Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

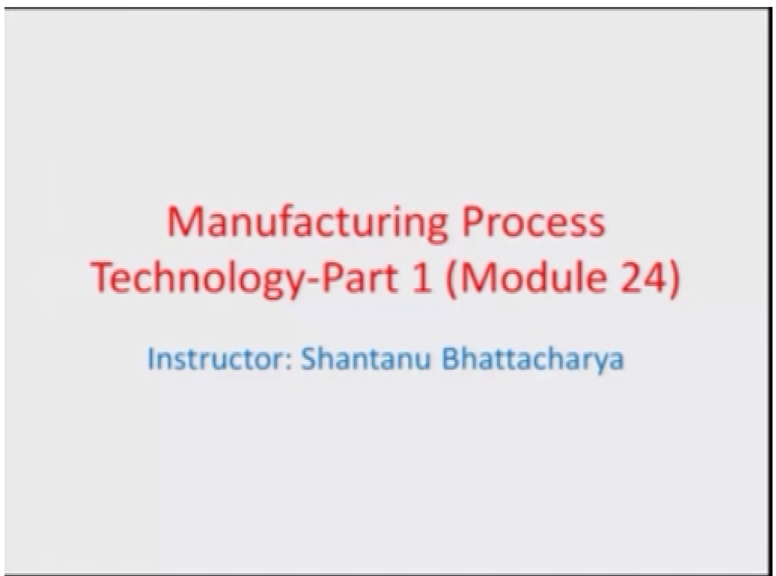
Course Title
Manufacturing Process Technology- Part- 1

Module-24

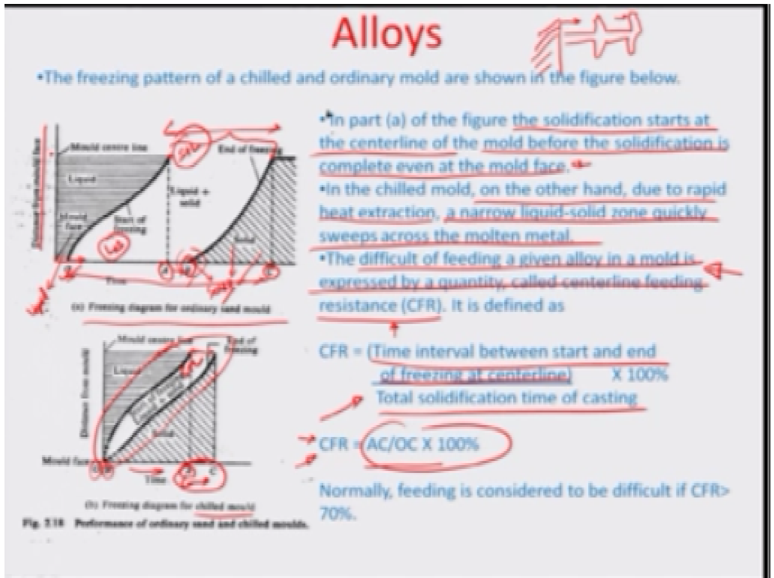
By
Prof. Shantanu Bhattacharya

Hello and welcome to this manufacturing process technology part 1 module 24.

(Refer Slide Time: 00:17)



Just a quick recall of what we did in the last lecture we were talking about the various issues related to solidification of molds particularly the fact that what difference will it make by the presence or absence of a chill and we also found out that there is a dendrite growth which happens because of a change in concentration across the solid-liquid boundary as in the solid phase the amount of liquid amount of solid that is really allowed may be lower than the liquid phase solubility of the solid phase therefore may be different in the solid as well as the liquid phase so having said that now we would like to just understand the consequences.
(Refer Slide Time: 01:02)



So the freezing pattern of a chilled an ordinary mold are shown right about here this is the freeze diagram as you can see here this is the distance from the mold phase therefore this line here right here really is the phase of the mold okay and you can very well see that the phase you know of the mold is completely frozen at the point B so there is a liquid line which is somewhere here this is the liquid part this part here is the liquid plus solid part.

So therefore there is a partial solidification partial liquefaction and then finally the whole mixture freezes here to the complete solid state as you can see here at the point B so between this O and B the available phase is a mixture of the liquid and the solid and beyond B there is a complete freezing so therefore this all portion right here are the solid portions of the more so in this figure the solidification starts at the center line of the mold before the solidification is complete.

Even at the mold phase obviously because I think we are illustrated earlier that the dendrite growth structure you know would take place from the mold face onwards and therefore you can say that the center line of the mold may start the solidification. so somewhere let us say you know if this is the mold centerline and you can see the star of the solidification point here where there is a solid plus liquid formed here.

It is much before the point B where complete freezing has happened actually so in the chilled mold on the other hand the due to rapid heat extraction this line this region of the solid plus liquid gets narrowed down because of which obviously the center line or the mold phase would have completely solidified before the center line would start the solidification look at the point for example O and B are more or less similar.

So it is an immediate sort of freezing of the mold phase and then the point where the solidification would start to happen on the center line is at a so a is far beyond in terms of time so it is at a much later point of time in comparison to O and B so a narrow liquid solid zone quickly sweeps across the molten metal and the difficulty of feeding a given alloy in a mold is expressed by the quantity or centerline feeding resistance.

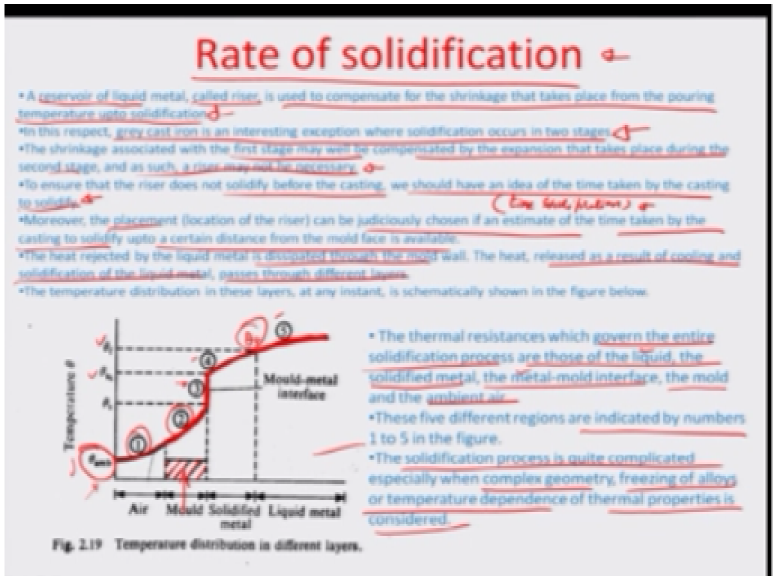
I think this we had done in the last lecture as well so obviously if there are solidification fronts which are formulated till and until the mold faces not even solidified it is very difficult for the liquid metal to kind of feed in because it has to pass through all these different solid boundaries okay so think of it as the liquid is coming through the gate region into a mold and it is facing so much of solidification solidified front as it moves from the mold face onto the inner centerline of the mold.

So there is something called center line feeding resistance which would develop and typically this is the time interval between the start and end of freezing at the center line as a positive of the total solidification time of the casting so if supposing there is a start and end of freezing here at Point A the centerline starts partially freezing and there is a solid plus liquid mixture developed here until it ends at the Point C.

So this particular interval of time whatever it takes as a function of the total solidification time as a percentage of the total solidification time which really is you know the point where the solidification has happened starting O all the way to the point see where the end of freezing cycle has happened so AC / OC, AC is this much time okay and OC is the total time of freezing so that the whole casting can be completely frozen.

So this expressed as a percentage is CFR so obviously if this time interval were narrow for example in a case of this chill time interval becomes lesser so the total CFR which would be typically obtained would be lesser because obviously this time interval AC is much lower in comparison to this where the chill was not used so this is a case where there is a chilled mold which is they are so chill has definitely lot of advantages because it prevents the dendrite growth as well as reduces the center line you know feeding resistance.

(Refer Slide Time: 05:49)



So having said that now if we look at what really is happening for you know in an event wise basis the rate of solidification can be really calculated as a function of what is the heat transfer across the mold phase and obviously in the design itself of the casting we had discussed that there is a reservoir of liquid metal called riser which is actually used to compensate for the shrinkage that takes place from pouring temperature up to the solidification temperature.

I think we had illustrate it very clearly that as the liquid metal is poured inside the mold there is going to be a change in the volume of the liquid metal per se here is going to be a change because the liquid metal solidifies so there is some change of state from liquid to solid and then obviously the solid contracts because the temperature comes from the freezing temperature all the way to the room temperature.

So in order to do this compensation for this shrinkage there is always a reverse flow which is generated because riser on one hand is an indicator whether the mold has been filled and on another hand is basically a reverse flow generator which would create more shrinkages or which would reduce the shrinkages by a backflow of the material from the riser into the mold so in this respect gray cast iron is an interesting exception where solidification occurs in two stages the shrinkage associated with the first stage may well be compensated by expansion that takes place during the second stage.

As such a riser may not be necessary and to ensure that the riser does not solidify before the casting we should have an idea of the time taken by the casting to solidify and therefore the time of solidification and how you estimate it is also an integral part of the designing of the casting as to where the riser exactly would be placed based on the time of solidification so you can place the riser in a judicious manner and if you are correct time estimate can be obtained of solidification up to a certain distance from the mold face the heat released as a result of cooling and solidification of the liquid metal passes through different layers.

Let us look at what happens here so you have one side the distribution of temperature across the mold phase so this right here is the mold so this right here is the mold and you can see that there is a temperature drop in the region one in the you know between let us say in the air there is one temperature drop region two in the mold itself so this is actually the outer surface temperature of the mold and this is the θ ambient which is actually the temperature of the environment in which this mold is present.

You have one temperature across the mold face so you have a higher temperature at the mold phase here there is obviously a solidified metal region so there is going to be a change of state over this region right here and there is going to be from the solid region almost a difference in temperature from the mold wall all the way to the freezing point of the liquid θ_F where the state conversion would happen between liquid and solid and beyond this it will be liquid metal obviously liquid metal also.

As it is more and more away from the mold would have different temperature so the temperature distribution here is really the resistance that the air the ambient environment in which the mold is present offers to take the temperature down you know there is a resistance offered by the mold itself so there is obviously from the face to the wall I mean from the from the mold wall all the way to the in the outside phase of the mold there is some kind of drop in the temperature because of the resistance offered there is of course the resistance offered by the solidified metal which is at the mold phase.

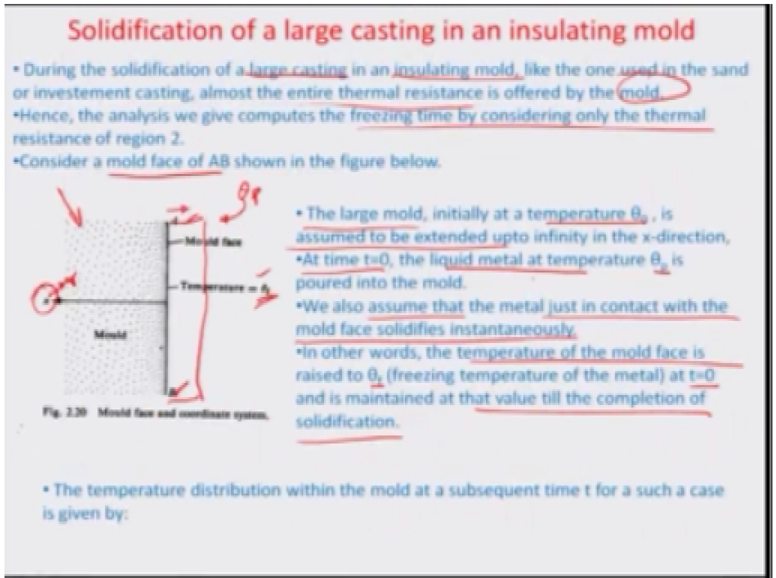
So there is going to be some change in the temperature some small minuscule change in the temperature from the at the mold phase itself there is also going to be a change in temperature beyond the solidified metal all the way to the liquid metal liquid solid interface and we assume that the liquid solid interface is at the θ freezing temperature beyond which again there would be a change in the temperature in the liquid metal itself.

So the resistance of the to the temperature here is offered by the metal here it is offered by the solid metal here again it is basically a function of the way that the solid is coupling with respect to the mold wall there may be a small film for example formulated and we will do all these cases independently when we do problem examples there may be another case here where the mold itself is sort of thermal inertia I provides thermal inertia thermal resistance I am sorry and then it basically again takes down the temperature.

And then obviously the air around the ambience around the mold wall which is also responsible for the same so the thermal resistances which governed the entire solidification process can be summarized to be those of the liquid the solidified metal the mild the metal mold interface the way that the solid film is coupled with respect to the mold wall in case the mold wall is rough there may be a near film in between in case supposing there is some kind of a you know proper like in case of in you know aluminum mold and pressure die casting there is a proper boundary between the metal to metal which is formulated.

They are also the estimate would be completely different then the mold itself that provides the resistance and the ambient here okay. so the five different regions are indicated by 1 to 5 in this curve here and obviously the solidification process is quite complex especially when complex geometry freezing of alloys or temperature dependence of thermal properties is considered so having said that let us now look at a simple example of how to calculate the temperature loss.

(Refer Slide Time: 12:04)



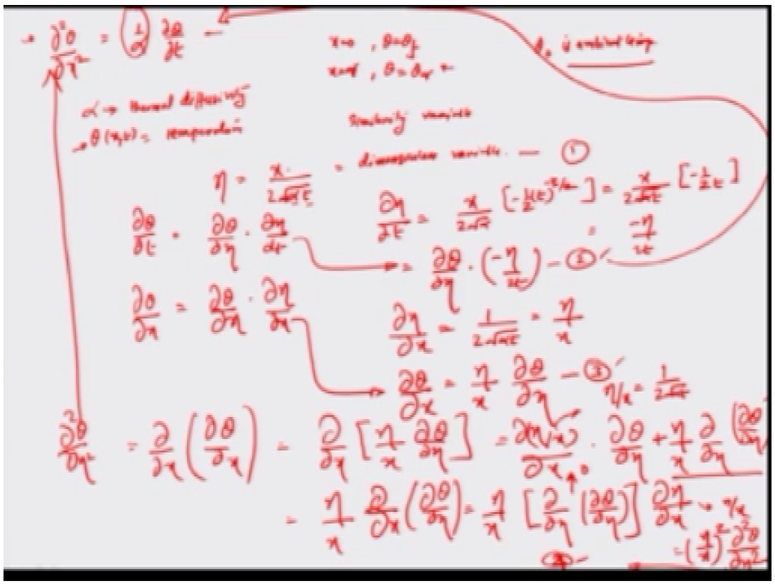
Let us say during solidification of a large casting in insulating more like the one in sand mold for example this is a sand mode or maybe in even investment casting almost the entire thermal resistance in this particular case is offered by the mold itself because the sand itself is a great heat sink okay and so therefore whatever resistance is maximum offered is offered by the mold so the analysis we give computes the freezing time by consider only in considering only the thermal resistance of the region two of the graph shown earlier.

So this particular region where it talks on one side the ambient air on another side the liquid metal and there is a mold resistance which is the principle thermal resistance being considered . so let us assume some boundaries here we consider that the mold face of AB is as shown here is held at a temperature θ freezing where the liquid is getting converted into solid so that is called the freezing temperature the large mold initially at a temperature θ_0 is assumed to be extended all the way up to ∞ in the X direction.

So you can assume this mold to be very large in comparison to dimensionally in comparison to the casting the casting may only be a very small slab in comparison to the width of this sand mold so X ▭ ∞ and at time T =0 we assume the liquid metal to be a temperature θ_P and it has been poured into the mold the moment it meets the face it comes to the temperature θ_F which ensures that it starts to freeze and then solidification front moves ahead you know with as a function of time as a function of distance from the mold as well.

So we assume that the metal just in contact with the mold phase solidifies instantaneously in other words the temperature of the mold phase is raised to a freezing temperature of the solid liquid boundary time T=0 and is maintained at that value till the completion of the solidification process so let us look at what is the temperature distribution in such a case which would happen okay.

(Refer Slide Time: 14:20)



$$\frac{\partial^2\theta}{\partial x^2}=\frac{1}{\alpha}\frac{\partial\theta}{\partial t}$$

$$\frac{\partial^2\theta}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial x}\right)=\frac{\partial}{\partial x}\left[\frac{\eta}{x}\frac{\partial\theta}{\partial\eta}\right]=\frac{\eta}{x}\frac{\partial^2\theta}{\partial\eta^2}$$

So the way that we do it is to try to solve you know the heat transfer one dimensional heat transfer equation $\frac{\partial^2\theta}{\partial x^2}=\frac{1}{\alpha}\frac{\partial\theta}{\partial t}$ where α seems to be the thermal diffusivity so θ is basically a function of X and time T and this is temperature and we want to first try to solve what is the variation of what is the function or variation of θ with respect to X and with respect to t by solution to this equation we already know that the boundary conditions allow this heat transfer at X=0 the θ becomes θ freezing.

And you can say that at x =∞ that means the mold is infinitely large the θ may become equal to the θ ambient or θ at ∞ you can say okay so that is how you do this calculation so that is how you represent this and then we assume that θ_0 is the ambient temperature in which this whole heat transfer is taking place so having said that now let us actually use a similarity variable method for solving this equation.

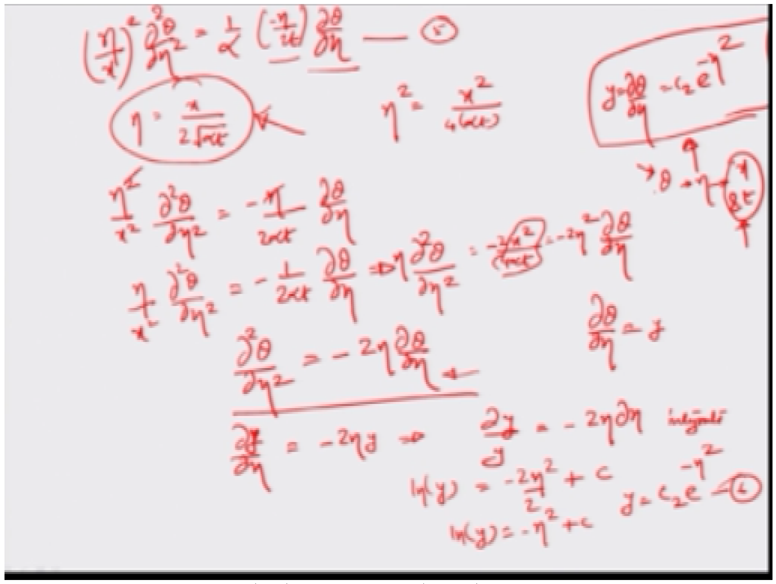
So we will create a variable here called η where η is basically X /2 √α T and η is a dimensionless variable this is a dimension less variable we can compare the distance units with respect to the α and time T so we first try to find out the ∂θ /∂T in terms of ∂θ/∂ η times of ∂ η /∂T okay and then on the other side we derive ∂θ / ∂x with respect to ∂θ / ∂η times of ∂η by ∂X so from this equation right here equation-1 ∂ η / ∂t = X /2√α-1/2*$T^{-3/2}$ and which again means this is equal to X /2αT hold on to the √x -1/2-1/2T.

So I can say that ∂/∂ t is nothing but – η/ 2t so in other words this expression right here changes to ∂θ /∂ η x – η/2T similarly we would like to find out the expression for θ or ∂η by ∂X so the ∂ η by ∂ X is 1 by 2 √ α T and this can be again represented as η/X and so I can say that similarly in this case ∂θ /∂X becomes equal to η/x * ∂θ / ∂η okay so this can be equation-2 this can be equation-3 and we need to now find out the second derivative of this so we just do ∂ / ∂ X of ∂θ /∂X which make sure we have ∂ / ∂X of η / x times of ∂θ / ∂η.

So again we apply the chain rule here so we have ∂(η / x)/ ∂x* ∂θ / ∂η + η / x * ∂/ ∂x of ∂θ / ∂η and we can now change and differentiate this guy right here because obviously this η / x is only a function of time as you could see here and so this would go to 0 and we are only left with η /x times of ∂ / ∂ x times of ∂ θ / ∂ η so let us just again apply the chain rule here we have ∂ / ∂η (∂θ / ∂ η)* η / ∂ X and ∂ η / ∂ X again comes out to be equal to n / x so we are left with the term n / x^2 ∂^2 θ / ∂x^2

So having said that now let this be regarded as equation-4 so from Equation-2 3 and and 4 we can actually put the value of $\partial^2\theta$ / ∂ x^2 into the heat transfer equation right here and same you can put the value of ∂ θ / ∂ η from to back into the heat transfer equation here.

(Refer Slide Time: 21:44)



$$\left(\frac{\eta}{x}\right)^2 \frac{\partial^2\theta}{\partial\eta^2}=\frac{1}{\alpha}\left(\frac{-\eta}{2t}\right)\frac{\partial\theta}{\partial\eta}$$

$$\eta^2=\frac{x^2}{4\alpha t}$$

$$\frac{\partial^2\theta}{\partial\eta^2}=-2\eta\frac{\partial\theta}{\partial\eta}=-2\eta y$$

$$\ln(y)=\frac{-2\eta^2}{2}+C$$

$$\ln(y)=-\eta^2+C$$

$$y=C_2 e^{-\eta^2}$$

So we are left with η / X^2 x ∂ 2 θ / ∂ η 2 equals1 / α * ∂ θ / ∂T which is actually equal to ∂ θ / ∂ η times of - η / 2t so we have putting this value here 1 / α x of - η / 2 θ x ∂ θ / ∂ η. So having said that it is you know little simpler to solve this equation and what we can do then is then we can actually try to see what it have also η was estimated from the term X / 2 √ of α T and if I just wanted to solve this equation that I formulated.

Let us call it equation 5 here which is square of η X^2 (X$\partial^2\theta/\partial\eta^2$ becomes equal to -η /2 αT x ∂θ/∂η okay further this η goes away okay. And we are left with η / X on one side either / X square once I times of ∂2θ /$\partial\eta^2$ and this becomes equal to - 1/2αT ∂θ / ∂η so obviously square of η can be represented as $X^2/4$ * αT so if I were to just substitute here or convert this equation here we get ∂ θ / ∂ η2 times of η equals - of X^2 .
Let us put it $2X^2/4\alpha T$ and this term right here is η^2so it is - $2\eta^2$ okay and this can be estimated as ∂θ /∂ η on the side so we are now finally left with a term or an equation of the form completely converted into the similarity variable η here okay so everything in terms of η and I call it

$\partial 2\theta/\partial\eta 2$ equals - of 2η $\partial\theta/\partial\eta$ so with this is the ordinary differential equation with one variable and we would like to just solve it by integrating again.

So let us say if we assume $\partial\theta/\partial\eta$ to be variable or value equals to let us say why here so we have $\partial y/\partial\eta$ equals $-2\eta * y$ okay so we can easily solve this by taking why on the other side and making it equal to2 η ∂ Nita we integrate on both sides so after integrating this becomes in y when this becomes - $2\eta^2/2$ plus the constant of integration C. so in other words this can be written down as in y equals square of η - plus C or Y becomes equal to some constant c2 times of e to the power of - $\eta^{2.}$

So let this be equation 6 so as our Y is actually equal to $\partial \theta / \partial \eta$ which can be written down as C_2 e to the power of - η^2 so we can find out a solution for now θ as a function of η and η basically involves the variables x and t and I am going to do that probably going to close this module now in the interest of time but in the next class I am going to actually try to find out what is this θ you know considering all the temperature boundary conditions etcetera so with this I would like to end this session thank you so much you.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram

Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production