Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Manufacturing Process Technology –Part-1

Module 22

By
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Hello and welcome to this manufacturing process technology part 1 module 22.

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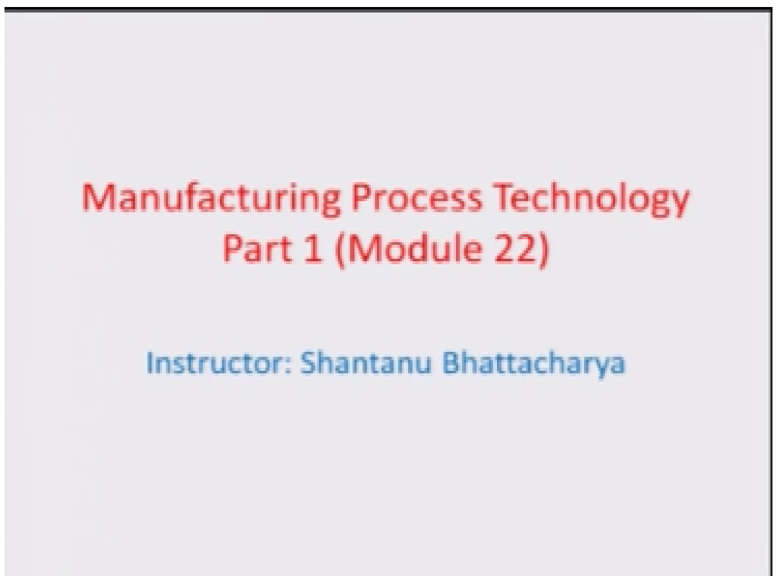

We were talking about how to estimate the time of filling in case of a complex problem as shown particularly here where talk about.

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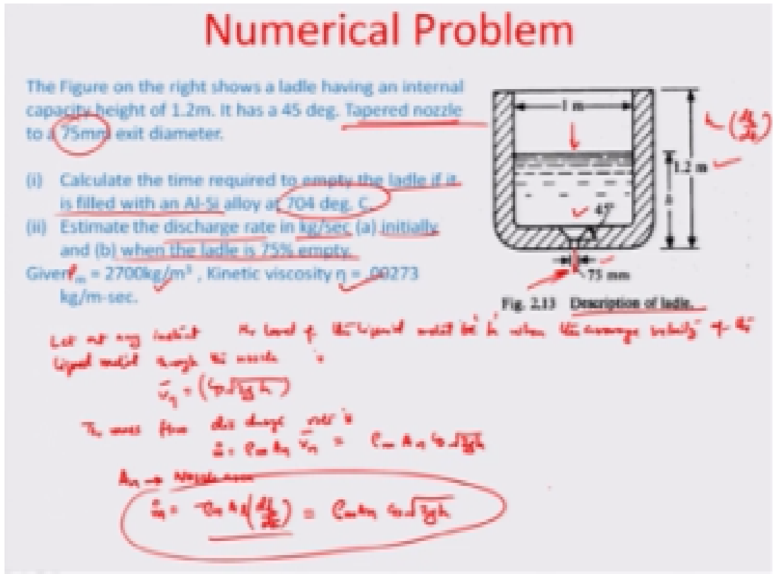

A ladle which is typically used to pour metal into the mold so the figure on the right shows this ladle and it has an internal capacity height of 1.2meters has 45 degrees tapered nozzle so that is like a fitting and the exit diameter of this nozzle is about 75 mm as illustrated here we will have to calculate the time required to empty the ladle if It is filled with an aluminum silicon alloy at 704 degree Celsius.

Which is right about the melting point of the alloy and we will also have to estimate the discharge rate in kg per second initially when the ladle is full and then case when the ladle is about 75% empty .so we will have different parameters to estimate the density rho m of the aluminum silicon Alloy a law in the liquid phase has been given to be 2700 kg per meter cube the kinematic viscosity 0.00273 kg per meter second so on so forth. so let us solve this problem so let at any instant the level of the liquid metal be *h* when the average velocity of the liquid metal through the nozzle is V and equals C_D *root of 2gh.* The mass flow discharge rate is m dot = rho$_m$ times of A_m times of V_m dash. which is rho$_m$ times of A_m C_D *root of 2gh* . so A_n is the nozzle area and m dot here equals the total density of the metal times of let us say the area of the ladle times of the rate at which the nozzle is emptying.

$$\acute{v}_n = C_D \sqrt{2gh}$$
$$\acute{m} = \rho_m A_n \acute{v}_n = \rho_m A_n C_D \sqrt{2gh}$$

So if this height be considered to be h and the rate at which emptying is considered to be dh by d t, then A_l times of dh by dt would really be the amount of flow out of the liquid metal which would happen as a function of time at the instants T and minus rho m times of Al times of dh by

dt is the mass flow out of the nozzle here as shown so this really is equivalent to the area of the nozzle times of the discharge velocity which is C_D *root of 2gh* okay.
So that is how you balance obviously you need to assume continuity here as whatever changes have taken place within the ladle as whatever flow out is happening of the nozzle also there is nothing no mass that is being created or destroyed you know within the system so that is how we formulate the characteristic equation to solve this problem.

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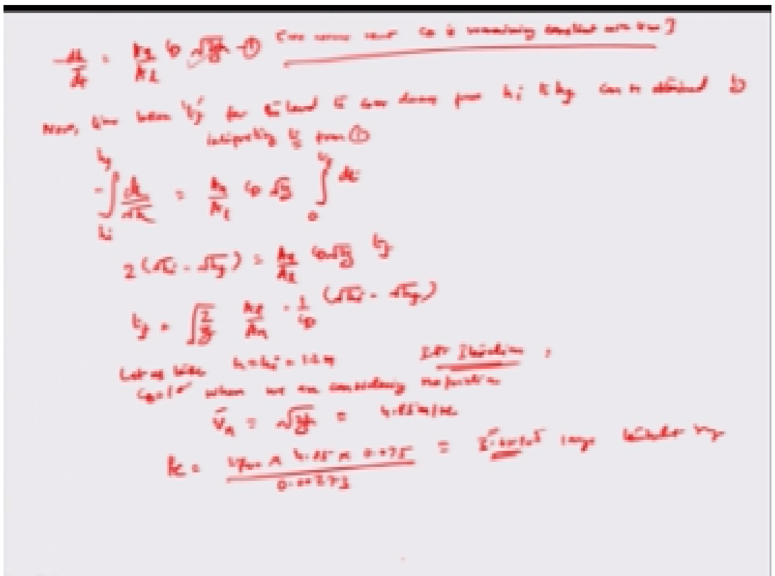

And we say that minus dh by dt = A_n by A_l* C_D times of *root of 2gh* . so here we assume that C_D is remaining constant with time which is really not the case because as the you know as the height of the liquid in the nozzle reaches to almost this bottom portion where there is going to be very less flow coming out the flow actually changes regimes and goes into laminar from the turbulent regime.

$$\frac{-dh}{dt} = \frac{A_n}{A_l} C_D \sqrt{2gh}$$

I will just show you this in by virtue of calculation that corresponding to a very small height of the liquid within the ladle the flow would change from turbulent to laminar okay and there the C_D definitely would be changing with respect to time appreciably so we are not assuming any such changes at this time and we are assuming that the ladle is still almost completely filled and we are only concerned with the time of estimating at when it is completely filled and time of filling time of out flowing when it incompletely filled versus when it is 75% empty okay.

So but we will do investigate the case where the C_D becomes very different by virtue of the flow regime changing between the turbulent to the laminar flows so now the time taken t_f for the level to come down from h_i to h_f can be obtained by interpreting t from this equation here equation one from one so let us look at it we have integral integrating on both sides integral and dh as the height varies or divided by root of h as borrowed from equation 1.

With the height varies from an initial height to a final height hi is the initial height and hf is the final height becomes equal to A_n by A_l time of C_D times of root 2g time of 0 to filling t_f dt . so this becomes equal to twice root of hi minus root of hf becomes equal to A_n by A_l times of C_D times of root 2g * t_f and thus t_f becomes equal to twice by g whole under the root A_l by A_n times of 1 by C_D times of root of hi minus root of hf .

$$-\int_{h_i}^{h_f} \frac{dh}{\sqrt{h}} = \frac{A_n}{A_l} C_D \sqrt{2g} \int_0^{t_f} dt$$

$$t_f = \sqrt{\frac{2}{g}} \frac{A_l}{A_n} \frac{1}{C_D} \left(\sqrt{h_i} - \sqrt{h_f} \right)$$

So let us take h equals hi equals 1.2 meters so C_D equals 1 when we are considering no friction. I think I had Illustrated it sufficiently earlier and Vn for the first iteration so this is the first iteration can be estimated as root of 2gh as 4.85 meters per second just as in the previous example let us calculate the Reynolds number here its 2700 times of 4.85 velocity times of 0.075 divided by 0.00273 which becomes 3.6* 10^5.

$$\acute{v}_n = C_D \sqrt{2gh} = 4.85\, m/sec$$

$$\Re = \frac{2700 * 4.85 * 0.075}{0.00273} = 3.6 * 10^5$$

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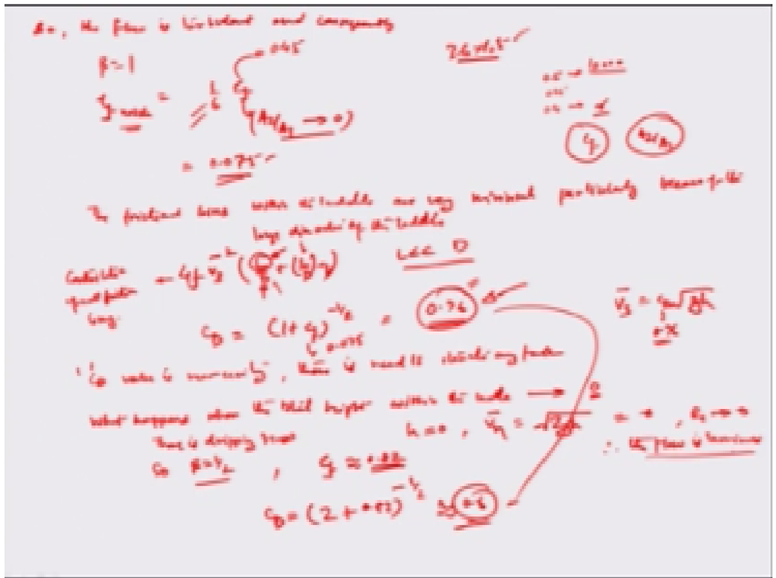

So very large and the flow is in the turbulent range again. so by virtue of the flow being in the turbulent range again the Reynolds number is amazingly large actually in this case so the flow is turbulent and consequently the β obviously would be taken as one. e_f in case of this nozzle which we are talking about with the 45 degree bend is actually 1by 6^{th} of the ef value at that area ratio A2 by A1 more or less tending to 0.

$$e_f = \frac{1}{6} e_{fs} = 0.075$$

I think I had Illustrated this earlier and for this kind of large value of Reynolds number I am not going back into the figure to look into the e_f value is estimated again as almost 0.45 minutes is more or less close because you know as you see in that particular graph the value recorded as 0.5 is corresponding to 10,000 and the value recorded as 0.4 is corresponding to infinite.

So you can say that it is more or less near to this in between so 0.45 was so that we are estimating . if you may remember the e_f versus A2 by A1 trend that we had discussed in the last example problem so here nozzle is sixth of that 0.45 value and that becomes equal to 0.075 these are very easily available.

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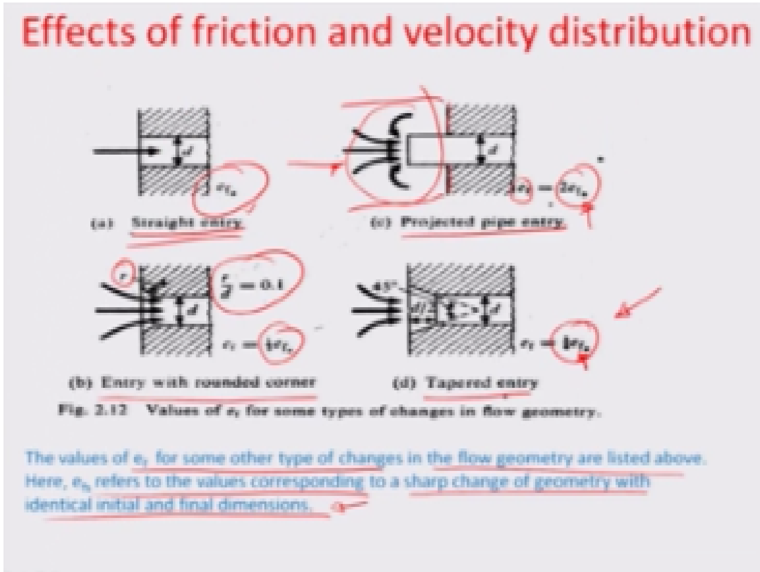

Let me just go back and recall the geometrical considerations that we had made just a few slides back in this particular case where I had Illustrated that for a 45 degree angle nozzle something like this which is actually the case in this case of the nozzle bottom ladle the actual e_f which would be the loss because of the particular fitting would come out to be a sixth of the loss that is read from the normal case okay.

e_{fs} is the loss from the normal case normal case meaning thereby just a contractor expansion so you see that for the straight entry case the value is e_{fs} and so it is a sixth of this loss that would be there if there is a nozzle so these are all empirically calculated and I do not want to delve into the details of the fluid mechanics where the empirical calculations are sort of Illustrated.

But they will be available as data tables or you know information given to you in the problem statement for making it easier for you to solve it .so here I would just go back to the slide that I was continuing we want to calculate corresponding to this 3.6* 10 to the power of 5 the actual e_{fs} value sixth that is one-sixth of e_f being 0.45 so this becomes equal to 0.075.

And so the frictional losses within the ladle the frictional losses within the ladle are very miniscule particularly because of the large diameter of the large diameter of the ladle so physically what happens is that as you know that this frictional factor would be given as 4*f* v_3 square times of L by D plus L by D equivalent right so in this particular case I guess the length of the smaller area which is the nozzle is too short you know.

So L is much smaller in comparison to the D and so all even if it is l by equivalent or L by D, I think the values are insignificant so the contribution off wall friction is really negligible so having said that here the C_D value can be just calculated as 1 plus e_f to the power of minus half so it becomes equal to 0.96 you know e_f being 0.075.

$$C_D = (1+0.075)^{-1/2} = 0.96$$

So C_D is near unity so C_D values near unity and there is no need to iterate any further so approximately you can say that from the frictionless case there is almost a four percent change in the C_D value in that frictionless case it was considered to be one. so the essence here is that you have been able to estimate what is the v3 here which is C_D times of root 2 g h and that way the C_D is 0.96 as can be seen or obtained in this particular case so having said that the other issue that we need to consider is what happen when the total height within the ladle approaches to almost zero meaning there by now let us look at the geometry the liquid is very close to the surface here there is hardly any liquid in the ladle and you can estimate this h to be equal to 0 . so in that case still there is some height which is leftover in the in the in the nozzle and because of this the flow would be very slow and this is also known as a dripping flow.

So let us estimate what happens to the C_D value in that particular case so what happens to the ok. so there is dripping flow and the regime would change here because obviously now the h is approximately 0 meaning there by the velocity of the nozzle which is root 2gh on the first iteration basis is also 0 and thus the Reynolds number Re is also 0.

So therefore the flow now is laminar so the β becomes equal to half because of the laminar of the flow and the e_f in this particular case would be taken as 0.82 which is a very high value of e_f ok so thus the C_D value completely gets changed as 2 plus 0.2 to the power of minus half which is actually equal to 0.6 so you can see that the C_D has reduced by almost 40% from the ideal case in that particular case.

$$C_D = (2+0.82)^{-1/2} = 0.6$$

But it is of not much consequence because obviously when the dripping flow of the ladle is there unless you are specifically asked to examine that condition in the practical case we consider that flow to be almost stopped for doing so what we would like to now do is to sort of estimate what is going to be the value of the filling time in this particular case. so let us say the height be equal to 5 centimeters.

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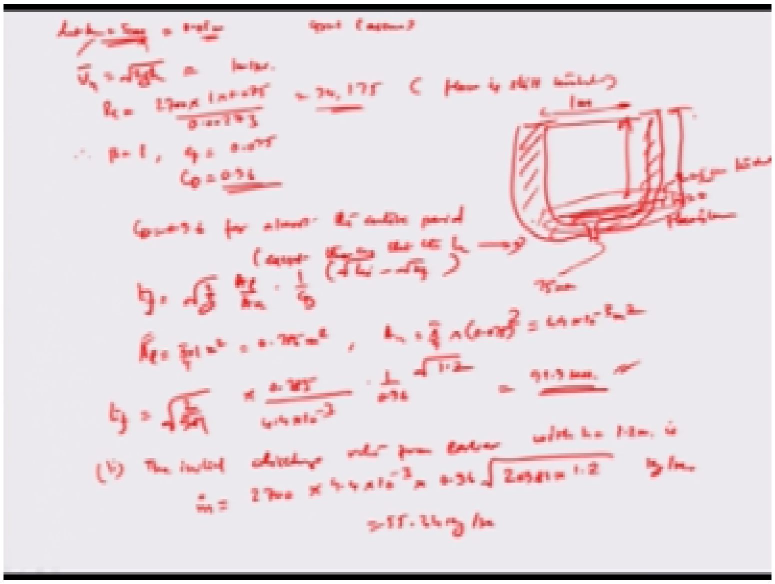

Which is still some height within the within the mold and 0.05 meters we assume the C_D to be equal to 1. and we estimate that corresponding to this value and the case that we showed earlier that is h is almost 0 is there going to be any change or is the flow still going to be turbulent or laminar so that now you have a scenario where the ladle has completely emptied in one case where the flow becomes laminar and there is some small height 5 centimeters which is kept there in the ladle and in that scenario.

What is going to be an you will see that for even a small height case the Reynolds number is going to be still in the turbulent range. so let us say V_n in this particular case is root 2 g h assuming no friction as 1 meter per second and with one meter per second if I estimate what is the Reynolds number value with such a high density of metal okay it comes out to be equal to 74175 so the flow is still turbulent.

$$\acute{v}_n = C_D\sqrt{2*9.81*0.5} = 1\, m/sec$$

$$\Re = \frac{2700*1*0.075}{0.00273} = 74175$$

So therefore in the example problem that we have where we are considering this ladle with a nozzle at the end the scenario tells us that corresponding to absolutely zero height equals zero the flow is laminar but even if you consider a very small height of metal inside which is height equal to 5 centimeters the flow is still turbulent . so the transition between turbulent to laminar is quite abrupt in this particular case.

So obviously in that case we would now assume you know with this kind of a flow condition what is going to be the C_D so the C_D here with β equal to 1 because it is turbulent and e_f equal to 0.075 still comes to be equal to 0.075 . so the C_D does not really change across the whole ladle till the value comes to very small and then we will neglect this in this particular question for further analysis.

$$e_f = \frac{1}{6} e_{fs} = \frac{1}{6} * 0.45 = 0.075$$

So if the C_D remains 0.96 for almost the entire period except the case that the height h tends to 0 and t_f had been earlier obtained as twice by g whole under the root A_l by A_n times of 1 by C_D root of h_i minus h_f and A_l being equal to pi by 4 times of the diameter square , diameter being so this is the area of the ladle so diameter here being one meter so one meter square this becomes equal 0.785 meter square that of the nozzle is pi by 4 times of this75 millimeters.

$$A_l = \frac{3.14}{4} * 1\, m^2 = 0.785\, m^2$$

$$A_n = \frac{3.14}{4} * 0.075 * 0.075\, m^2 = 4.4 * 10^{-3}\, m^2$$

So this becomes equal to almost 0.075 square that is 4.4* 10 to the power of minus 3 meter square and so the tf value comes equal to twice by 9.81 whole under the root times of of 0.785 divided by 4.4* 10 to the power of minus 3 times of 1 by 0.96 root of 1.2 and this typically is equal to 91.9 seconds. so that is how the initial discharge would be when the height is completely at the top-level here and then let us say the initial discharge rate from earlier with h equal to 1.2 meters is given as m dot equals the density times of the area of the nozzle.

$$t_f = \frac{\frac{\sqrt{\frac{2}{9.81} * 0.785}}{4.4 * 10^{-3}} * 1}{0.96} \sqrt{1.2} = 91.9\, sec$$

Which has just been calculated times of the velocity which is C_D times of twice 9.81 times the 1.2 kg per second so this becomes55.34 kg per second so the discharge rate when the ladle is 75% empty as has been asked that is the height within the nozzle is about 25% of 1.2 millimeter meters is about 0.3 meters.

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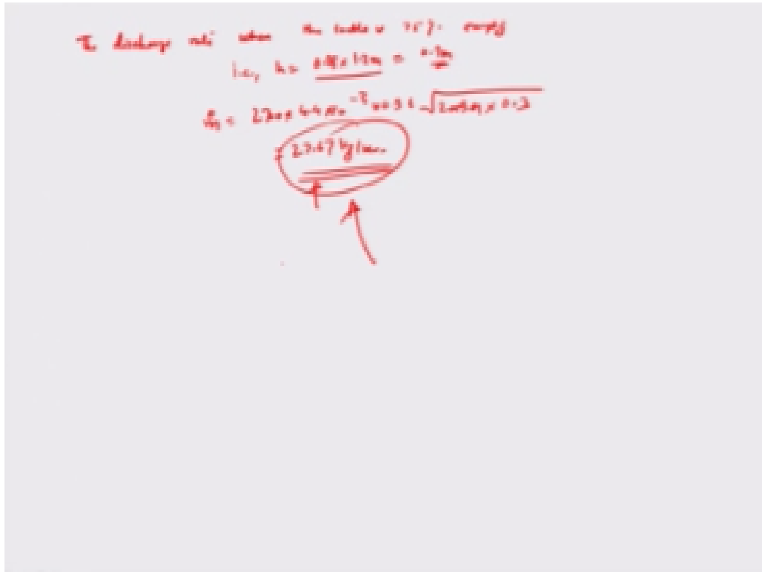

So in that event the m dot would again be 2700 times of the area of the nozzle times of the C_D value root 2g, g in this case is 9.81 meters per second square. so two times of 9.81 meter per second square times of 0.3 meters . so that becomes 27.67 kg per second so that is how the mass discharge rate would vary so earlier at the beginning of the flow and the ladle is completely full charge rate is 55.3 kg per second and it actually comes out to 27 kg per second.

$$\dot{m}=2700*4.4*10^{-3}*0.96*\sqrt{2*9.81*1.2}=55.34\,kg/sec$$

So it is about a half almost a half or fifty percent of the flow as the height varies between full height and I mean twenty-five percent of the height. so obviously the flow rate is going to be different instances and that is why h is a function of time that has been assumed at the beginning of this problem. so I think I have more or less completed how you calculate iteratively such examples of flow discharge and is very important to estimate the flow of time because you know the total flow the time of flow.

And time of filling because the casting problem is really about a timely solidification so that a desired grain structure can be obtained so this time of flow has to be balanced with the transfer and the grain growth time which am going to now from next module onwards to a lot of calculations and illustrations.

So the solidification time should be such that all the flow timing should be typically lesser by orders of magnitude than the solidification time our number one number two is that you should ensure that there is to be a flow delivery before the metal gets solidified into the mold . so with this I would like to end this module and in the next module we are going to learn some things about what would be the heat transfer at the mold metal interface and how that varies as a function of different boundary conditions of the of the mold so thank you so much.

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