other.

$$k_{s} \frac{\partial \theta}{\partial x_{x=\delta}} = \rho_{m} L \frac{d\delta}{dt}$$
$$\theta(\delta, t) = \theta_{f}$$

So having said that now this is the 2^{nd} condition that has to be satisfied that the amount of heat that is liberated from the solid okay, it will ∂ is the amount of the heat that flows out into interface okay solid metal liquid interface and then obviously the 3^{rd} condition that needs to be obeyed is this ∂ , t at which this temperature should be = to the freezing point of the material

otherwise there is no point having the interface between the solid and liquid. So this is actually necessarily = θf , so all the above equations.

(Refer Slide Time: 10:26)

Solidification with predominant resistance mold and solidified metal

Can be combined to give $\theta_f - \theta_s$ times of Cs/ L $\sqrt{\pi} = \zeta e^{\zeta^2}$ in a similar manner as I have solved earlier and also the $\theta_{\infty} - \theta_s$ times of Cs divided by L $\sqrt{\pi} = \zeta e^{\zeta^2}$ and $\theta_s - \theta_0 / \theta_{\infty} - \theta_s = (Ks \rho_m c_s / k\rho c)^{1/2}$ which is actually = Φ constant in this particular case okay. So these are the result of all, you know manipulation of $\partial \theta / \partial x$ at x + 0 or at x-0 so on so forth. So that all the 3 conditions are earlier can be satisfied okay and so the 3 unknowns namely θ_s , θ_{∞} and ζ can be solved from these 3 equations $\theta_s - \theta_0$ times of Cs / L $\sqrt{\pi} = \zeta e^{\zeta^2} \Phi$ and $\theta_f - \theta_0$ times of Cs / L $\sqrt{\pi} = \zeta e^{\zeta^2}$ err function $\zeta + \Phi$ oaky.

$$\frac{\theta_{f} - \theta_{s}}{L\sqrt{\pi}} = \zeta e^{\zeta^{2}} erf(\zeta)$$
$$\frac{(\theta_{\infty} - \theta_{s})}{L\sqrt{\pi}} = \zeta e^{\zeta^{2}}$$
$$\frac{\theta_{s} - \theta_{0}}{\theta_{\infty} - \theta_{s}} = \left(\frac{k_{s}\rho_{m}C_{s}}{k\rho C}\right)^{0.5} = \emptyset(const)$$

So you can just compare all this by looking at this side here so obviously we have θ_{∞} - θ_s being recorded here from equation 3 as $\theta_s - \theta_0 / \emptyset$ that is exactly what we are substituting here in this particular equation θ_{∞} - θ_s and so you are getting this 1st equation from the solution of this, by submission of θ_s - θ_{∞} the value obtained here. And the other equation here is obtained by simply

adding the π value from this particular equation right about here to obtain you knows. So here for example you are trying to solve ($\theta_f - \theta_s$) Cs / L $\sqrt{\pi} = \zeta e^{\zeta^2}$. So already we know that this $\theta_s - \theta_0$ times Cs/ L $\sqrt{\pi}$ is this $\zeta e^{\zeta^2} + \Phi$.

$$(\theta_{s} - \theta_{0}) \frac{C_{s}}{L\sqrt{\pi}} = \zeta e^{\zeta^{2}} \emptyset$$
$$(\theta_{f} - \theta_{0}) \frac{C_{s}}{L\sqrt{\pi}} = \zeta e^{\zeta^{2}} [erf(\zeta) + \emptyset]$$

So let us call this equation 1 and 2 if we add 1 to 2, so adding 1 and 2, we get $\theta f - \theta_0$ times Cs/ L $\sqrt{\pi}$ is this ζe^{ζ^2} err function $\zeta + \Phi$, which is actually this particular equation obtained here okay, just adding 1 to 2 okay. so you find out now that in the case where the resistance comes from predominately from the solidified as well as the the mould solid boundary there is a slight modification to the equation that was earlier there only for the predominately mould boundary case.

Solidified metal case, in this case the only addition that has been happened actually is to the right hand side and this new constant Φ the ratio between the Ks pm, Cs and K pc, those associated with the properties of the mould that is in question. So this new Φ gets added up in this expression which would give predominately resistance in mould as well as solidified metals.

(Refer Slide Time: 15:54)



So let us do some problem example now, so one of the numerical problems for example here we determined the solidification time of the slab shaped casting considered in the earlier example.

Casting is done in water cool copper mould, so basically this is the case corresponding to the constant casting surface temperature meaning there by that this is the case of water cool thin copper mould and the other one is the thick copper mould, where the predominant resistance would come from the solidified metal portion as well as the mould metal interface okay.

In both the cases assume more resistance at the mould metal interface by the thin film of air or something, the mould itself you know having itself a good joint with the metal there is hardly any contact angel issue at the interface. So the following data is given for the iron the solid metal the specific capacity is 0.67 kJ/kg-K. The thermal conductivity of the solid iron metal is about 83 W/m-K, θ s is the freezing temperature 40^o c all these basically the 272kJ/kg and the density of the liquid metal or the solid metal both are about 7850kg for the iron metal.

The solidified metal and for the mould side the copper similar observation as been made for the thermal specific heat capacity, the density of the material and the thermal conductivity of the material. So having said that let us now try to sort of go ahead and solve this problem.

(Refer Slide Time: 17:45)



So in the 1st case we know that the we consider the case of thin copper mould with water cooling, obviously the casting surface temperature will be constant, I can say θ_0 and θ_s are similar to each other. So in that instance if we apply in the corresponding equation, just as illustrated here, where this $\zeta e^{\zeta 2}$ * error function of $\zeta = \theta_s / \sqrt{\pi}$. So we have θ_s as room temperature here we assume that to

be 28°c. The height of the casting is 0.1m about 10cm that is the thickness so the $\zeta e^{\zeta^2} *$ error function of $\zeta = \theta_s / \sqrt{\pi} \text{ Cs/ L}$.

Where this is the specific heat capacity of the solid metal, this is the heat of solidification of iron, so we just put the values here the freezing temperature 1540 degrees – 28 times of 0.67 which is the specific heat capacity of the solid metal divided by 272 these are Kj we do not need to illustrate converting these into joules anymore oaky. So there is the kilo joule in the numerator and denominator oaky. So this becomes the = 2.1, so if we really want to see by plotting a graph of ζ with this term ζe^{ζ^2} , where these two would correspond to you know the whole expression here being corresponded to = 2.1.

$$\zeta e^{\zeta^2} erf(\zeta) = \frac{(1540 - 28) * 0.67}{272\sqrt{3.14}} = 2.1$$

So corresponding to ζ value 0.98 which we can read from these tables as been illustrated earlier, so 0.98 is somewhere close to 1, so the error function ζ is 0.8427. so infact you know what I am trying to say here is that 0.8427 which is the value of the error function, this error function of ζ times of e ⁽²⁾ so 0.98 times of 0.98, so this calculation should be = 2.1 or nearby. So this is equated process for estimating the ζ value by again checking the different values of ζ corresponding to which the LHS right hand side of this particular what you balance to each other.

So therefore the ζ value corresponding to 0.98 would definitely call for the solidification time t_s now, for that we need to calculate what is α_s so α_s here in this case is heat conductivity or thermal conductivity of the solid metal/ density of the solid and liquid metal capacities of Cs of the solid metal. So this becomes = 83/7850 the density times of 0.67 into 10^3 m². Please be very careful of the units. So this comes out to be = 15.8 x 10 ⁶ m²/sec and the solidification time therefore K_s would happen to be h²/16 times of 15.8 10 ⁻⁶ and this comes out to be 0.0115 hour okay.

$$\alpha_{s} = \frac{\kappa_{s}}{\rho_{m}C_{s}} = \frac{83}{7850 * .67 * 1000} = 15.8 * 10^{-6} m^{2} / sec$$
$$t_{s} = \frac{0.1 * 0.1}{16 * 0.98 * 0.98 * 15.8 * 10^{-6}} = 0.0115 hr$$

So that is how the value of t_s can be calculated in this particular case. So as regards the estimation of the time of solidification for the 2^{nd} case we will see that there is slight variation in the time of solidification, it is going to be a liitle more than this case because of this we are

adding more resistances to the heat flow path by taking also the mould as a sink in the in the thick cooper mould case.

(Refer Slide Time: 22:28)

umerical Proble

So let us do the problem now, so for the thick copper mould without water cooling, I would like to find out first what is going to be the Φ value here. Φ is obviously ratio $(K_s\rho_mC_s/k\rho c)^{1/2}$. So this comes out to be 83 times of 7850 the density of the liquid metals times of the specific heat capacity of the solid metals divided by 398 the thermal conductivity of the mould, times the density of the mould material times the specific heat capacity, so this becomes = 0.57 and corresponding to θ_0 of 28°C, ζ e ⁽² + 0.57 which is the Φ value here, should be equal to the freezing minus room temperature θ_0 divided by this specific latent heat of solidification times of $\sqrt{\pi}$ times of the c.

$$\varnothing = \left(\frac{83 * 7850 * 0.67}{398 * 8960 * 0.376}\right)^{0.5} = 0.57$$

$$\zeta e^{\zeta^{2}} [erf(\zeta) + \varnothing] = \frac{(1540 - 28) * 0.67}{272\sqrt{3.14}} = 2.1$$

$$\zeta = 0.70$$

The specific heats capacity of the metal, solidified metals 0.67 values. So this becomes again 2.1 okay and this particular case as if we calculate the based on you know the equations that we have done earlier, the equation gets balanced, the left and right side corresponding to ζ value of 0.815. So again a graph of ζ versus ζ e ^{ζ} needs to be plotted in order to a certain corresponding ζ would 12

actually give you the LHS and RHS as equality. So having said that now therefore times of the thermal diffusivity 15.8 *10⁻⁶ as calculated before.

This comes out be 0.0165 hours, so obviously it is little more because now you are seeing that the applying resistance in comparison to you knows over all process. So therefore the resistance of the heat flowing increase here and it needs more time for the temperature to come to the freezing point and therefore the time of solidification in this case is more in comparison to the other case where the mould acts like as a heat sink and there is constant casting surface temperature because of that.

$$t_{s} = \frac{0.1 * 0.1}{16 * 0.1 * 0.1 * 15.8 * 10^{-6}} = 0.0226 hm$$

$$\frac{\theta_{f} - \theta_{s}}{L \sqrt{\pi}} = \zeta e^{\zeta^{2}} erf(\zeta)$$

$$\frac{(1540 - \theta_{s})0.67}{272 \sqrt{\pi}} = 0.7 e^{0.49} erf(0.7)$$

$$\theta_{s} = 982 \, degree \, C$$

So the surface temperature of the metal θc or θs , this can be estimated as 1540 minus 854 that is 685.4° C, so that is what the value will before the surface temperature of the metal in this particular case. So obviously how we have estimated gate is from the again the error function of by just calculating the θ_s value. Which can be done by any of these equation, θs in this particular case would be $\theta_s - \theta_0$ times = $\zeta e^{\zeta^2} \pi L \sqrt{\pi} Cs$ and $\theta s = \theta_0 + \zeta e^{\zeta^2} \pi L \sqrt{\pi} Cs$ okay.

So that is how the 685.4 can be calculated, so having said that I am kind of done with heat transfer theory so in summary we have seen what happens when there is the case where the sand mould is completely is the only resistive part of the heat transfer or the heat flow. In case 2 where the thin film formulated because of differences in contact angels and incomplete with respect to the solid surface and case 3 again where we have considered a thin mould with water cooling, so that you know you have this predominate interfacial.

So the only resistance left in the whole picture is basically the resistance of the solid metal part and then in the finally case 4 where without water cooling, so that the two components that is the mould resistance as well as the solid metal both of them are predominate nature. So in this case you have also seen that the time of solidification also based on more and more resistance added to the heat flow goes higher because now more time is needed for the freezing point. So having said that I think I am done with all the heat transfer issues and castling how to design the casting for the solidification, so think of it now as to be with the times scales on the riser on the idea is the back flow to sustain within this time so that the metal get solidified fully and then there is always possibility of refer, so you have to design from an overall sense all these problems together considering all these cases together for a realistic casting design. That we are going to do in probably in the next module up till then good bye thank you.

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