Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology –part-1

Module 27

By
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Hello and welcome to this manufacturing process technology part 1 module 27.

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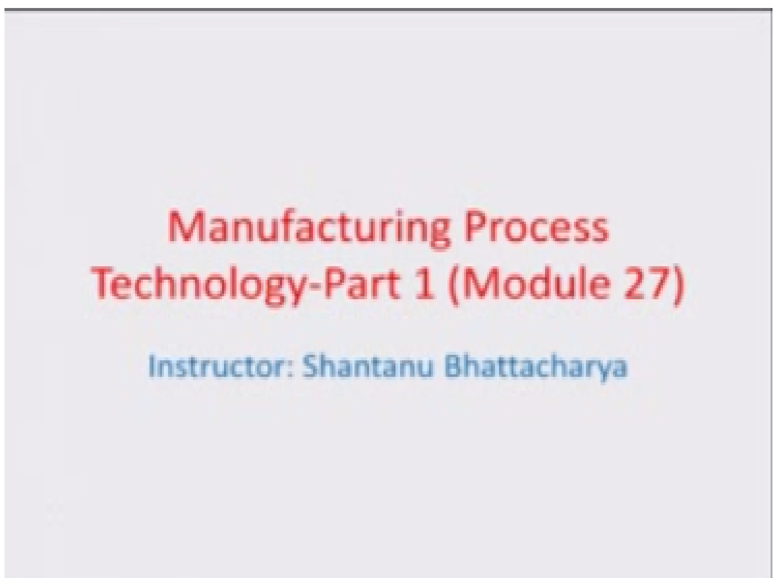

Brief recap we were actually trying to look into the heat transfer across a mold phase given the temperature boundary condition of the mold phase they would be equal to the freezing temperature of the material and then we would also assume that at X equal to infinity all the way from the mold phase the room temperature conditions exist that is theta equal to θ_0 which exists actually.

So here we would like to discuss another numerical problem where we want to determine the solidification time of the following two cast iron pieces or iron castings.

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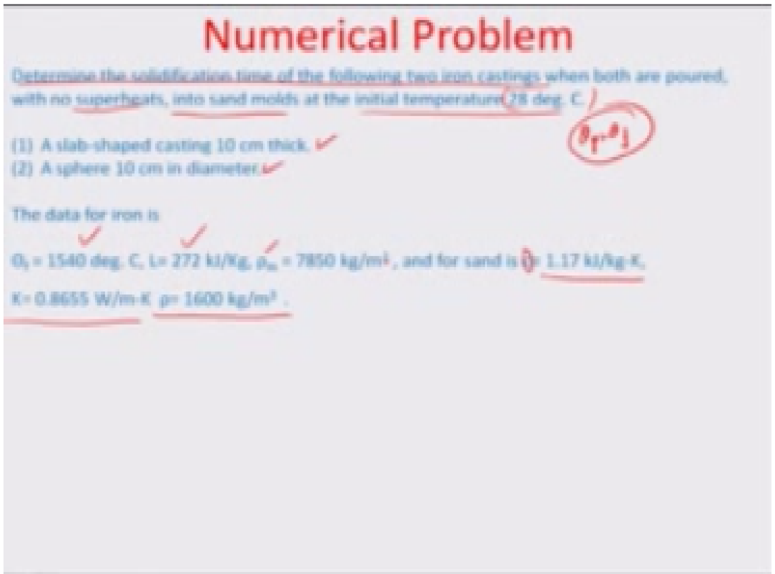

$$V = lbh$$

$$A = 2(lb + bh + lh) \approx 2\,lb$$

$$\frac{V}{A} \approx \frac{h}{2} = 5 * 10^{-2} m$$

$$\lambda = \frac{(1540 - 28) * 1600 * 1.17 * 10^{3}}{7850 * 272 * 103} = 1.3256$$

$$\beta = \frac{2}{\sqrt{\pi}} * 1.3256 = 1.4957$$

$$\alpha = \frac{k}{\rho c} = \frac{0.8655}{1600 * 1.17 * 10^{3}} = 0.46 * 10^{-6}$$

$$\alpha\, t_s = \frac{\left(\frac{V}{A}\right)^2}{\beta^2}$$

$$t_s = \frac{25 * 10^{-4}}{2.24 * 0.46 * 10^{-6}} = 2430\, sec\, 0.675 hr$$

(ii)

$$\beta = 1.4957 + \frac{0.4419}{\beta}$$

$$\beta^2 - 1.4957\,\beta - 0.4419 = 0$$

$$\frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{5}{3} * 10^{-2} m$$

$$\alpha\, t_s = \frac{\left(\frac{V}{A}\right)^2}{\beta^2}$$

$$t_s = \frac{25 * 10^{-4}}{9 * 3.05 * 0.46 * 10^{-6}} = 198\, sec\, 0.55 hr$$

When both are poured with no superheats that means in this particular case the theta pouring and theta freezing are equal there is no super heating as such of the material so they are poured into sand molds and the initial temperature of the sand mold or the room temperature could be about 28 degree Celsius. so in two cases one of the shapes that is being produced as lab shape at casting tensile fatigue the other was about 10 centimeter diameter sphere.

So in both the cases there would be definitely different amount of heat transfer because of the geometry and I think I had earlier mentioned about the geometry very clearly so the data of the iron is given that the freezing is that means freezing temperature of solidification temperature is 1540 degrees Celsius this can be also the mold wall temperature and then you have the latent heat of solidification is 272 kilo Joule per kg.

The density of the liquid metals about 7850 kg per meter cube and you know the density for the specific heat capacity for the sand, C 1.17 kilo Joule per kg Kelvin the k the thermal conductivity of the sand and the density of the sand has been given out to 0.8655 watt per meter Kelvin and1600 kg per meter cube so having said that let us actually try to see how will solve a problem so let us do part one first.

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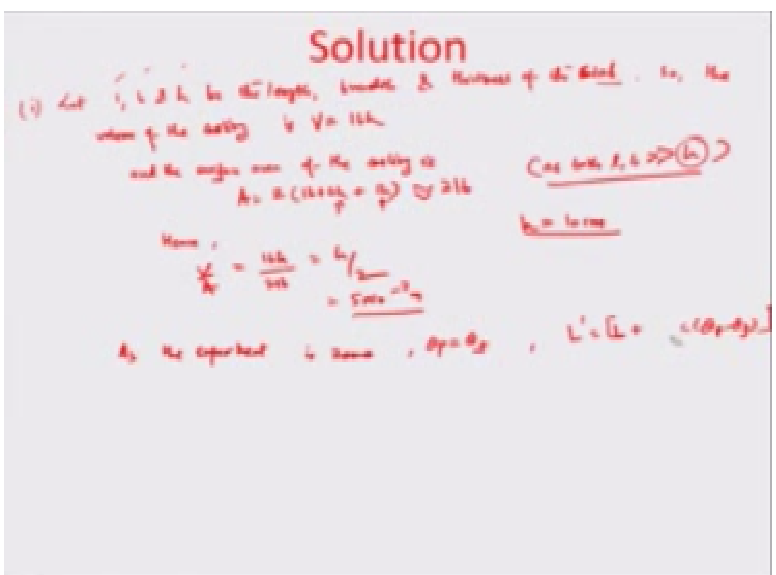

So let us assume l, b and h be the length breadth and thickness of this labs in the slab casting case that is how we have assumed this okay so the volume of the castes V equal to $V = lbh$ and the surface area of the casting is a equals $A=2(lb+bh+lh)$ so we can assume that both the length and breadth of the casting are very high in comparison to the thickness which is the case of a thin slab okay.

And that is what we had illustrated earlier also so this can actually be written down as twice because wherever there are the h components they can be safely neglected. so hence the V/A in this particular case would be L be h by twice lb that is h by 2 and in this particular case the height h thickness h of the casting has been provided earlier to be 10 centimeters so h by 2 becomes equal to 5 *10 to the power of -2 meters okay 5 centimeters.

$$V=lbh$$
$$A=2(lb+bh+lh)\approx 2\,lb$$
$$\frac{V}{A}\approx\frac{h}{2}=5*10^{-2}m$$

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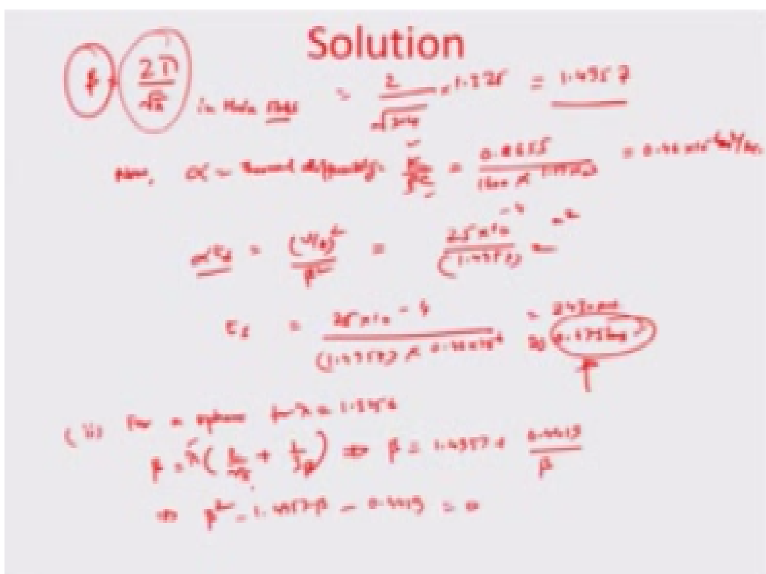

So as the superheat level is 0 we have condition the θ pouring equals θ freezing therefore which is equal to $L+C_m(\theta_p-\theta_f)$. it becomes equal to L as these two guys are same to each other so you have the value of L as 272 kilo joule per kg. Further you have theta zero equal to 28 degrees Celsius.

$$\lambda=\frac{(1540-28)*1600*1.17*10^3}{7850*272*103}=1.3256$$

So therefore we can write down both the coefficients λ and β. λ as per the earlier the deviation or proof is theta f- theta zero / by Rho ml - times of C if you may remember we had brought out two quantities β and λ and had correlated them with respect to each other as a function of the geometry for the spherical case it would have some relation for the cylindrical thin cylinder case it would have some other relation so on so.

For the so here we can write this down as 1540 -28 degrees Celsius times of density of the sand mold times of specific heat capacity of the sine mode which is 1600kg per meter cube times 1.17 to the power of three 3 J/kg-K. so that is how you find out rho and C and this you have to divide by the density that is density of the liquid metal so 7850 times of the L dash which is actually in this case equal to L, 272 times often to the power of 3 Joule per kg-K okay.

So that is how you arrived at the λ value. λ is coming out to be 1.3252 in this particular case and we also know further that the β is estimated as twice λ by root of particularly in case of thin slabs. I think I had done this derivation or proof earlier how this relationship works out and so this can be written down as twice by root of 3.14 times of 1.325 and that becomes 1.4947.

$$\beta = \frac{2}{\sqrt{\pi}} * 1.3256 = 1.4957$$

So that is how β coefficient can be calculated or correlated. now for the thin slab case let us calculate what is the thermal diffusivity k/rho C where k is the conductivity across the mold and C are the density of the mold and the specific heat capacity of the mold k has been already given as 0.8655 watt per meterK divided by the density with the 1600 kg per meter cube times of the specific heat capacity of the sand mold which is $1.17*10^3$ Joules per kg Kelvin.

$$\alpha = \frac{k}{\rho c} = \frac{0.8655}{1600 * 1.17 * 10^3} = 0.46 * 10^{-6}$$

So this comes out to be equal to 0.46 into 10 to the power of minus 6 meter square per second. Alpha t_s becomes equal to V /A square by square of β as per the earlier expressions and this V /A square is $25*10^{-4}$ meter square divided by the square of β which is 1.4947 square. so therefore the value of t_s comes out to be $25*10^{-4}$ / 1.4967 times of the value of the alpha will just come out to be 0.46*10 to the power of minus 6 meter square per second.

$$\alpha t_s = \frac{\left(\frac{V}{A}\right)^2}{\beta^2}$$

$$t_s = \frac{25 * 10^{-4}}{2.24 * 0.46 * 10^{-6}} = 2430\, sec\, 0.675\, hr$$

And this comes out to be equal to about 2430 seconds approximately 0.67 hours so that's how much it would take about close to 40 odd minutes for the casting to solidify as far as the thin castings case goes in this particular case. We just do the same for the sphere .so for sphere we already know the λ value to be equal 1.3256 and the relationship between β and λ is governed

by the expression we try go to $\beta=\lambda\left(\frac{2}{\sqrt{\pi}}+\frac{1}{3\beta}\right)$.

So we can actually substitute the values of λ and π and find out expression for β equals 1.4957 + 0.4419/ β. so this brings us to a quadratic equation β square -1.4957 β- 0.4419 equals to zero. if we take only the positive real root of this equation to find β then β can be written down as β = -b ± root over b Square- 4 ac by 2a okay.

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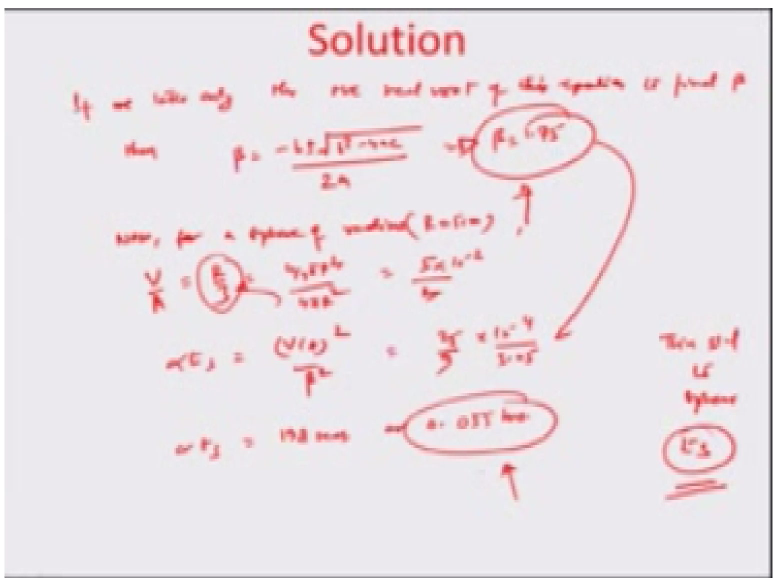
$$\beta=1.4957+\frac{0.4419}{\beta}$$

$$\beta^2-1.4957\beta-0.4419=0$$



So calculating this β comes out to be equal to 1.75. now for a sphere of radius R equals 5

centimeters the V/A are really equals $\frac{V}{A}=\frac{\frac{4}{3}\pi R^3}{4\pi R^2}$ okay. so R by 3 and this can be written

down as $\frac{5}{3}*10^{-2}m$ because the radius is 5 centimeters so α t_s in this particular case would be V/A square by square of β and this becomes to 25 by 9 times of 10 to the power of minus 4 divided by square of β s 3.05 is borrowed from the solution of here.

$$\frac{V}{A}=\frac{\frac{4}{3}\pi R^3}{4\pi R^2}=\frac{5}{3}*10^{-2}m$$

$$\alpha t_s=\frac{\left(\frac{V}{A}\right)^2}{\beta^2}$$

$$t_s=\frac{25*10^{-4}}{9*3.05*0.46*10^{-6}}=198\,sec\,0.55\,hr$$

or the t_s value comes out to be equal to 198 seconds or 0.055 hours. So you can think about the reduction in thecae from thin slab to sphere in the t_s values time of solidification values so that is how we basically try to arrive at solutions regarding t_s or solidification time of the casting based on geometry.

We still need to develop two more cases where we would like to illustrate particularly the you know concept of thermal resistance and thermal capacitance so for that I would just urge you to look into this electrical analogy of all you know thermal problems where there is a mass M of specific heat capacity C.

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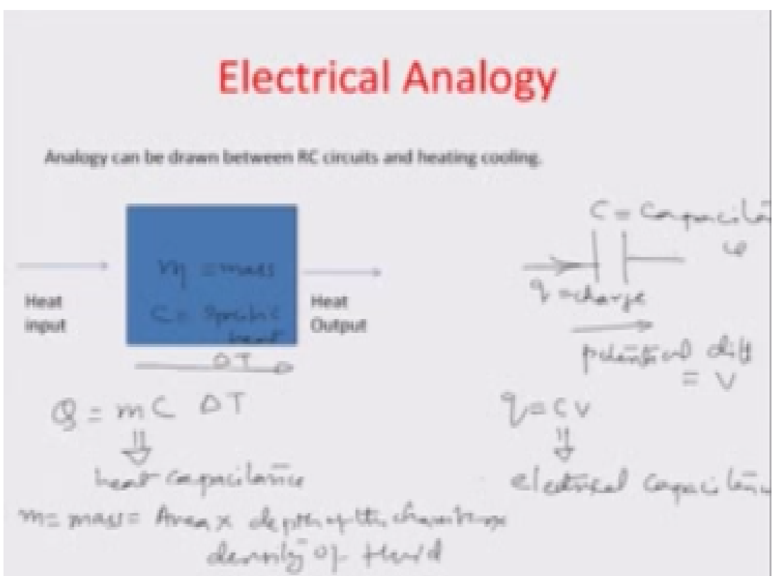

$$Q=mC\,\Delta T$$

And there is a heat input from one side output from the other and there's temperature gradient which exists Delta T across this mass so we all know that the total amount of heat transferred would be equal to $Q=mC\,\Delta T$ and mC is basically called the heat capacitance because this can be an analogy between capacitor so in a capacitor if you have a charge Q flowing and capacitance is C people have Q equal to C V relationship between the charge of the voltage here the heat is the charge flow it is the analogy for charge flow and the voltage here.

The capacitance is having an analogy delta T which is the driving factor of the potential for the heat to flow so obviously there has to be a temperature gradient this side has to have a higher temperature and this to a lower temperature for the heat to move from the high temperature to the low temperature so this is the gradient just like voltage gradient which carries forward charges in electrical circuit shaving said that mC therefore is basically the capacitor equivalent C equivalent in the thermal circuit.

So in our future analogies we may be needing to use this electrical analogy for understanding what is the thermal capacitance or heat capacitance similarly we have the thermal resistance and analogy where if we use throne-dimensional 4-years law the Q dot or the del Q by Del T the rate of heat flow across this mass would be related to the temperature gradient delta T by X where X is the length across which this delta temperature gradient exists and it will be proportional to this temperature gradient and the proportionality constant here is the thermal conductivity times area.

So there is A very famous for Fourier's law of heat conduction one dimension deep connections I would like to just draw an analogy here between a resistor which is having current I across the resistor is potential V because of which the currents formulated. so I which is actually rate of charge flow same to rate of heat flow here becomes V by R that means whatever is a potential divided by the R. so in this particular case the heat resistance comes out to be equal to R equals X by K A.

So this is V this is the analogy and this X by K A becomes the resistance R okay so that's how we get the thermal resistance so therefore just as the electrical circuit there is a you know there is a potential drop that is also going to be a temperature drop as the heat front meets new material or additional material.

So just for the sake of analogy please keep this in mind we will close this module here but in the next module we will like to do again you know a different variation of the problem by seeing what is going to be the boundary change if supposing there is an issue of the liquid not spreading across the whole surface okay and so that is the air film which is formulated between the solid boundary of the metal and the sand boundary okay so with this we like to end thank you.

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