

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**

**Manufacturing Process Technology – Part -1**

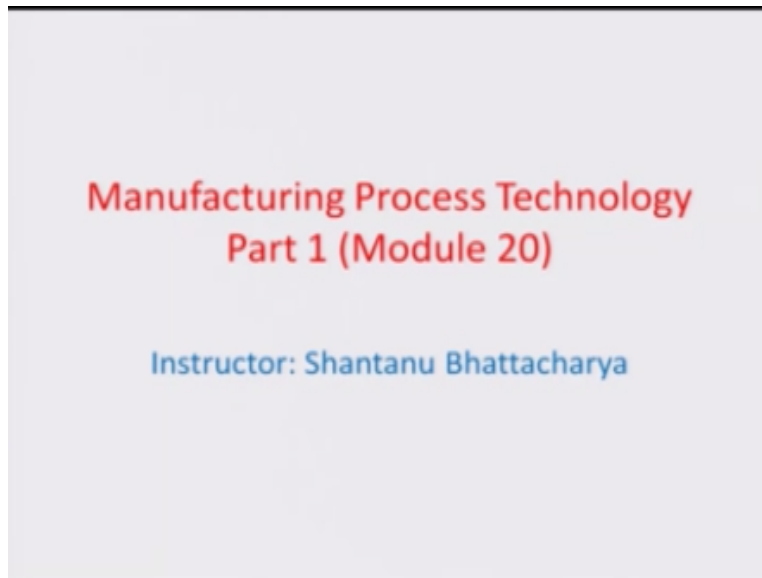
**Module – 20**

**by**

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Hello and welcome to this manufacturing process technology part 1 module 20.

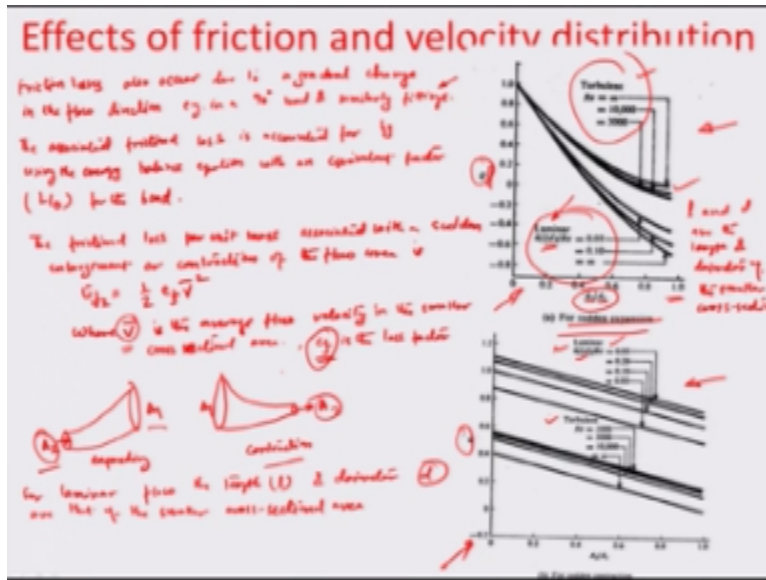
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We covered wall friction as well as the velocity distribution the effect the kinetic energy effect because of velocity distribution and the last module here we would like to cover the energy loss because of you know certain change of direction or even you know entrance effects which would happen typically when a cross sectional area of the conduit suddenly varies it expands a contract you know?

So that is also sort of directly related to the floorage and there may be different entrance effect imposed because the flow obviously being laminar and turbulent , so we would like to now sort of estimate this empirical area and will borrow some of the principles from fluid mechanics.

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So friction losses is also occur due to a gradual change in the flow direction example in a 90° bend and similarly fittings. so fittings may be expansive or contracting a nature as we will illustrate just little bit later. So the associated frictional loss is accounted for by using again the energy balance equation Bernoulli's equation with an equivalent factor  $L/D$  for the bend the frictional loss per unit mass associated with a sudden enlargement or contraction of the flow cross section or flow area is let us say  $e_{f2}$  just as we had  $e_{f1}$  for the wall friction effect earlier.

So this can be recorder as  $E_{f2} = \frac{1}{2} e_f v^2$  where again  $v$  is the average velocity average flow velocity in the smaller cross sectional area supposing the flow is expanding let us say like this you know where the area is changing from smaller area  $A2$  to a bigger area  $A1$ . So the average velocity would be the entrance velocity to this expanding flow and for the contracting case or the flow is coming from a bigger area to a smaller area again if the root be the discharge flow that we will be talking about.

So the value  $A2$  always which is the smaller area is the value cross section where the average velocity needs to be recorded and obviously  $e_f$  is the friction factor is the loss factor I would say because of this bending and you know the expansion contraction effects etc. and the way that you look at  $e_f$  is really plotted right here and this is again two very important trends which are borrowed from elementary fluid mechanics where we talk about the energy loss because of bending or contraction as a function of the area ratio.

So saw for the sudden expansion case as you can see there is a region corresponding to the turbulent Reynolds number varying between 3000 above and the  $ef$  is plotted here as a function of the area ratio it to by  $A1$ . So obviously for sudden expansion the  $A2$  is going to be the entrance area and  $a1$  like you could see here and  $A1$  is the discharge area and so therefore this  $A2/ A1$  can be maximum 1 and otherwise it is showing to be less than one.

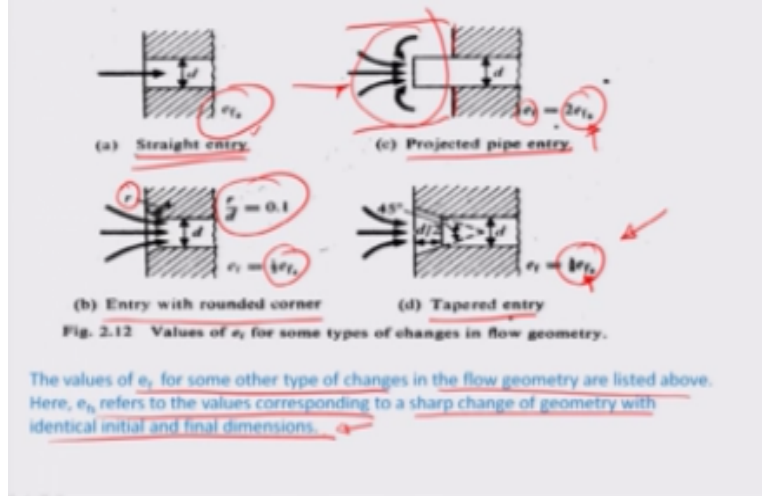
So the same thing happens for Reynolds numbers which are in the laminar zone. So the  $l/d$  which the term which is coming here particularly for the laminar flow case is you have to remember this  $l$  and  $d$  are the length and diameter of this smaller cross section that is how the data has been recorded and these are sort of empirical data's which are recorded again through you know just looking at the energy balance across such expanding contracting through it channels.

So for the sudden contraction case again the same is recorded as laminar and turbulence as you can see and this  $ef$  is really is the loss factor that comes because of the bend or the change in the cross section. So for laminar flow the length  $l$  and diameter  $d$  as you can see in most of the cases whether it is an expansion of contraction you have a inclusion of this  $l/d$  factor here, are that of the smaller cross sectional area the value of  $ef$  depends on whether the flow area is enlarging or contracting in the flow direction and the value of  $ef$  for sharp change in the flow cross section can be recorded as in these two graphs shown here.

So obviously it depending on the type of entrance there is also a entrance effect which is added on to this friction factor where this  $ef$  term gets slightly modified because of such entrance effects.

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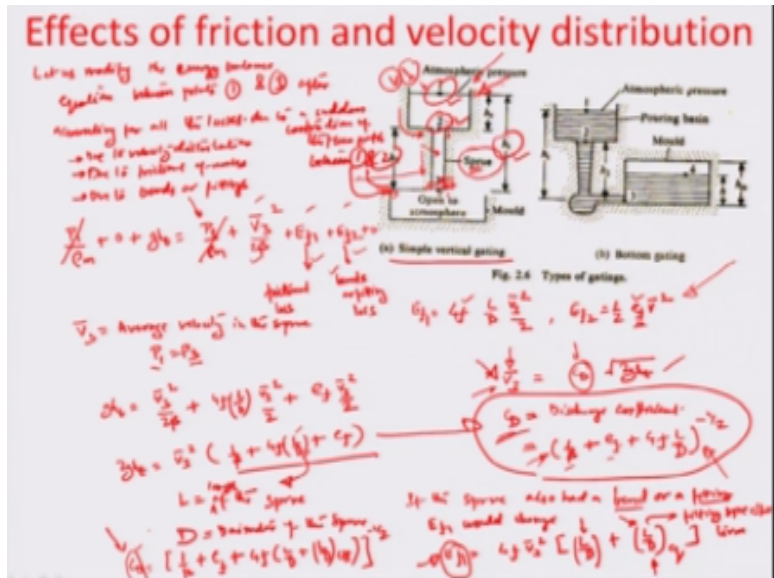
## Effects of friction and velocity distribution



So let us look at this entrance effects so for a straight entry case  $e_f$  obviously is the  $e_f$  that you are recording, so whatever  $e_f$  data you are you know recording with respect to the area ratio is for the straight entry case value of the actually loss factor and the value of projected pipe entry as you can see here where this pipe comes out you know from this entrance in to the mold, so may be the pouring basin as write about here and the flow is happening in this direction. So in this kind of a situation the  $e_f$  that is recorded really is twice the value of the  $e_f$  or that it absorb from the change in the area ratio as in the last figure for the case where it is entry with rounded corners and there is some kind of radius effect which is involved here and  $r/d$  is typically 0.1 so for that the  $e_f$  is recorded as the third 1/3 of the value which is red from the plot on the area ratio versus  $e_f$ .

And then for the tapered entry again the modification that you have is a sixth of the  $e_f$  value that is red from the graph sudden expansion and sudden contraction effects. So I am going to use these more appropriately when we actually numerically design this problem and try to see what is going to happen to the overall energy balance between the point one and three in both the vertical and the horizontal gaining system, so as of now the values of  $e_f$  for some other type of changes in the fluid geometry where  $e_f$  is refer to the value corresponding to the sharp change in the geometry with the identical initial and final dimensions.

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Now let us go back to the again the vertical simple vertical gating system and try to modify the energy balance here, so let us modify the energy balance equation between points one and three after accounting for all the losses, losses are principally three kind one is due to change in kinetic energy, due to velocity distribution, change in kinetic energy due to friction of walls and that due to bends or fittings, so these losses are because a sudden contraction of the flow path between one and two as you can see here.

So the energy balance at one can be written down as  $p_1 / \rho_m +$  velocity head is 0 as assumed earlier the velocity  $v$  here at the top of the particular poring basin is assumed to be 0 and the metal is too large a quantity for any substantial change in the height to happen at this particular juncture plus the potential head which is  $gh$  in this particular case  $ht$  being the height of the top of the poring basin from the mold okay.

So that become equal to  $p_3 / \rho_m$ .  $P_3$  bring the pressure at the juncture three right here plus the kinetic loss which happens because of velocity distribution effect, so write this as square of the velocity by  $2\beta$  where  $\beta$  again would have a different value when it is laminar and another different value when it is you know turbulent in nature the flow is turbulent tin nature plus the loss  $e_{f1}$  which happens because of the frictional loss of the wall plus the  $E_{f2}$  which is the loss because of the bends or the fitting.

$$\frac{P_1}{\rho_m} + 0 + gh = \frac{P_3}{\rho_m} + \frac{v_3^2}{2g} + E_{f1} + E_{f2}$$

So that is how the energy balance would finally happen as you can see here there is no potential head really, so height of the mold as assumed here at point 3 is basically 0 it is top gating system so that is how we have been doing this analysis earlier as well and the case where there was no friction. So on the right side at the juncture three you are accommodating for the loss due to the velocity distribution the loss because of wall friction and the loss because of the sudden contraction and expansion in this particular case between one and two there has been contraction and because of that there is a loss so this loss has to be taken in the overcall energy balance equation for good analysis and  $v_3$  is really the average velocity in this prove and I also further assume that the mold is open to the atmosphere so  $p_1$  and  $p_3$  are similar to each other.

$$2gh_t = v_3^2 \left( \frac{1}{\beta} + e_f + 4f \frac{l}{d} \right)$$

$$v_3 = C_D \sqrt{2gh_t}$$

So because of the pushes the pressure heads get cancelled out and I can further write down the  $e_f$  as the total amount of friction loss which is actually given by  $4f l / d v_3^2 / 2$ ,  $v_3$  is the velocity equal to  $v_2$  of this particular section of the top gating system which is the sprue section you know that this is a continues area constant area geometry where whatever be the inflow would be the outflow for mass continuity to hold sprue and  $e_f$  is basically the frictional loss associate with the sudden contraction in this particular case the contraction happens from the section one to the section two and beyond two there is no other such contraction which takes place.

$$C_D = \left( \frac{1}{\beta} + e_f + 4f \frac{l}{d} \right)^{-0.5}$$

So we are just considering the potential the energy just at the point of exit of the metal from the sprue to the mold and so this  $e_f$  comes out of be  $\frac{1}{2} v^2$  where  $e_f$  again is has to be obtained from the plots for the energy loss  $e_f$  with respect to the area ratio for sudden expansion or sudden contraction case and these all values for example  $e_f$  or  $f$  depend again on the Reynolds number of flow or whether the flow is laminar or turbulent in nature.

So the final expression which emergence in this particular case comes out to be  $gh_t$  on one side and then we have  $v_3^2 / 2 \beta + 4f l / d$  times of  $v_3^2 / 2 + e_f$  times of again  $v_3^2 / 2$  in other words if I just simplify this how it becomes  $gh_t = v_3^2$  times of  $1 + \beta + 4f l / d + e_f$  and I think we have earlier clearly illustrated that this length  $l$  is the length of the smaller diameter of the sprue and similarly the  $d$  is the diameter of this sprue.

$$E_{f1} = 4f \dot{v}_3^2 \left[ \frac{l}{d} + \left( \frac{l}{d} \right)_{eqv} \right]$$

So in this particular case it is really a case of contraction where from one the flow is submerges or converging in to the section 2 and as you can see that there is a contraction in the area and in all such cases you always have to assume the length and diameters of the smaller area of cross section. So in this case the smaller area of cross section is are two and so we assuming the length and diameter of this sprue for calculating this loss contraction loss of the fluid.

So from this particular equation now then you have  $v_3' =$  some coefficient  $C_d$  times of  $2 \sqrt{h}$  and this  $C_d$  is very popularly known as the discharge coefficient from elementary fluid mechanics and this is represented as  $1 / \beta$  the *loss* term because of the kinetic energy change due to velocity distribution the *loss* term because of bending contraction expansion so on and so forth plus the frictional *loss* and the pipe which is  $4fl/d$  time of  $3^{-1/2}$ , so that is how you defined the discharge coefficient and I will say I will kind of show you in the following iterations that the discharge coefficient here can be generated in a iterative way because at the outset you probably just have a first guest of the Reynolds number and that may not be the accurate velocity because the velocity has to go through all these you know different terms.

$$C_D = \left[ \frac{1}{\beta} + e_f + 4f \left\{ \frac{l}{d} + \left( \frac{l}{d} \right)_{eqv} \right\} \right]^{-0.5}$$

So the earliest assumption that you can make if there is no *loss* factor that is  $C_d = 1$  is that  $v_3$  it converges in to  $\sqrt{2} \text{ ght}$  which is in fact the same terms as you had seen in the analysis where we neglected the frictional effects. And then starting from there and then trying to see what is the flow region whether it is laminar or turbulent we can try to estimate iteratively what is the  $C_d$  value so that the actual values of  $v_3$  could be slowly sort of converged at and so with two or three iterations you are able to and come to a point where you can have a perfect convergent of the  $v_3$  term here while doing calculations for such molds.

So if supposing in this analysis we would assume a bend or a fitting so if this sprue also had a bend apart from the normal contraction *losses* that you can see here which is typically the case in most of the applications where you want to avoid putting all the metal on the top of the mold, mold is very, very fragile you know mold is a very, very fragile contusion and typically made up of refractive material.

So it is sometimes are visible to not have the gating arrangement which is includes the poring basin or the sprue just over to the mold rather it is preferable that you have a misaligned poring basin and sprue somewhere here and you have bending providing in the pipe which connected and still makes a top gating because the gating provides the material flow at the top of the mold write about here and you know everything else can be starting you know half shifted manner from the actual mold cavity.

So from a engineering perspective that becomes a pretty good exercise, so if the sprue also had a bend or a fitting then this factor  $efl$  would change and the change is again represented in terms of the equivalent loss that would be effected by the fitting itself. So this is the  $l/d$  of the wall friction of the smaller cross sectional area as I had discussed before and this  $l/d$  equivalent which is a new term which has been added here this is actually a fitting specific term and or a bend specific term and there are many experimental analogy is which talks about what this  $l/d$  equivalents is going to be.

So basically if supposing there is bending of a certain dimension there are may be data tables which are available about it supposing there is a particular kind of bend or a kind of fitting what would be the  $l/d$  equivalent in that particular case. So this would be definitely contribute when increase in the total frictional loss and thus the whole equation now of the  $C_d$  would be written down as  $C_d = 1/\beta$  as I had illustrated earlier plus the small  $ef$  which is the loss because of the bending or the fitting and then there is a frictional loss imparted by the bending or the fitting which would be actually  $4f$  times of the  $l/d$  wall friction effect plus the  $l/d$  equivalent loss due to friction at the bend or friction at the fitting which would come up to the power of  $-1/2$  and so that is how you typically make the discharge coefficient and iteratively progress to find out the range of the flow whether it is laminar or turbulent and then arrive at a certain value of  $C_d$ .

So I am going to now close this particular module in the interest of time but in the next module I am going to definitely try and tell you some practical problem in which we can estimate what is the  $C_d$  value and also what is the velocity of flow. So thank you so much.

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