Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology – Part- 1

Module- 25

by
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Hello and welcome to this manufacturing process technology part1 module 25

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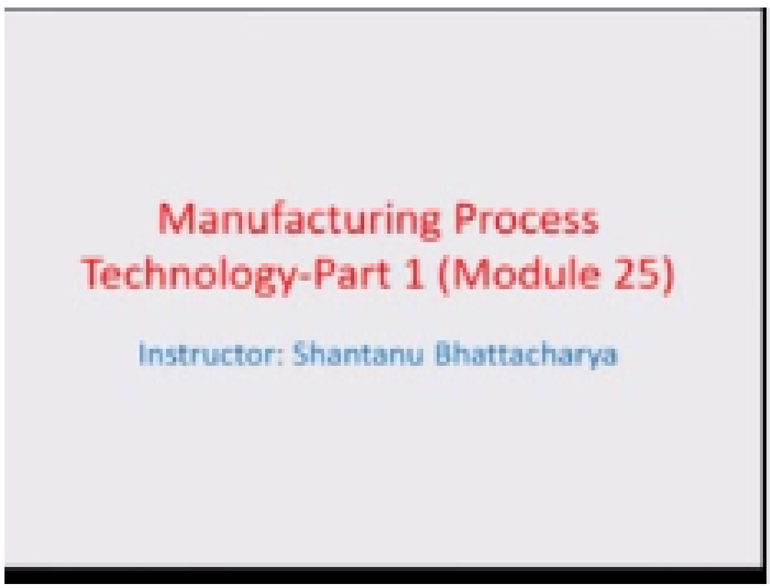

We were talking about the temperature distribution of a sand molding and one side of the molding on the temperature boundary condition was that equal to the freezing temperature of the material the other side it was open to the atmosphere and we were trying to decide for what would be the distribution of θ per sale with respect to time and temperature. So for that we actually done this similarity variable analyses yesterday.

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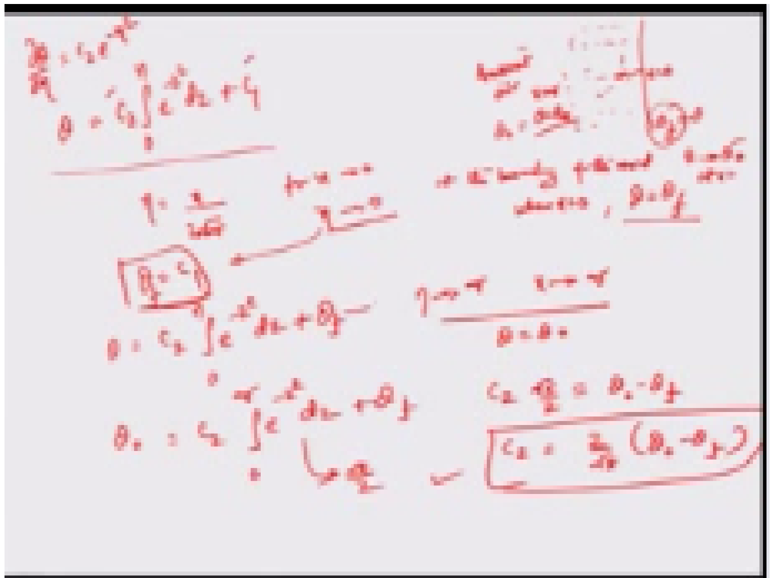

Through which we got a variable η and then try to convert the transfer problem in time and space to the variable η okay and so what we found out was his expression $d\theta/d\eta=y=c2\eta^2$ or just writing that back here $d\theta/d\eta=C2e^{-\eta 2}$ meaning there by that θ can be represented as if you just integrate θ you know $d\theta$ it can be represented as integral 0 to η, e^{-z2} dz+c1 and let's now decide for what are C2 and C1 through the normal boundary conditions so if supposing η which is actually $x/2\sqrt{\alpha t}$ for x tending to 0 also becomes 0.

So the η tends to 0 here and that there would be actually equal to C1 and at the boundary of the mold where x is 0 you already know that the θ has been defined as freezing temperature θf. It may just recall this mold was the something like sand mold okay on one side of it the boundary was corresponding to the start of x=0 as x would proceed all the way to ∞ add x to ∞ and there is ambient air which should be all round this particular mold okay.

So $\theta=\theta_f$ here and the other thing would correspond to eventually when for the large amount of time the θ would tend to be equal to θ_0 at $x=\infty$ meaning there by that you know it is always sort of it is actually equal to room temperature obviously it's very, very large distance away so it should be equal to ambient temperature so this is θ_0 okay.

So having said that now let us now try to evaluate θf becomes equal C1 corresponding to $\eta=0$ at this particular phase you know $\theta= \theta_f$ here and therefore θ can be represented as C2 integral 0 to η, $e-^{e2}dz+ \theta_f$

And let's look at the case when η is ∞ that means corresponding to x trending to infinity when θ becomes equal θ_0 so you have θ_0 here this is C2 integral 0 to ∞ $e^{-e2}dz + \theta_f$ and this is actually integral whose value is estimated as √pi/2.

So that is how you know you can estimate what is going to be the C2 value for in this particular case the C2 times of √Π/2 happens to be equal θ_0-θ_f and so the C2 would come out to be equal twice by √Π (θ_0-θ_f). so having found out what is C2 and what is c1 through these two equations I substitute this back into the final form of the equation here which we call 7 and try to find out what is the distribution of θ.

So the θ then becomes equal C2. let's write the equation once more C2 integral 0 to η $e^{-e2}dz + \theta_f$ which is actually the temperature boundary condition along the phase of the mold on c2 here becomes equal to again if you just look back C2becomes twice by √Π (θ_0-θ_f). so twice by √Π (θ_0-θ_f) times of integral 0 to η $e^{-e2}dz + \theta_f$ so that is how θ is represented on another words if I just wanted to add and delete θ_0 so basically let's say we had and believe θ_0 here.

So I would have much simpler form of the expression as $\theta_0 + \theta_f - \theta_0$ times of (1-2√ η) 0 to η $e^{-e2}dz$ and this is nothing but the error function of the term η okay and η basically is nothing but numerical integral so η basically nothing but x/2√αt and so therefore the value of θ comes out be equal to $\theta_0 + \theta_f - \theta_0$ times of 1-error function of x/2√αt so that is how I would I to finally express the θ calling it equation 8 and then you know this is the only case where I have in the interest of just

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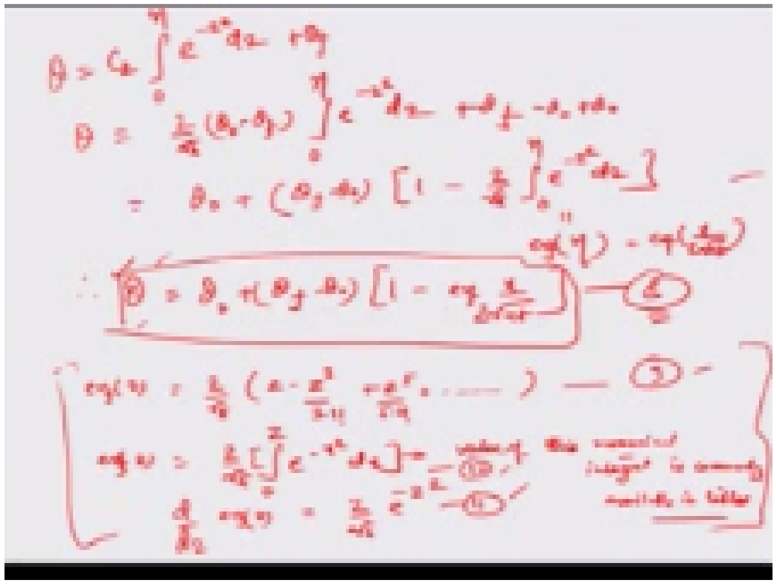

Detail knowledge I try to prove what is the value of θ you have to understand the all other condition that are trying to happen including the condition where there is resistance at the you know the mold wall because of the thin air fill developing between the liquid metal and the mold or in the case when there is a liquid to liquid or a solid to solid complete welding you know because of a solid mold the only things which vary there are the boundary conditions.

So we will try to sort of evaluate and various boundary condition and decide what is going to be the transfer equation in those cases analyse there is curvature etc these are the normal modality which are followed, so that is how you basically represents the temperature distribution function of the mold as the function of distance and here it is probably for me to define the error function so the error function of the function z can be written down series as twice by $\sqrt{\Pi}$ times of z-z cube /3! so this is from common knowledge of mathematics I'm not going to go into the details of all this but just to the present because we need these illustration of these sort of you know proof the various boundary condition etc.

And then also there are function of z as you know can be written down as the 2 by root pi times of integral of 0 to z e^{-x^2} dx and obviously the value of this integral of this numerical integral is commonly available in tables through which you would like to sort of illustrate or actually try to find out what are the real times of solidification etc based on distribution of etc. so also that d/dz of this error function should be able to get represented as $2\sqrt{\Pi}e^{-z^2}$ so these are some things we call it equation 9,10 and 11 it's the some of the basic mathematical forms of the error function that we would like to investigate or like to use basically later on from our analyse point of view and we already know how this error function is related with respect to θ with the function of x.

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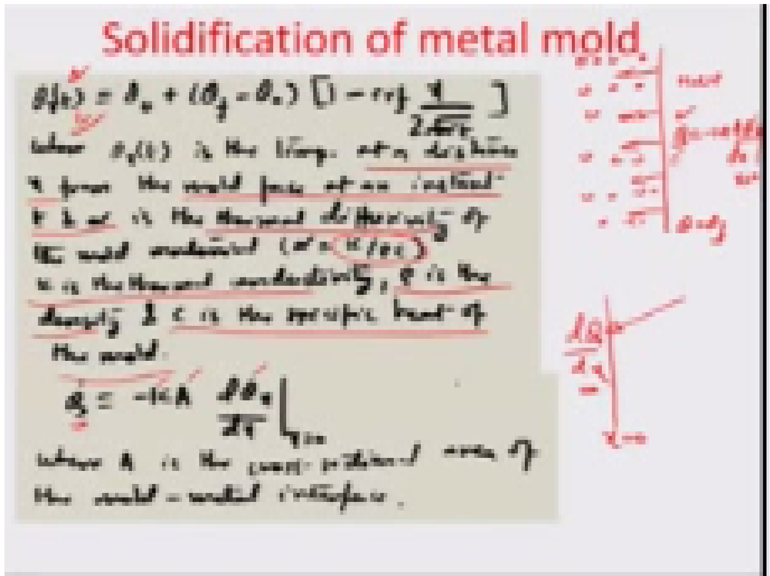

$$\theta_x(t)=\theta_0+(\theta_f-\theta_0)\left[1-erf\,\frac{x}{2\sqrt{\alpha t}}\right]$$

$$\dot{Q}=-kA\left(\frac{d\theta_x}{dx}\right)_{x=0}$$

So this is an actual summary of whole form of representation of θ so θx is now the temperature at a distance x from the mold phase at the distance t and α is the thermal diffusivity which is normally given my thermal conductivity k/row c that is how it was defined in the heat transfer equation the thermal conductivity, rho is the density of the liquid metal.

And C is specific heat capacity density of the mold I'm sorry and heat is the specific heat capacity of the mold as well this temperature distribution mind you in mold so it's actually the sand properties that we are evaluating here while doing the θ distribution within the sand mold and therefore now you already know that the rate of heat flow which would happen from the metal to the mold would really depend on –k , the thermal conductivity times the area of the mold interface with respect to the liquid metal times of difference ingredient with respect to x so this dθ/dx is something that we have to look at boundary x=0 so you already know that the sand mold as I illustrated earlier had a boundary where the θ value that was there was corresponding to the freezing temperature.

And the heat transfer that was happening was typically in this direction across the area of the mold metal interface so the amount of heat quantity now heat rate Q dot that is minted from the obviously the metal which is at the higher temperature to the mold is actually is depend on –ka $d\theta x/dx$ at x=0. so that is how you basically try to decide for the Q dot or the rate of heat transfer so having said that now the obvious idea that would come into picture is how to sort of calculate what is this Q dot okay.

And try to then illustrate everything in terms of temperatures θ_f, θ_0 so on so forth so we have to somehow now be able to find out mathematically what is this $d\theta x/dx$ from this expression θx so that it should be able to calculate what is the rate of heat flow, so let's do that. so here you have side dq/dt which is actually equal to $-kAd\theta x/dx$ from this simple four year law add x=0 and the θx as a function of x has been given out as θ_0 times of θ_f-θ_0 times of 1-the error function of $x/2\sqrt{\alpha t}$ and therefore the $\delta\theta x/\delta x$ here comes out equal to θ_f-θ_0 with the minus sign of course times of $2\sqrt{\Pi}$ times of you know $1/\sqrt{\pi t}\ e^{-x2/\alpha t2}$.

So that is how you calculate the error function because already I have defined d/dz of the error function of z as equal to $2/\sqrt{\Pi}\ ^{e-z2}$ okay. So in this case z off course is $x/2\sqrt{\alpha t}$ and so the d/dz of the error function of $x/2\sqrt{\alpha t}$ would actually become $2/\sqrt{\Pi}\ ^{e-x/2\sqrt{\alpha t2}\ so}$ it is basically $1/2\sqrt{\alpha t}$ so having said having said that.

$$erf(z)=\frac{2}{\sqrt{\pi}}\left(z-\frac{z^3}{3.1!}+\frac{z^5}{5.2!}-\frac{z^7}{7.3!}+\ldots\right)$$

$$erf(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}dx$$

$$\frac{d}{dz}erf(z)=\frac{2}{\sqrt{\pi}}e^{-z^2}$$

So that is how we substituted this value right about here so let's now look at what really that would be so this is actually the two goes on$\sqrt{}$ here and we are left with at x=0 if you want to calculate the $\delta\theta\ x/\delta x$ x=0 that becomes equal to θ_0-θ_f so the minus is taken inside times of $1/\sqrt{\Pi\alpha t}E^0$ which is actually can be again further written as θ_0-$\theta_f/\sqrt{\Pi\alpha t}^{1/2}$ so that is how $d\theta x/d$ and therefore the dq/dt that I was talking about –k times of a times of θ0-θf divided by again the in another words it can be written down as k θf-$\theta 0/\Pi\alpha t^{1/2}$ so that is how the dq/dtx=0 is going to be defined.

So having said that now the total quantity of heat flow across the mold phase up to a certain time t_0 is qt_0 the total amount of heat is $Q_{t_0} = \int_0^{t_0} \acute{Q} dt$ okay so if I really want to put this integral here to solve the Q_0 had earlier the Q dot and come out to be $\acute{Q} = \frac{kA(\theta_f - \theta_0)}{\sqrt{\pi \alpha t}}$, so this value would then actually be equal to $Q_{t_0} = \int_0^{t_0} \acute{Q} dt = \frac{2kA(\theta_f - \theta_0)}{\sqrt{\pi \alpha}} \sqrt{t_0}$.

So again so $t^{-1/2}\int -1/t1$ that is how you calculate this integral okay so that is how your total heat from time 0 to t_0 across the mold phase would actually flow okay. next calculate the heat that the liquid metal would reject in order to solidify so if the liquid metal as latent heat l , a specific heat capacity to be C_m and density ρm then the heat rejected by the liquid metal converting to solid can be written down as ρm times of v obviously the mass times of the mass that has been converted into solid times of the latent heat l which goes on detected because there is a phase change times of C_m the specific heat capacity of the metal from the poring temperature to the freezing temperature. So you assume that the solid that has been formulated is at the θ freezing point or θ freezing temperature.

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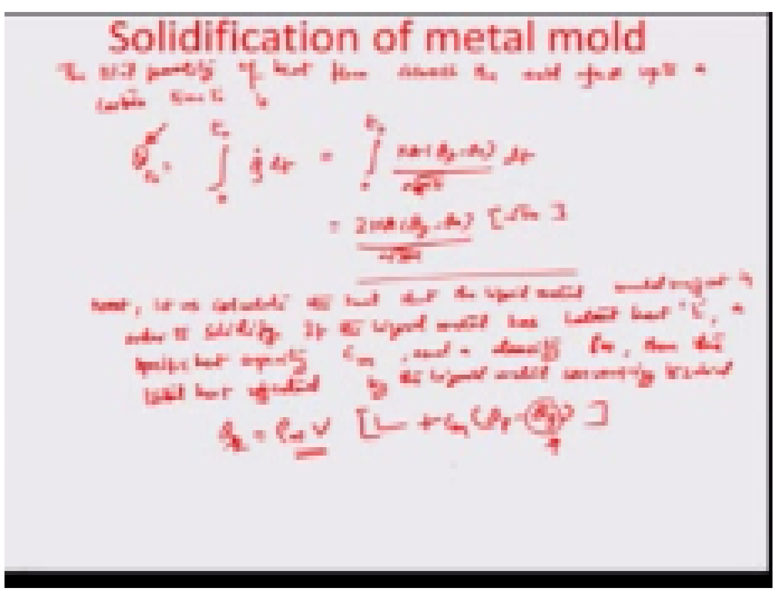

And the poring temperature is different than the freezing temperature meaning there by you are super heating the material in order to induce into the mold just that the time of solidification can be delayed a little bit and you don't have any problems in filling the mold with the reverse flow of the riser side, so that in that situation you having the total amount of heat reject Qr=ρ_mv[L+ Cm (θ_P-θ_f)].

$$Q_R = \rho_m V \left[L + C_m \left(\theta_p - \theta_f \right) \right]$$

So have said that now if I incorporate this into our expression earlier so the total amount of heat rejected should be the total heat that flows out so we have an equation KA/θf-θ0 times of √Πα times of √t0 that is the amount of time this heat would take for translating okay so this I actually call t0 at this point and I will let u know what exactly this value means should be equal to ρ_mv[L+ Cm (θ_P-θ_f)].

$$\frac{2kA\left(\theta_f - \theta_0\right)}{\sqrt{\pi\alpha}} \sqrt{t_s} = \rho_m V \left[L + C_m \left(\theta_p - \theta_f \right) \right]$$

So this time t_0 is nothing but the total amount of time that the liquid metal would need to solidify and so therefore we can record this as the solidification time okay so this is how you principally in principal calculate this solidification time of a certain mass of a liquid getting converted into solid because of the heat transfer across the mold wall assuming that there is a temperature distribution with the function of x starting at the mold wall onwards into the particular sand mold.

So having said that I think I have sufficient justification for you to be able to estimate now we took at practical problem so for that we need to probably break here and then go back into the next module later on thank you so much.

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