**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**
**Manufacturing Process Technology – Part- 1**

**Module- 26**

**by**
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Hello and welcome to this manufacturing process technology part 1 module 26.

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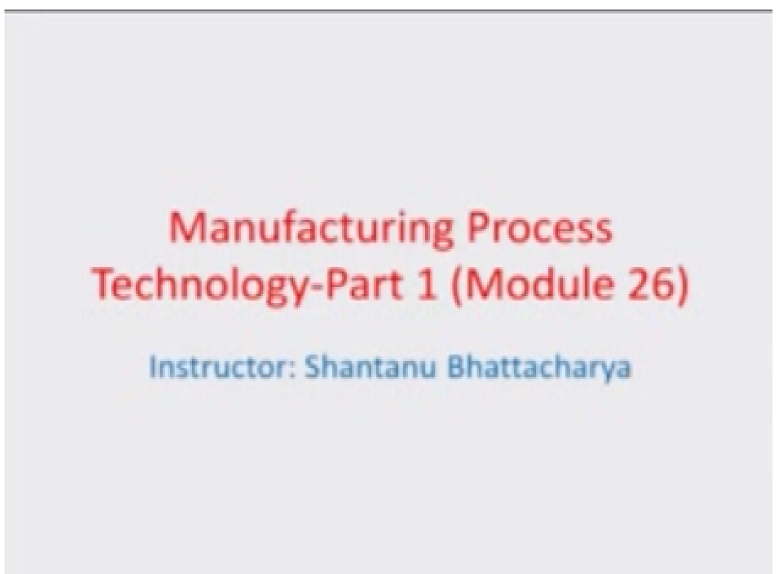


We will talking about then solidification time of a casting particularly a sand casting with the certain temperature boundary condition observed rated and then we found out that the total amount of time.

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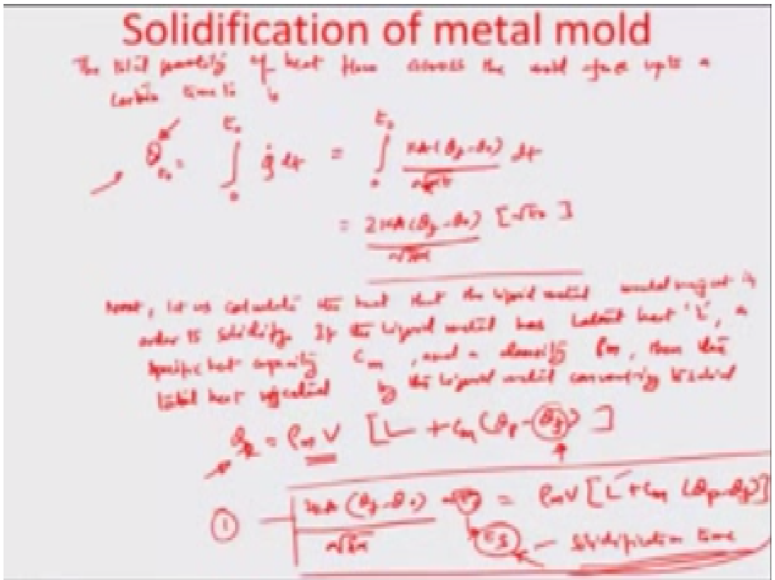


$$\dot{Q} = -kA\left(\frac{d\theta_x}{dx}\right)_{x=0}$$

$$\dot{Q} = \frac{kA\left(\theta_f - \theta_0\right)}{\sqrt{\pi\alpha t}}$$

$$Q_{t_0} = \int_0^{t_0} \dot{Q}\,dt = \frac{2kA\left(\theta_f - \theta_0\right)}{\sqrt{\pi\alpha}}\sqrt{t_0}$$

That is really needed is defined by this particular equation here given as equation 1 probably okay which talks about how the heat rejected is going to be transport across a mold phase into the mold.

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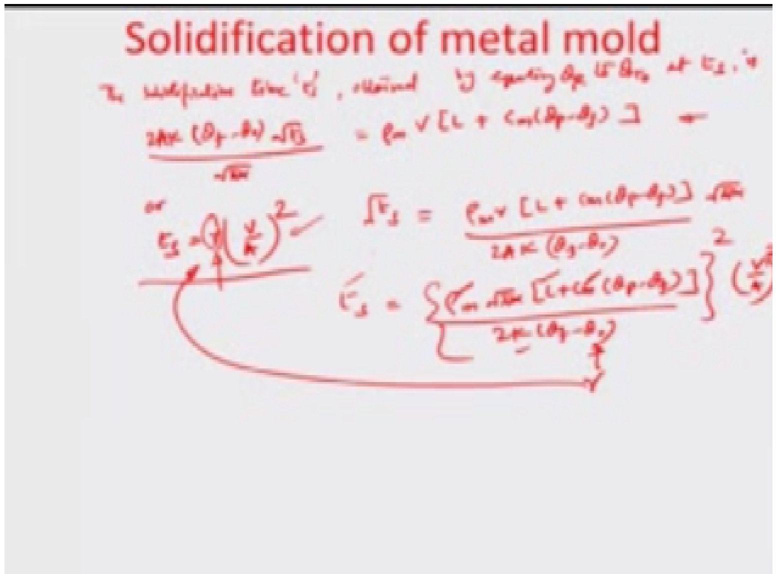


So in other words we solidification time $t_s$ and by equation the heat rejected $Q_R$ to the heat flowing across the mold phase is 0 at $\frac{2kA(\theta_f-\theta_0)}{\sqrt{\pi\alpha}}\sqrt{t_s}$ and that equal to $Q_R=\rho_m V\left[L+C_m(\theta_p-\theta_f)\right]$ okay or the total time of solidification Ts can be proportional to the volume per unit area square or the square of the you know length dimension, okay and this γ here write here is a constant which is generated from this particular equation okay. So in any even less just do how we arrived that it so obviously the Ts in this particular case?

$$\frac{2kA(\theta_f-\theta_0)}{\sqrt{\pi\alpha}}\sqrt{t_s}=\rho_m V\left[L+C_m(\theta_p-\theta_f)\right]$$

Could be √ $t_s$ in this particular case could be recorded as $\sqrt{t_s}=\frac{\rho_m V\left[L+C_m(\theta_p-\theta_f)\right]\sqrt{\pi\alpha}}{2kA(\theta_f-\theta_0)}$ so on so forth. Or I can say $t_s=\frac{\{\rho_m[L+C_m(\theta_p-\theta_f)]\}^2\pi\alpha}{\{2k(\theta_f-\theta_0)\}^2}\left(\frac{V}{A}\right)^2$ i.e. $t_s=\gamma\left(\frac{V}{A}\right)^2$ where,

$\gamma=\left[\frac{\rho_m\sqrt{\pi\alpha}\{L+C_m(\theta_p-\theta_f)\}}{2k(\theta_f-\theta_0)}\right]$ so this exactly as what I mean by γ .it is material property which depends on the thermal conductivity of the mold to the density of the liquid metal the specific heat capacity of the liquid metal and the latent heat of fusion, or solidification of the liquid metal so therefore this it is responds and then.

The solidification time here can be assume to be proportional to the square of the length parameter V / A volume by area that is a length okay, so having we said that it should be noted that the forming analysis.

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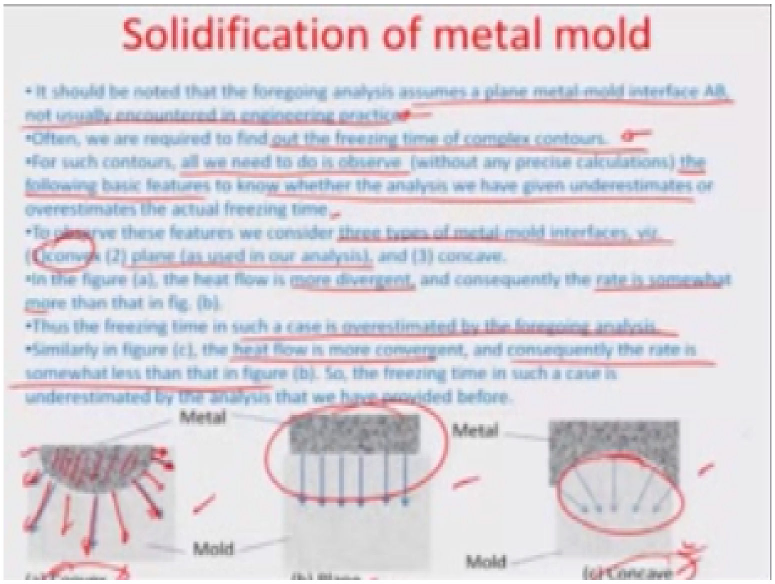


Assumes the plane metal mold interface AB okay is we will not encountered in engineering practice because obviously there are seldom situations where there is actually a straight metal phase, rather the metal that is being caste or of difference sizes and shapes that it can be it is spherical ball that you are producing or may be it can be a concavity or convexity on the surface that it could be so often we are required to find out the freezing time of complex contours in case of actual metal solidification process.

And for such contours we all need to do is observe without any precise calculations the following basic features to know whether the analyses is given in underestimate or an overestimate of the actual freezing time so observe this features we considered three different types of mold metal interfaces, mainly the convex which can be recorded here , a plane interface which we have already done as used in our analyses, and a concavity which is here so in this particular case as you can see.

That the heat flow is more divergent in nature okay in the convex right because this is how the hot metal is placed and basically trying to conduct the heat outside in the mold in a divergent direction okay, similarly in a concavity you have other way round that is all the heat is being focused into a different into a one particular region so therefore it is convergent okay, so thus the freezing time in such cases particularly if it is more divergent the rate obviously would be somewhat more than hydrate be in a straight case.

Right because obviously you are not letting it out the heat flow on to a larger volume okay so therefore you can say that with the time of analyses that we had in our planner case this would be a little over estimated the freezing time in such a case would be kind of over estimated okay because freezing would happen earlier in case the heat flow is divergent, but if the heat flow is more convergent as in the concavity case so obviously we are underestimating so it is consequently somewhat less than that in figure P.

So therefore the freezing time has to be different at least intuitively that is what we can look at and see from these different shapes here and how the convexity or concavity may be able to underestimate or overestimate the freezing time okay, so the quantitative for results.

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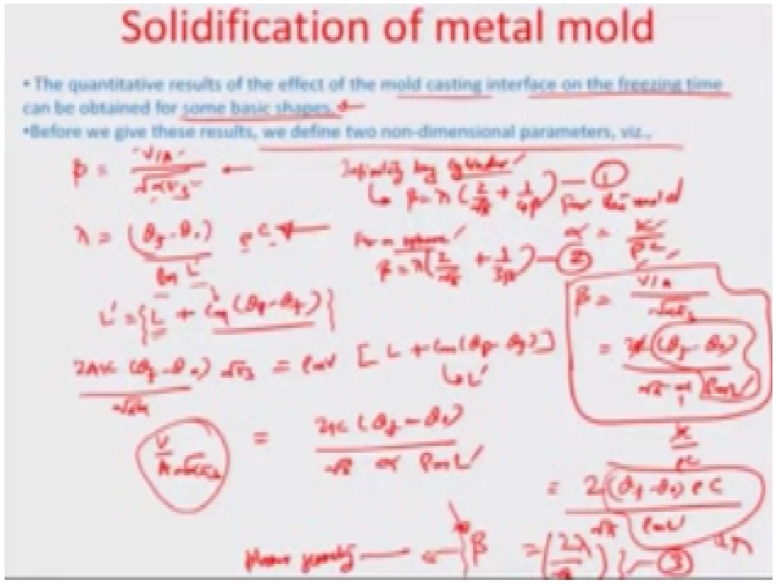


Which are of the effect of the mold casting interface on the freezing time as they can be obtained or some basic shapes and I am not going to go into the thermal engineering theory behind it and I am going to just mention it has hand and thumb rules which we could actually follow to do the designing which is a more important part here, write about here so before we gave these results we define to non dimensional parameters namely one parameter we define is β which is actually equal to $\beta = \frac{V/A}{\sqrt{\alpha t_s}}$ where V is the volume is area α is again the thermal diffusivity $K/\rho C$ of the mold.

And Ts is the time of solidification and the other parameter that I would like to define is λ which is actually given by $\lambda = \frac{(\theta_f - \theta_0)}{\rho_m L^{'}} \rho C$ where ρ and C are the density and the specific heat capacity of the mold whereas ρm is density , the is L dash is the latent heat of solidification of the liquid metal, and this L dash is something which can be related to the difference between the difference between the pouring and freezing temperature, this is basically the term $L^{'} = L + C_m(\theta_p - \theta_f)$ that had been earlier illustrated in last analyses okay.

So this is the modified latent heat because of some you know issue of a higher pouring temperature than the freezing temperature and just so hest losses by virtue of that for the pouring temperature changing into the freezing temperature and the other part is go because of the solidification which is completely done by latent heat, so obviously if this θP and θ for to be equal to then L = L dash so that is how would estimated so in terms of these parameters, now let us write the equation that we had seen earlier.

So V on one hand and $\frac{2\,Ak\left(\theta_f-\theta_0\right)}{\sqrt{\pi\alpha}}\sqrt{t}$ that is a heat total rejected across the mold phase, and the other one was related the solidification process, so density of liquid metal times of volume of the metal plus the modified $L^{'}=L+C_m\left(\theta_p-\theta_f\right)$ which was actually g modified L dash okay, so in other words I can arrange these terms together as $\frac{V}{A\sqrt{\alpha t_s}}=\frac{2k\left(\theta_f-\theta_0\right)}{\sqrt{\pi\alpha}\,\rho_m L^{'}}$ , so we know that for the mold the α thermal diffusivity is actually K / ρC where k is the thermal conductivity.

Of the mold and ρ and C is or the density of the mold and the specific heat capacity of the mold so therefore the β value here, can be recorded as the $\beta=\frac{V/A}{\sqrt{\alpha t_s}}$ which is actually the parameter β is at has been illustrated here, and the V / A value again you know actually is nothing but this right so this can be written down as $\frac{2k\left(\theta_f-\theta_0\right)}{\sqrt{\pi\alpha}\,\rho_m L^{'}}$ okay so that is how β can be defined and the θf – θ0 / ρm L dash this whole value times of ρC okay so if I were to multiply this with let say you know juts by mentioning one by this expanding this term α is K/ρC.

I would actually be able to write this down as the K cancels out we have $\frac{2\left(\theta_f-\theta_0\right)\rho C}{\sqrt{\pi}\,\rho_m L^{'}}$ dash okay so this guy is nothing but λ as you can see from this equation write here, so we have the case where β can be defined in case of plain geometry, as $\beta=\lambda\frac{2}{\sqrt{\pi}}$ so this is in case of planner geometry and the only difference that the concavity and the convexity would be made will be actually in terms of this relationships and I am going to empirically now just the state all this relationships so though getting into the integrity of proving details in fact if you need any details about these expressions you can actually ask a separate question.

Question and maybe we will send you the interest of time the complete details but for infinitely long cylinder, the relationship that would be given as $\beta=\lambda\left(\frac{2}{\sqrt{\pi}}+\frac{1}{4\beta}\right)$ and for a sphere which are generally the shapes you know more or less this expression would be $\beta=\lambda\left(\frac{2}{\sqrt{\pi}}+\frac{1}{3\beta}\right)$ okay, so that is how we are going to record for cylinder case and sphere case and overall for a planner geometry this is what it is so this is a planner geometry, so these are the three assumptions 1 2 and 3.

That we would have for us be enabled to do problems solving as I will illustrate in the next question.

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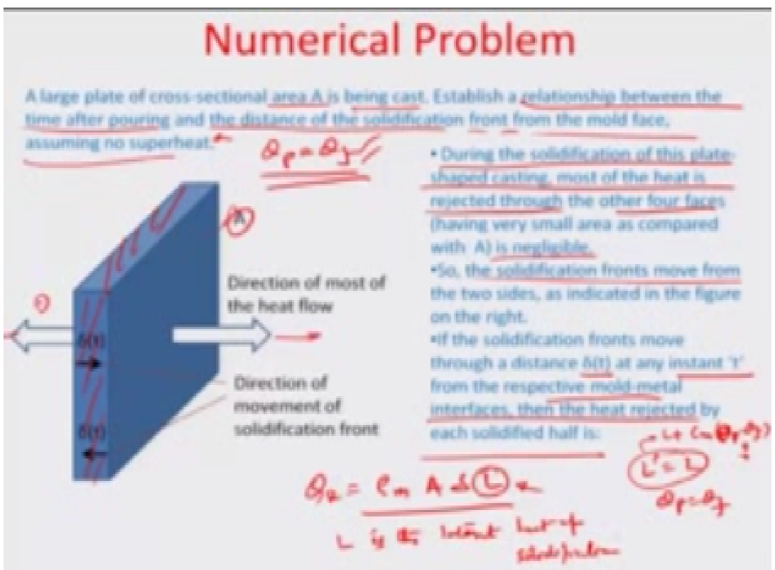


So here for example there is a large plate of cross sectional area A which is being cast and I would like to numerically now start estimating the time of solidifications, so you have established or the relationship between the time after a pouring and the distance of solidification front from the mold phase, assuming no super heat so we assume the pouring temperature is the same as the freezing temperature so during the solidification of this plate shape it casting most of the heat is rejected through the other four faces.

Okay and we can see that this kind of negligible okay, so you can see that most of the heat is rejected and cross the face A and the other phases here or here are very, very small so their rejection rate is probably not considerable here, so the solidification fronts starts moving mostly in both the directions either in this direction or in this direction it is obviously the most of the majority of the heat flow is in this direction, and in the solidification front moves through a distance let say $\delta$T.

At some point of time t or it some instant of time T from the respective mold metal interface and the heated rejected by each solidified a ½ will be now estimated as $Q_R = \rho_m A \delta L$ obviously the L dash is L here because θP and θf or same we are assuming that the and picture has been the pouring temperature is nothing but the freezing temperature, so the L dash terms which was either $L' = L + C_m(\theta_p - \theta_f)$ .
This would actually be the 0 component l and L dash will be similar so therefore it is only the heat of heat of solidification, which is actually to come out and it has to be rejected so the L is the latent heat of solidification okay.

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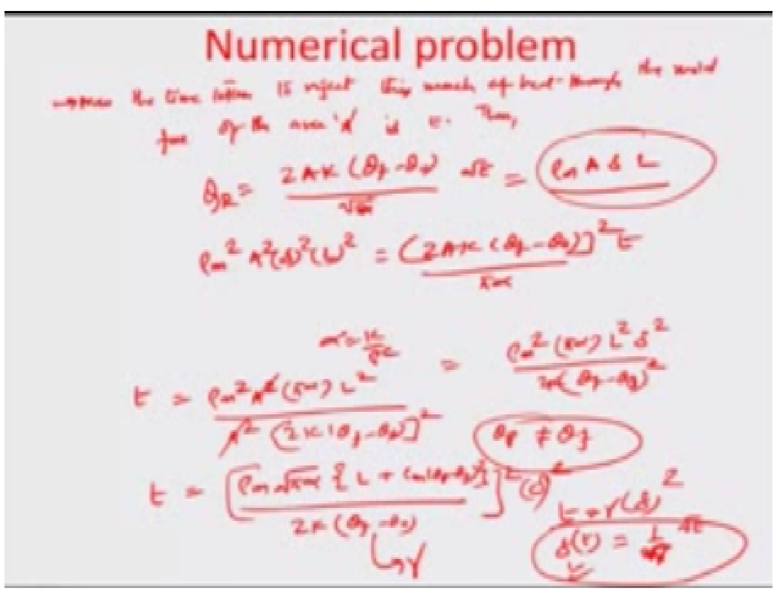


Now the time taken to reject this much of heat through the mold face of the area A let say this is t okay, then the Qr that is the heat rejected should be equal to $Q_R = \frac{2Ak(\theta_f - \theta_0)}{\sqrt{\pi\alpha}}\sqrt{t}$ where t is the total time that is needed okay, so therefore we can always say that the total amount of heat

rejected which I has earlier calculated here that is $Q_R = \rho_m A\delta L$ is equated to this particular amount of heat that is you know actually rejected.
So this is the heat translated for solidification to happen this is the heat which is rejected to equal to each other, and so therefore we can say that $\{\rho_m A\delta L\}^2 = \left\{\frac{2Ak(\theta_f - \theta_0)}{\sqrt{\pi\alpha}}\sqrt{t}\right\}^2$ , α = K / ρC and it would therefore me in that for the total times that is needed, for the whole process here to happen is actually equal to $\rho m^2$ times of πα times of $L^2$/ 2Ak let say we just take out the square of A so we have $A^2$ times of 2K $[\theta f - \theta 0]^2$ okay so in this being the as calculation here for the suppress of T.

And these guys go off and we are left with actually for term or $t = \frac{\{\rho_m A\delta L\}^2 \pi\alpha}{\{2Ak(\theta_f - \theta_0)\}^2}$ and supposing then the θp or not equal to θf on the pouring temperature was a different and there was some degree of super heat and I would say that T then in that we were would be written down as Pm or ρm times of 8πα times of $L + C_m(\theta_p - \theta_f)$ . that is L dash value okay, and divided by $\{2k(\theta_f - \theta_0)\}^2$ okay $L^2$ of $\delta$ and another words that is what the γ is going to be and t is actually $t = \gamma\delta^2$ or $\delta$t is a function of the time of solidification.

Becomes $\delta(t) = \frac{1}{\sqrt{\gamma}}\sqrt{t}$ okay so that is how the functional relationship would be obtained between the time of pouring the distance of solidification front of the mold phase so the $\delta$ is a function of time is $\delta(t) = \frac{1}{\sqrt{\gamma}}\sqrt{t}$ okay so that is how we do these questions so I will actually close on here the and the interest of times so we just would like to end this particular module and the next module we will do few numerical problems regarding this estimation of the time solidification thank you.

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