

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title

Manufacturing Process Technology- Part- 1

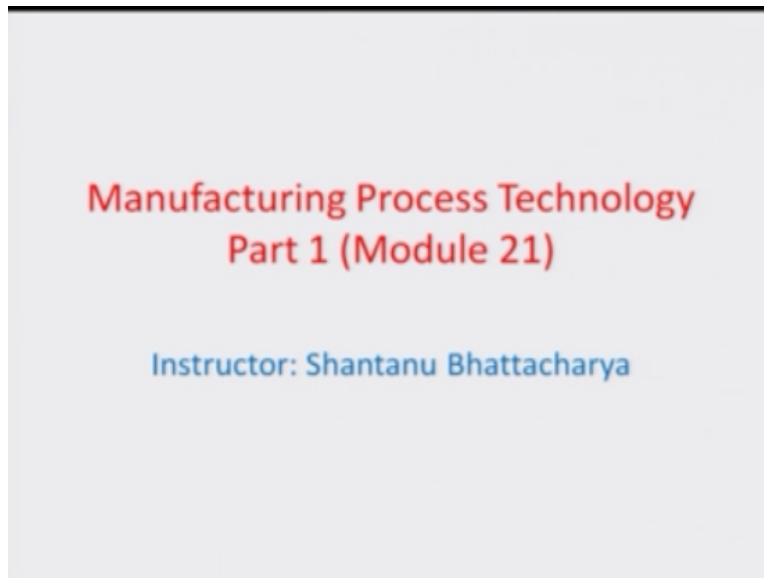
Module-21

By

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Hello and welcome to this manufacturing process technology part 1 module 21.

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We were talking about frictional losses in realistic situations where wall friction would play substantially a big role in determining the energy balance particularly in case of sprue and runners and that with bends and fittings so we are going to look at a problem example today where we would like to do this calculation.

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Numerical problem

Numerical Problem: A gating design for a mold of 50cm X 25 cm X 15 cm is shown in Fig. below. The cross-sectional area of the gate is 5 cm². Determine the filling time for this design by including the friction

and velocity distribution effects. The liquid being poured is molten Fe with the properties $\rho = 7800 \text{ kg/m}^3$, Kinetic viscosity $\eta = .00496 \text{ kg/m-sec}$. For the 90 deg. Turn at the sprue end.

$(L/D)_{eq} = 25$

The diagram shows a mold with a sprue. The mold dimensions are 50 cm (length), 25 cm (width), and 15 cm (height). The sprue has a diameter of 12 cm and a height of 12 cm. The cross-sectional area of the gate is 5 cm². Handwritten notes include: "It should be noted that β , f and e_f are should determine the flow as laminar or turbulent. The case for a kinetic process. As a first approximation the average velocity is computed with an assumption of $C_d = 1$. $V_s = \sqrt{\frac{2gh}{C_d}} = 1.716 \text{ m/sec}$. (as calculated before) Now get velocity loss $v = C_d V_s = \frac{1}{2} V_s$ $d = 0.012 \text{ m}$. $R_e = \frac{7800 \times 1.716 \times 0.012}{0.00496} = 38,000$ determining the flow as turbulent."

So there is a gating design for a mold which is 50cm x 25 cm x 15 as shown in this figure right here and the cross-sectional area of the gate is about 5 cm² which has been given here and you have to determine what is the filling time for this design by including now the frictional effects and the velocity distribution effects particularly so the liquid being poured is molten iron with the properties of liquid density or liquid metal density about 7800 kg/m³ kinematic viscosity η about 0.00496 kg / m sec.

And for a 90 degree bend as you can see right about here in this particular bounce proved the L x D equivalent which has been obtained from standard data is 25 you know so that is the contribution to the wall friction effect because of the bend obviously there will be a bending effect because of which there would be an energy loss so these all terms have to be taken together to compute the Cd.

So it should be noted that to obtain the β value which is actually the distribution function the kinetic and the velocity distribution the energy loss because of the velocity distribution function and also the friction coefficient F and the e_f which three of them are very important for determination of the Cd so we should determine the flow as either laminar or turbulent so this calls for an iterative process.

So iterative because at the outside you have to decide sort of find out that without including all these frictional effects what is going to be the range of the velocity and then from there you look at whether the velocity range corresponds to a laminar range or a turbulent range and then try to

estimate the different factors β , f and e_f using that strategy and so as a first approximation the average velocity is computed with an assumption of the C_d value being equal to 1.

So obviously the v_3 average velocity would be $\sqrt{2gh}$ and in this case as we had calculated before the H being equal to this 12 centimeters we have a v_3 value which is approximately 1.716 m/s and this I think we had calculated in one of the earlier modules where we talked about no wall friction and only an idealized condition so now the gate region has a total area of 5 cm² and if we assume the gate diameter to be D we can say $\pi/4d^2$ is actually equal to 5 cm² or in other words in the gate region the diameter would be 0.0252 m.

So let us now calculate what is going to be the Reynolds number as you know Reynolds number is $\rho v d$ by μ where ρ is the density of the metal in this case fluid metal $\rho_m v$ is the velocity of the metal d is the diameter of the cross section which is actually the gate dia in this particular case and μ is the kinematic viscosity of the material. Let me just write it a little more properly $\rho_m \times v \times d / \mu$ okay.

So Re becomes equal to in this case 7800 kg /m³ which is actually given to be the density of the liquid metal times of the velocity which is 1.716 m/sec times of 0.0252m diameter divided by the viscosity which is 0.00496 kg /m sec and so that becomes equal to a Reynolds number of about 68,000 which is actually determining the flow as turbulent even though there is no wall friction flow as turbulent so this is very interesting that the flow is seen to be turbulent by the first estimation without the use of any friction now let us start the analysis.

$$Re = \frac{\rho_m v_3 d}{\eta} = \frac{7800 * 1.716 * 0.0252}{0.00496} = 68000$$

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Numerical Problem

If the flow is turbulent. Thus, $\beta = 1$.
 → $C_d = 0.45$. [Check on area ratio A_2/A_1 and the velocity in the laminar region and out of the gate from the graph showing condition conflict]

→ $f = \frac{0.0791}{(Re)^{0.25}} = 0.0048$ (Use $C_d = 0.45$)

$Re = \frac{\rho V D}{\mu}$ where $\mu = 0.0122$ kg/m·s
 $C_D = [1 + 0.45 + 4 \times 0.0048 \left(\frac{0.1}{0.0122} + 25 \right)]^{-1/2} = 0.7$

$\bar{V}_2 = \sqrt{\frac{2gH}{C_D}} = 0.7 \times 1.716 \text{ m/s} = 1.202 \text{ m/s}$

Recheck \bar{V}_2 again using the modified velocity = 1.20 m/s.
 For second approximation: $Re = \frac{\rho V D}{\mu} = 57,600$

$f = 0.0791$, $C_d = 0.46$, $f = \frac{0.0791}{(57,600)^{0.25}} = 0.0049$

$C_D = [1 + 0.46 + 4 \times 0.0049 \left(\frac{0.1}{0.0122} + 25 \right)]^{-1/2} = 0.69$

$\bar{V}_2 = 0.69 \times 1.716 \text{ m/s} = 1.184 \text{ m/s}$

Conversion of $\frac{1}{2} \rho V^2$ to $\frac{1}{2} \rho V^2$

So the flow is turbulent as the at the first iteration and thus obviously the first value of β that you would take is one because as I told you $\beta = 0.5$ if the velocity is in the laminar range and when it becomes turbulent the β becomes equal to 1 that is how the velocity losses or the kinetic energy losses because of the velocity distribution effect is understood from fluid mechanics theory also we need to find out what is going to be the ef value in this particular case with an area ratio A_2/A_1 almost tending to 0 as you can see here.

The area of the upper sprue right here is much bigger in comparison to the area 5cm^3 of the gate or the sprue which is being talked about so the A_2/A_1 where A_2 is the 0.2 and A_1 is the point you know area at the point 1, 1 and 2 have different consequences you may have just illustrated about this earlier as one here and two here right about this.

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Effects of friction and velocity distribution

Let us modify the Bernoulli equation
 Expansion between points 1 & 2 after
 Accounting for all the losses due to contraction of jet
 → due to velocity distribution
 → due to friction of water
 → due to bends or fittings

(a) Simple vertical gate (b) Bottom gate

Fig. 2.6 Types of gates.

$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \frac{p}{\rho g} + z + \frac{v^2}{2g} + h_{f1} + h_{f2} + h_{c1} + h_{c2}$

$\bar{v}_3 = \text{Average velocity in the sprue}$
 $\bar{v}_1 = \bar{v}_2$

$h_c = \frac{v^2}{2g} + 4f \left(\frac{l}{D}\right) \frac{v^2}{2g} + c_d \frac{v^2}{2g}$

$h_c = \frac{v^2}{2g} \left(1 + 4f \left(\frac{l}{D}\right) + c_d \right)$

$L = \text{height of the sprue}$
 $D = \text{Diameter of the sprue}$
 $f = \text{friction coefficient}$

$h_c = \frac{v^2}{2g} \left[1 + 4f \left(\frac{l}{D}\right) + c_d \right]$

If the sprue also had a bend or a fitting
 f should change to $\left(\frac{l}{D}\right) \frac{v^2}{2g}$ term

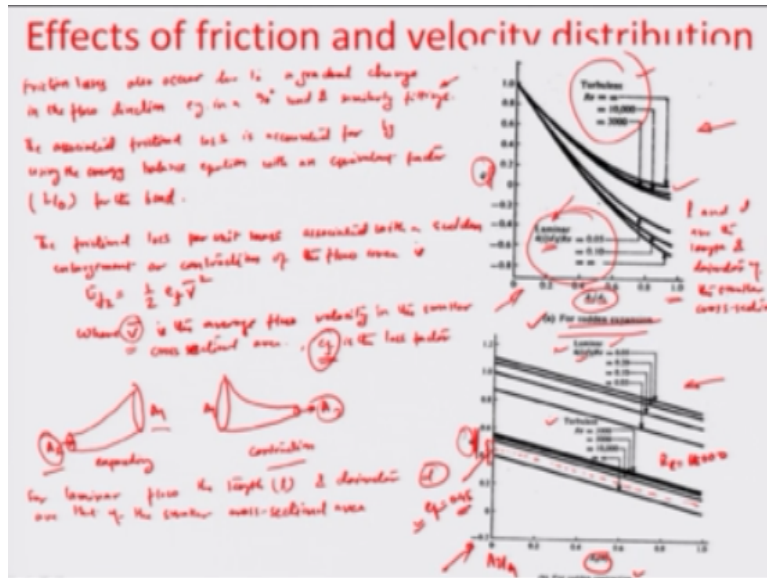
$h_c = 4f \frac{v^2}{2g} \left[\frac{l}{D} + \left(\frac{l}{D}\right) \frac{v^2}{2g} \right]$

$\frac{1}{2} \frac{v^2}{2g} = \frac{1}{2} \frac{v^2}{2g}$

$C_d = \text{Discharge coefficient}$
 $C_d = \left(\frac{1}{2} + c_d + 4f \frac{l}{D} \right)^{-1/2}$

So the two is always the lower cross sectional diameter so therefore the A_2/ A_1 corresponding to 0 and the flow to be in the turbulent region we have to read out read out the c_d value from the graph describing contraction conditions okay so A_2 is a cross-sectional area of the gate to be of gate and A_1 is the cross sectional area of the pouring basin. so let us look at that graph which we had borrowed from earlier.

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So let us go back to the particular two cases where for certain expansion and sudden contraction you have a listing of the values of the k_f of the friction loss and bending or fitting with respect to the Re 1 and also with respect to the area ratio A_2 / A_1 . so the A_2 / A_1 in our case is 0. So it is tending to 0 so we will have to read out really on this particular axis and we are also suggesting that the flow in our case is actually a turbulent flow and in the turbulent flow as we have shown here or illustrated here.

The Reynolds number that we have is close to about 68,000 which is actually somewhere in between these two values right so one value corresponding to the k_f at $A_2 / A_1 = 0$ happens to be close to about 0.5 and the other one is 0.4. So somewhere in between here by extrapolation of the k_f values you can have a $k_f = 0.45$ approximately so having said that now if I have this $k_f = 0.45$ let us go back put this back into the C_d equation and try to solve so f is taken as 0.45 from the graph okay.

$$f = \frac{0.0791}{68000^{0.25}} = 0.0049$$

And we also know that for a rep turbulent regime the friction factor of the wall friction basically is determined by the term $0.0791 / Re^{1/4}$ as I had illustrated earlier and this is actually plain and simple borrowed from fluid mechanics we are not going to the details of how you obtain these but we are wanting to see the application side of how you can design the casting once this is known.

So this happens to be equal to about 0.0049 so that is how the wall friction effect would come length obviously is the length of the sprue side because that is the lower a diameter side and in this case the total height that is there is about 0.12m about 12cm as has been illustrated here this total height from the pouring base into the end of the gate region is about 12 cm. So you are left with a value of Cd.

So we have a Cd value which is equal to 1 that is $1 / \beta + e_f = 0.45$ plus now you have four times of f which is in this case 0.0049 times of L x D which is the wall friction part of the problem. So I can write this down as the total length of the smaller side that is the sprue divided by length of the diameter of the smaller side that is diameter of the sprue which is 0.0252m as obtained earlier and there is a friction loss because of bending which has been given by this L/ D equivalent of 25 as earlier illustrated in the question.

$$C_D = \left[\frac{1}{\beta} + e_f + 4f \left\{ \frac{L}{d} + \left(\frac{L}{d} \right)_{eqv} \right\} \right]^{-0.5}$$

$$C_D = \left[\frac{1}{1} + 0.45 + 4 * 0.0049 \left\{ \frac{0.12}{0.0252} + 25 \right\} \right]^{-0.5} = 0.7$$

So you can see the cell by equivalent to be equal to 25 so that we add to this and compute what is going to be the value of Cd and this value comes out to be 0.7 so therefore the v3' or the velocity is going to be the Cd times of $\sqrt{2gh}$ as I had earlier illustrated formula with friction of the energy balance so this is 0.7 so it is a 70% of the ideal case which was 1.716 m /sec this comes out to be about 1.20 m/ sec .this is not the end of the story because we will have to illustrate to recalculate.

$$\dot{v}_3 = C_D \sqrt{2 g h_t} = 0.7 * 1.716 = 1.2 \text{ m/sec}$$

So actually the velocity is not 1.7 16 but 1.2 m/ sec let us now recalculate with this velocity to just iteratively see what is going to be the change because of this change in the velocity because of frictional effects so recalculate the V3 again V3' again assuming the modified velocity 1.20 m/sec okay so let us for second approximation calculate the modified Reynolds number.

So this becomes equal to obviously we can say it is just a velocity ratio 1.2 x 1.716 times of the earlier obtained Re 6800 and which is about 47500. so obviously you know that re equals ρmvd by μ so this V is changing from 1.7 16 to 1.2 so this is the new Reynolds number value and for this new Reynolds number value if we look at again the regime is turbulent so the β happens to be 1 the ef value happens to be about in this particular case slight improvements of slight change.

$$\Re = \frac{1.2}{1.716} * 68000 = 47,500$$

So we let us go back to that graph again and try to understand what is the year value so you have you had a value corresponding to 68,000 now you have a value corresponding to about 47,500 so you can iteratively manage this by calculating it and you know it will be slightly the higher side this is this was the Reynolds number that one which I am omitting is the Reynolds number 68000 case so this will be slightly closer to the 10,000 Reynolds number.

So let us say it is on a little higher side and you can extrapolate the value as that if in 68000 the value was 0.45 and in about 10,000 the value is about you know close to about 0.5 then around 47,500 the value should be about close 2.46 something to that light so just extrapolate between 10,000 and 68,000 and estimate what is the value of the Reynolds number at 40 a 47,500 so this comes out to be about point four six so 0.46 is the e_f the new e_f value we also have the wall friction f coming out at 0.0791 / the new Reynolds number $47,500^{1/4}$ from fluid mechanics theory so this is coming out to be 0.054 and if I compute what is the Cd coefficient here.

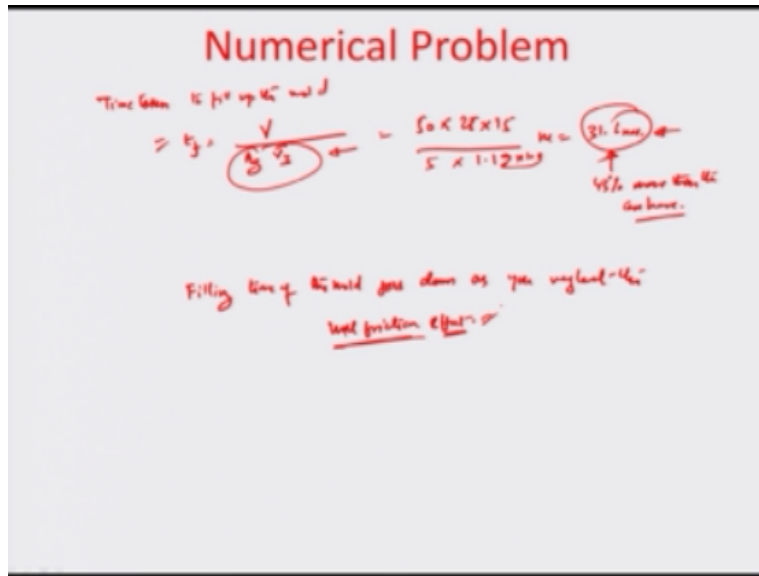
$$f = \frac{0.0791}{47500^{0.25}} = 0.0054$$

The Cd comes out to be $1 + e_f = 0.46 + 4$ times of the total friction factor wall friction factor 0.0054 times of the L by equivalent on the smaller diameter side which is 0.12 is the length and the diameter of the gate is 0.252 you know the bigger diameter is the pouring base inside as I had earlier illustrated plus the length equivalent which causes wall friction so that to the power of -1/2 and this comes out to be 0.69 so it is not really much different from 0.70 and you can say that the values have kind of converged.

$$C_D = \left[\frac{1}{\beta} + e_f + 4f \left\{ \frac{L}{d} + \left(\frac{L}{d} \right)_{eqv} \right\} \right]^{-0.5}$$

$$C_D = \left[1 + 0.46 + 4 * 0.0054 \left\{ \frac{0.12}{0.0252} + 25 \right\} \right]^{-0.5} = 0.69$$

So you can say that the velocity V_3 in this particular case is very close to as obtained earlier it is 0.69 times of the 1.716 m/sec so this is about 1.19 m/ sec which is quite close to 1.20 m/sec which was obtained earlier so this is the converging step of the iteration step of the iteration and I can say that iteratively I have arrived at a CD value close to about 0.69 which actually seems to be a reasonable estimation of the frictional factor in this particular case.
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Now let us estimate because of all these changes what is going to be the time of filling of the mold so let us say the time taken in this particular case to fill up the mold t_f comes to be equal to the volume of the mold / the gate area times of the velocity this is how the volume flow rate would be you know that there is a area of cross section A_g and through that there is a velocity emerging of the flow which is V_3 so A_g into v_3 is basically going to be the volume rate of flow.

$$\dot{v}_3 = C_D \sqrt{2gh_t} = 0.69 * 1.716 = 1.18 \text{ m/sec}$$

And so you having the overall volume of the mold divided by the volume rate of flow is basically the time of filling so this comes out to be equal to 50 cm times of 25 times of 15 cm³ / 5 x 1.19 or x 100 because we have to convert this into centimeters 119 cm and this is actually recorded as about 31.6 seconds so that is how the new filling time is and you can see that you know in the ideal case the filling time was about 43% more than the case here.

$$t_f = \frac{V}{A_g \dot{v}_3} = \frac{50 * 25 * 15}{5 * 1.18} = 31.7 \text{ sec}$$

So the filling time definitely has increased and that is because of the energy loss which happens because of the bending and the different fittings and the wall friction and other issues so definitely in this particular illustration it is shown that the filling time goes down okay so I can conclude here that the filling time of the mold goes down sorry goes down as you neglect the wall friction effect okay.

The filling time of the mold goes down as you neglect the wall friction effect which is actually a very important supposition here so in all future modules what we will do is will estimate the field

in time with reference to in iterative manner just as I have been done in this particular case with reference to wall friction and also friction because of bending losses or losses and fittings so with this I would like to end this module and in the next module will do another very complex problem where again all these effects are clubbed together to see what is the effect of the total time of discharge of a big pouring piece thank you so much you

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