Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology- Part-1

Module- 29

by
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Hello and welcome to this manufacturing process technology part 1 module 29, we will be now discussing a case of you know a thin section mold with the mold is metallic and primarily the thermal conductivity of the mold is much, much larger in comparison to the metal which is been solidify and we would also try to estimate in this condition what is going to be the solidification time, okay.

So we have so far covered two different cases just to recall one was a plane sand casting where we assume prefect welding condition and then we introduced a non-wetting condition and then try to estimate the heat transfer assuming interfacial resistance because of a thin air film formulated between the casting and the surface., so in this particular case we are having a small thin section mold with a large casting which is you know predominantly the region 4 on the curve that we shoed earlier would be the cause of all the temperature difference, okay. So the thermal resistance of more important consequence would be actually.

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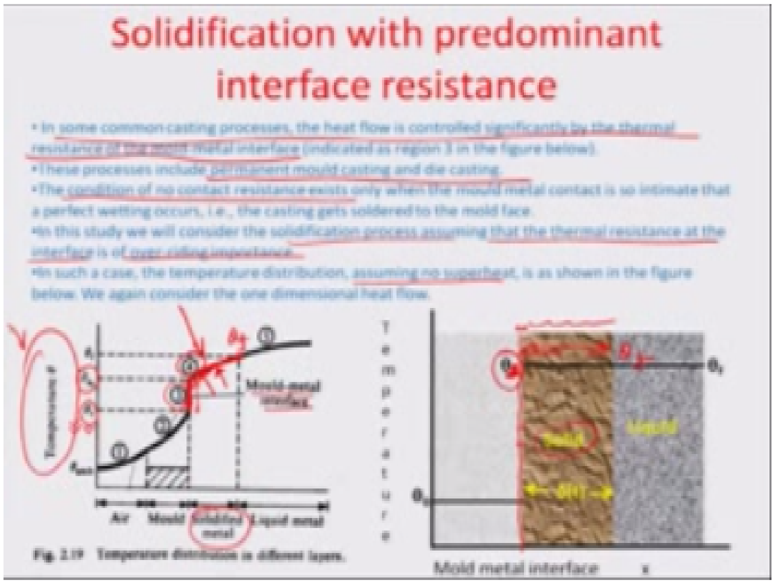

In this particular case only the region 4, as can be illustrated in this particular figure here so having said that let us look at what is going to be the surface temperatures also what is going to be the solidification time that such a process would actually need.

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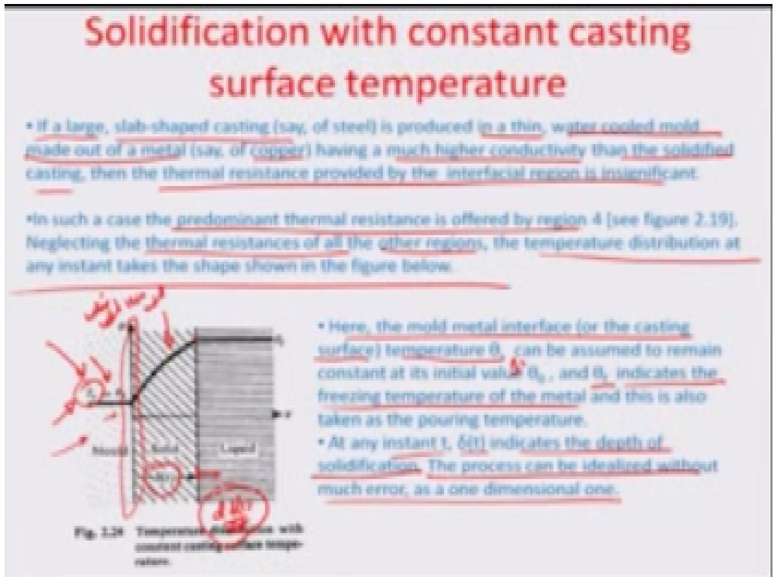

So if a large slab shaped casting is produced in thin water cooled mold made up of metal say copper having much larger conductivity then the solidified casting then the thermal resistance provided by the interfacial region between the solid and the mold would kind of be eliminated, okay, it is insignificant almost. So in such a case the predominant thermal resistance would be offered by the region 4 is I just shown, so neglecting the thermal resistances of all the other regions the temperature distribution at any instance takes the shape shown in the figure below right here, okay.

So here the mold metal interface or the casting surface temperature θz or θs can be assumed to remain constant in time particularly because the mold is also water cooled okay, so you have a completely water cooled thin mold so obviously there is going to be a you know constancy of the surface temperature of this particular metal, so let us assume that thermal temperature to be $\theta_s=\theta_0$ okay, and let us also assume the freezing temperature to be θ_f which is not different in the poring, so poring and freezing are same.

So it indicates the freezing temperature of the metal and at any incidents t, δt indicates the depth of solidification as you can see particularly here this is the solid liquid front and it is moving ahead at $d\delta(t)/dt$ and δt is the instantaneous thickness of the shell which is being developed here solid shell and the process can be idealized without much error as a one dimensional heat transfer problem particular because we are having almost a heat sink you know as a mold because some water cooled mold and so therefore the heat and transfer would be typically one dimensional.

The maximum resistance thermal resistance which is offered to this liquid temperature of the freezing temperature is really the solid, solidified portion of the mold and the other end of this is really a constant temperature surface or constant casting surface temperature θs. So let us see this kind of a problem what really happens or how, what would be the estimation of the solidification time.

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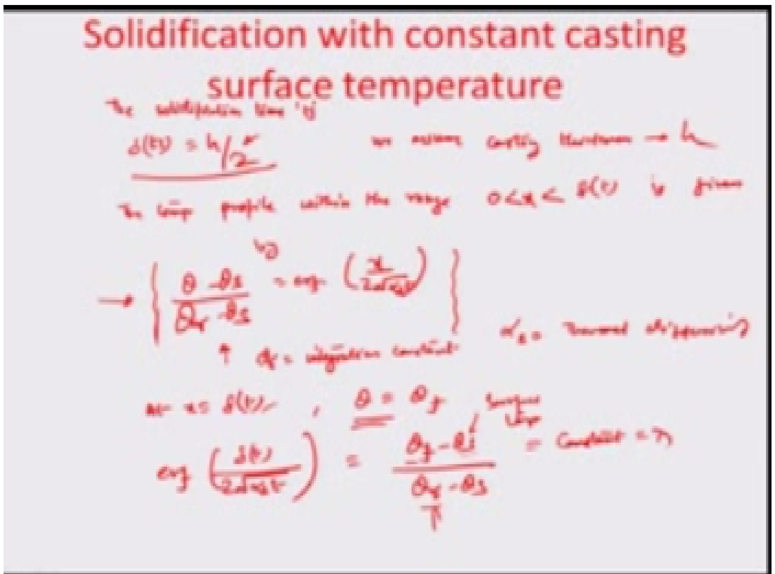

$$\frac{\theta-\theta_s}{\theta_\infty-\theta_s}=erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)$$

$$erf\left(\frac{\delta(t)}{2\sqrt{\alpha_s t}}\right)=\frac{\theta_f-\theta_s}{\theta_\infty-\theta_s}=\lambda$$

$$\delta(t)=2\zeta\sqrt{\alpha_s t}$$

$$k_s\frac{\partial\theta}{\partial x}_{x=\delta}=\rho_m L\frac{d\delta}{dt}$$

$$\frac{\partial\theta}{\partial x}=(\theta_\infty-\theta_s)\frac{d}{dx}\left[erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)\right]$$

$$(\theta_\infty-\theta_s)\frac{1}{2\sqrt{\alpha_s t}}\frac{2}{\sqrt{\pi}}*\exp\left[-\left(\frac{x}{2\sqrt{\alpha_s t}}\right)^2\right]$$

$$\frac{k_s}{\sqrt{\alpha_s}}(\theta-\theta_s)\frac{1}{\sqrt{\pi t}}\exp\left[-\left(\frac{x}{2\sqrt{\alpha_s t}}\right)^2\right]=\rho_m L\frac{d\delta}{dt}$$

$$\sqrt{k_s\rho_m C_s}\frac{\theta_f-\theta_s}{erf(\zeta)}\frac{1}{\sqrt{\pi t}}e^{-\zeta^2}=\rho_m L\sqrt{\alpha_s}\frac{1}{\sqrt{t}}$$

$$\zeta e^{-\zeta^2}erf(\zeta)=\frac{\theta_f-\theta_s}{\sqrt{\pi}}\frac{C_s}{L}$$

$$2\sqrt{\alpha_s t_S} = h/2$$

$$t_S = \frac{h^2}{16\alpha_s \zeta^2}$$

So let us estimates the solidification time ts, so we get dδ(ts) should be really equal to half the thickness of the mold. we assume mold thickness here or casting thickness here to be equal to h, so therefore there is heat transfer from both sides of the casting so therefore h/2 is really the plane along which the solidification trans from both sides with propagate and stop, okay. So the temperature profile within the range x varying between 0 and δt is given by $\frac{\theta-\theta_s}{\theta_\infty-\theta_s} = erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)$. I think you had earlier talked about this details when we did the thermal conductivity and temperature distribution across the sand mold.

So I am not going to illustrate how this equation came or you do already know that it was from a similarity variable analysis and one dimensional heat transfer problem which we actually solved to obtain in the case of sand mold okay, so in this case I will just mention that $\theta\infty$ is a integration constant and we will not bother about it because ultimately we need not use this value for our calculations okay, so this is the constant of integration if you remember the whole process of the solution there were many integrals which were formulated in order to estimate the real value of θ.

An α_s basically is the thermal diffusivity of the mold which was actually also recorded as the thermal conductivity of the mold per unit density transfer of heat capacity of the mold. So at x= let us say δ(t), θ the temperature is actually the freezing temperature right, θ_f so I would put this value of x and θ in this particular equation we have the error function of $\delta(t)/2\sqrt{\alpha_s t}$ is estimated as θ_f-θ_s/θ_∞-θ_s is remind you this is the casting surface temperature, okay you are assuming that the mold is water cooled and acts as a sink so this actually cannot change because the freezing point and the casting surface temperature are similar so are the constant of integration θ_∞ and θ_s . so therefore this is really a constant, λ okay. Let us call this constant λ, so we now have then.

$$erf\left(\frac{\delta(t)}{2\sqrt{\alpha_s t}}\right) = \frac{\theta_f-\theta_s}{\theta_\infty-\theta_s} = \lambda$$

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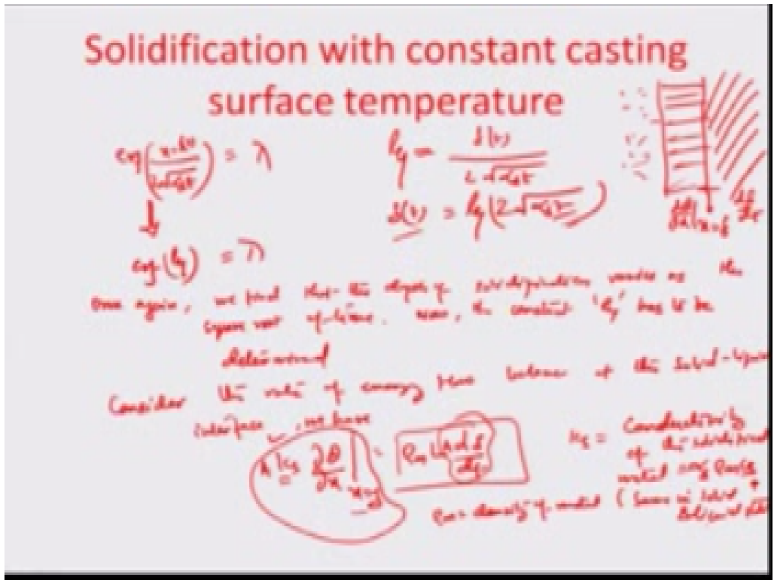

The error function of $(x/2\sqrt{\alpha_s t})=\lambda$ further let us $x=\delta(t)$. so further let us assume that there is another ratio defined as ζ which is actually equal to this $\delta(t)/2\sqrt{\alpha_s t}$ or in other words $\delta(t)$ is basically $\zeta.2\sqrt{\alpha_s t}$ and we have the term written as error function of $\zeta=\lambda$. Once again, we find that the depth of solidification varies as the square root of time as you can get an illustration here. Now the constant ζ has to be determined in order to find out the time of solidification okay, along these two relationships which are here.

$$\delta(t)=2\zeta\sqrt{\alpha_s t}$$

So let us consider the rate of energy flow or the energy balance at the solid, liquid interface we have $k_s\ \partial\theta/\partial x$ at $x=\delta$, equals to $\rho_m L d\lambda/dt$ so obviously at this front δ which is the you know this is the mold, this is the solid part and this is basically the liquid part so we are talking about this $x=\delta$ as function of times so if there is a depth the gradient $d\theta/dx$ which exists at $x=\delta$ that times of the thermal conductivity of the solid k_s should be equal to the total amount of heat rate that is liberated because of the moving front we assume that at incidence t this front is moving at a rate $d\delta/dt$.

$$k_s \frac{\partial\theta}{\partial x}_{x=\delta}=\rho_m L\frac{d\delta}{dt}$$

So the area terms get cancelled here, areas are on both sides actually so you have the rate of change of volume times of the specific latent heat okay, so that is times of densities basically the total amount of heat rejected out of the solidifying metal and $k_s a\ \partial\theta/\partial x$ at $x=\delta$ the total amount of heat which comes out of the liquefied metal into the solid liquid boundaries, so they should be heat balanced so k_s here in this particular case is the conductivity of the solidified metal and this can be written down as α_s $\rho_m(c_s)$, c_s is the specific heat capacity of the solidified material and ρ_m is the density of metal which is same in solid and liquid state okay, so this is the unified density does not change much with respect to solidification at the freezing temperature.

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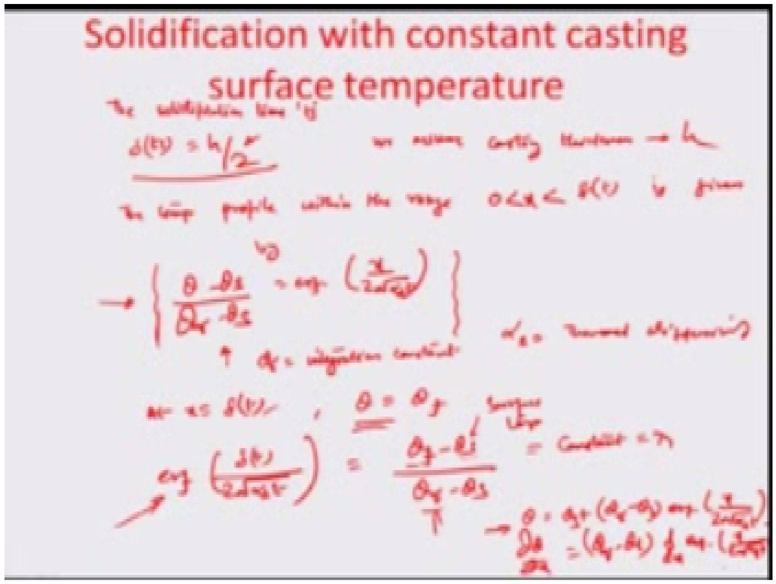

So therefore the $\partial\theta/\partial x$ should be defined as from there should be defined from the equation that we have considered earlier for the θ right about here okay, so here we had the θ to be equal to $\theta_s+\theta_\infty-\theta_s$ times of the error function of $x/2\sqrt{\alpha_s t}$ okay, so from here we can calculate $\partial\theta/\partial x$ as $\theta_\infty-\theta_s(d/dx)$ of the error function of $x/2\sqrt{\alpha_s t}$ okay, so let me just write this down that here again.

$$\frac{\partial\theta}{\partial x}=\left(\theta_\infty-\theta_s\right)\frac{d}{dx}\left[erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)\right]$$

$$\left(\theta_\infty-\theta_s\right)\frac{1}{2\sqrt{\alpha_s t}}\frac{2}{\sqrt{\pi}}*\exp\left[-\left(\frac{x}{2\sqrt{\alpha_s t}}\right)^2\right]$$

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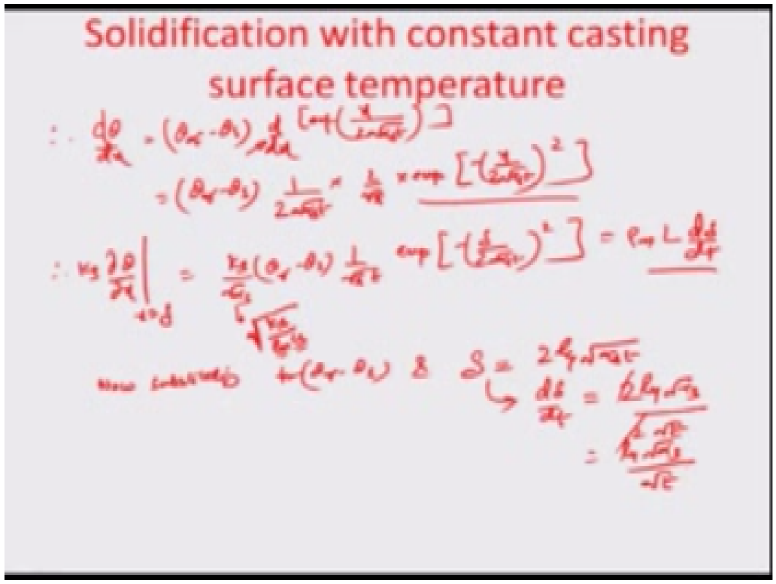

I am just borrowing it from earlier equation so $(\theta_\infty-\theta_s)$d/dx times of the error function $x/2\sqrt{\alpha_s t}$ and .therefore this can be you know defined as $(\theta_\infty-\theta_s)1/2\sqrt{\alpha_s t}$. I am going to just differentiate this error function times of again $2/\sqrt{\pi}$ exp$^{[-(x/2\sqrt{\alpha s t})2}$ okay, so that is how the error function is differentiated I think I have mentioned it in the last lecture and so this really can be put back into the equation that we were doing earlier as ks$\partial\theta/\partial x$ at $k=\partial$ we can actually put this as $ks/\sqrt{\alpha_s}$ $(\theta_\infty-\theta_s)1/\sqrt{\pi t}$ okay, exponential and this becomes $-\partial/2\sqrt{\alpha_s t^2}$ and that can be equated to $\rho_m L d\delta/dt$ okay, in order to find out what is going to be the $d\delta/dt$ in this particular case.

So we can substitute the value of α as here as $\sqrt{ks/\rho_m c_s}$ okay, where again it is a thermal diffusivity of the solid phase so it is can be defined as the thermal conductivity of the solid phase per unit the density of the solid or liquid metal similar to each other times of the specific heat capacity of the solid phase, okay. So this we substitute for $\theta_\infty-\theta_s$ so that is what we substitute the equation and δ which earlier was $2\zeta\sqrt{\alpha_s t}$ from what we have concluded here earlier okay, so we had assumed this ζ to be $\delta t/\sqrt{2\alpha_s t}$.

So when we substitute this back let us see what is going to be the final form so the first thing that I would like to get is expression for $d\delta/dt$ from this term so $d\delta/dt$ becomes equal to 2ζ being constant $\sqrt{\alpha_s}/\sqrt{t}$ this goes away so we are left with $\zeta\sqrt{\alpha_s}/\sqrt{t}$ and on the other hand we have the from equations done earlier here.

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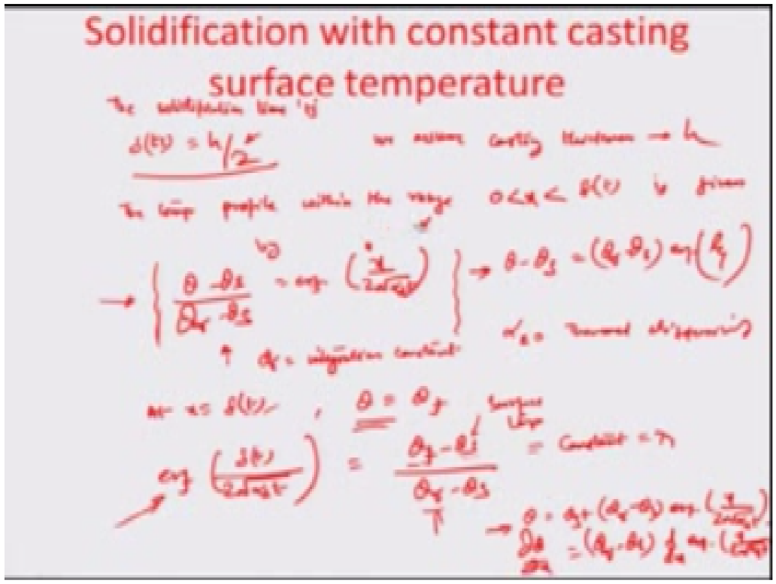

We have that the θ-θs really is a function of $\theta_{\infty}-\theta_s$ error function of the term ζ okay, this was ζ corresponding to x tending to δ if you may just remember okay. So if I just substitute this value here on the expression which I formulated in the last step this expression right here let us call it equation 1, then we have.

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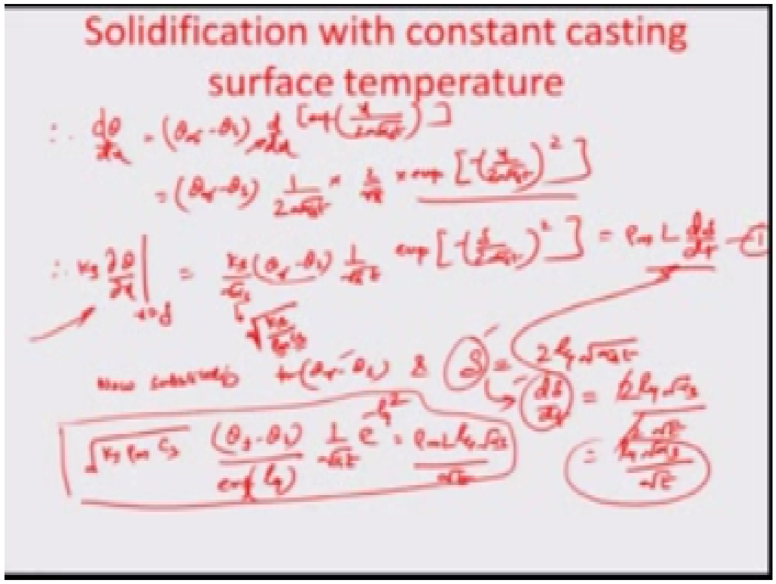

$\sqrt{k_s \rho_m c_s}\ \theta_f - \theta_s / errf(\zeta) 1/\sqrt{\pi t}\ e^{-\zeta 2} * \rho_m L \zeta \sqrt{\alpha_s}/\sqrt{t}$, so basically on one hand we are substituting the value of δ okay, $d\delta/dt$ from this expression here another hand this term $\theta_\infty - \theta_s$ is replaced by $\theta - \theta_s$ so therefore this is how the expression gets formulated after the substitutions of these two values respectively that the δ/dt and the $\theta_\infty - \theta_s$, so I will just now condense this expression.

$$\frac{k_s}{\sqrt{\alpha_s}}\left(\theta-\theta_s\right)\frac{1}{\sqrt{\pi t}}\exp\left[-\left(\frac{x}{2\sqrt{\alpha_s t}}\right)^2\right]=\rho_m L\frac{d\delta}{dt}$$

$$\sqrt{k_s \rho_m C_s}\frac{\theta_f-\theta_s}{erf\left(\zeta\right)}\frac{1}{\sqrt{\pi t}}e^{-\zeta^2}=\rho_m L\sqrt{\alpha_s}\frac{1}{\sqrt{t}}$$

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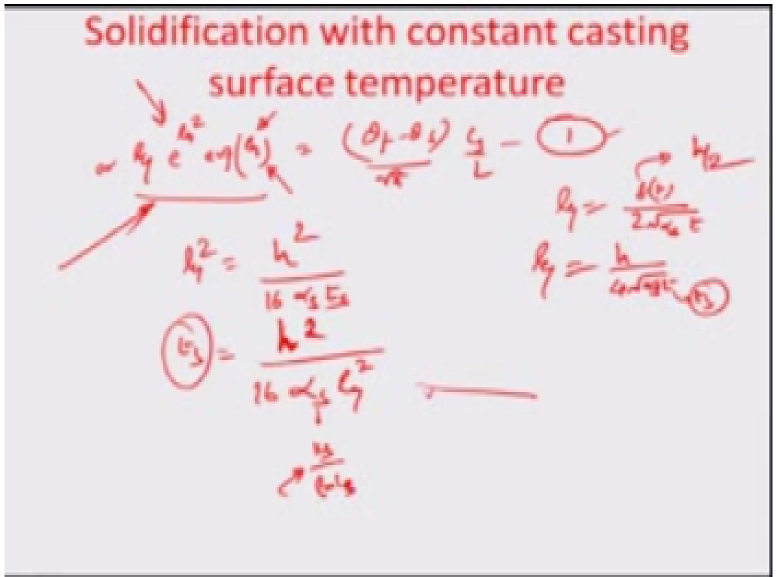

And write this as $\zeta e^{\zeta 2}$ error function of ζ on one side okay, is equal to θ_f-θ_s/$\sqrt{\pi}$ c_s/L, so typically if you wanted to find out what is going to be the value of ζ, so ζ as you already are aware is basically δt/2$\sqrt{\alpha_s}$t and this δ tends to h/2 if the casting is completely solidified so the ζ here would typically b=h/4$\sqrt{\alpha_s}$t or if I am able to find out the value of ζ all I need to do is to do ζ^2=h^2/16$\alpha_s t_s$ and obviously when δ is h/2 t becomes t_s that means it solidifying the whole casting at that one if point of time.

And the t_s becomes equal to h^2/16$\alpha_s\zeta^2$ so the trick here really is to find out what is this ζ from this equation right now here and this would need estimation for numerical integration method and there are standard tables which have to be obtained for the error function corresponding to the different values of ζ so that the left hand side and right hand side of this equation 1 can be away and from that you can have the time of solidification as the h^2/16α_s mind you this is the thermal diffusivity of the solid part which is k_s/$\rho_m c_s$ you had earlier seen.

$$\sqrt{k_s \rho_m C_s}\frac{\theta_f-\theta_s}{erf(\zeta)}\frac{1}{\sqrt{\pi t}}e^{-\zeta^2}=\rho_m L\sqrt{\alpha_s}\frac{1}{\sqrt{t}}$$

$$\zeta e^{-\zeta^2} erf(\zeta)=\frac{\theta_f-\theta_s}{\sqrt{\pi}}\frac{C_s}{L}$$

$$2\sqrt{\alpha_s t_S}=h/2$$

$$t_S=\frac{h^2}{16\alpha_s\zeta^2}$$

How we have substituted into the expression times of the ζ^2okay, let me just write this there little proper manner here okay, so this is times of ζ^2. So I think we now can iteratively sort of estimate what is going to be the solidification time and we will see in this particular case also numerically how the time is going to get varied because of this constant casting surface temperature. So in the interest of time we will close on this module and probably explore a little more of this processes in the next module, thank you so much. Bye.

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