Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 07
Key Concepts and Terminologies

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Hello, welcome of basics of finite element analysis, this is the second week of this course and we will continue our journey and what we will be doing this week is primarily get familiar with the with several mathematical concepts on which the whole finite-element theory has been constructed upon, and in specifically in today's lecture we will help you understand several important technologies as they relate and they are frequently used in finite element analysis methodology. So that is what we will

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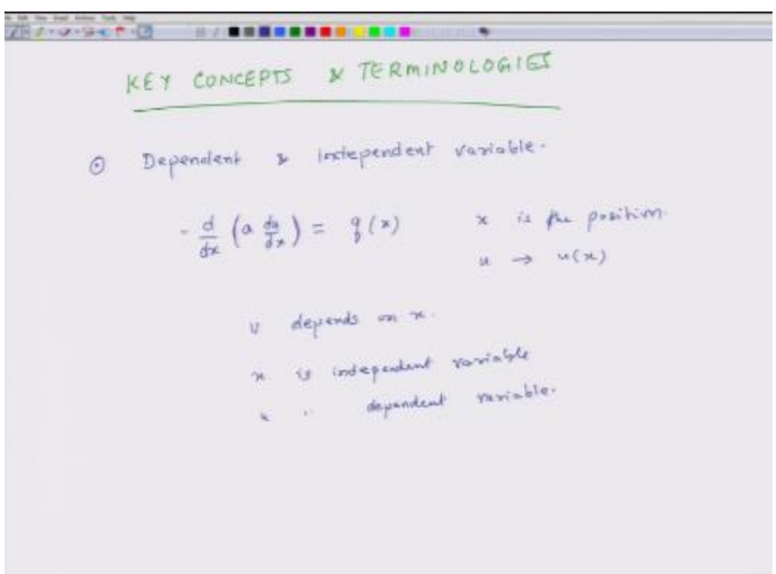

Discuss, key concepts and terminologies, so the first term is what is a dependent variable and independent variable okay. So let us say we have a differential equation d over dx, a d over dx equals let us say q_x okay, so this is a partial differential equation of second order a could be a function of x so that is why it has been kept inside the parentheses otherwise it would have worked out.

Now here x is the position, is the position, is the position and u is a function of x, so u depends on x, x does not depend u, u is x is independent does not matter what the value of u is x is a coordinate in space so that is why, so x is independent variable and u is dependent variable, it is dependent on x in this case for two-dimensional 2D problems for instance if I have relation for temperature, temperature is dependent on two independent variables x and y.

If I have a plate which is subjected to some external forces in that case there maybe three displacements at any given point u, v and w and each of these guys u, v and w they depend on two independent variables x and y, if it is a dynamic problem each of these values a variables u, v and w which are dependent variables they depend on three independent variables x, y and time, x, y and time, so that is what is about dependent and independent variable.

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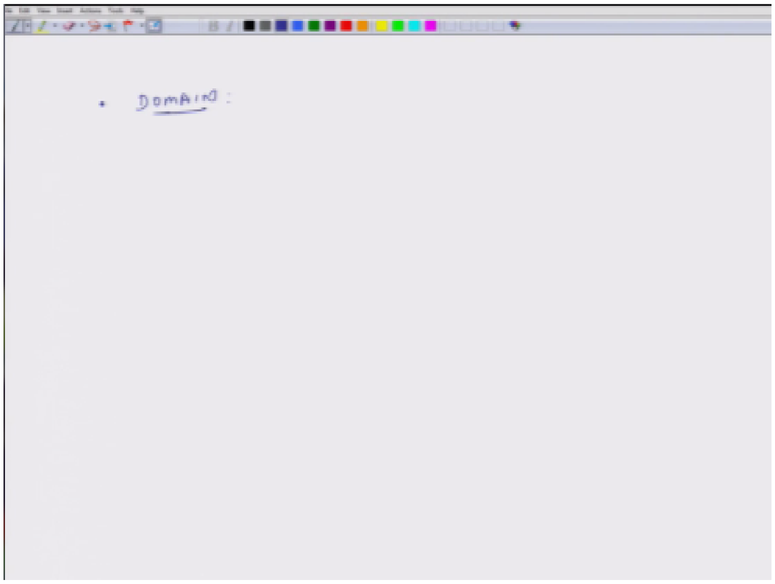

The second term is domain, so till so far we have discussed and used domain and I have said that from a practical standpoint it is the region in which we are interesting to find the solution of a problem right, from a mathematical standpoint the definition is a little much more regress, so what is domain it is a collection

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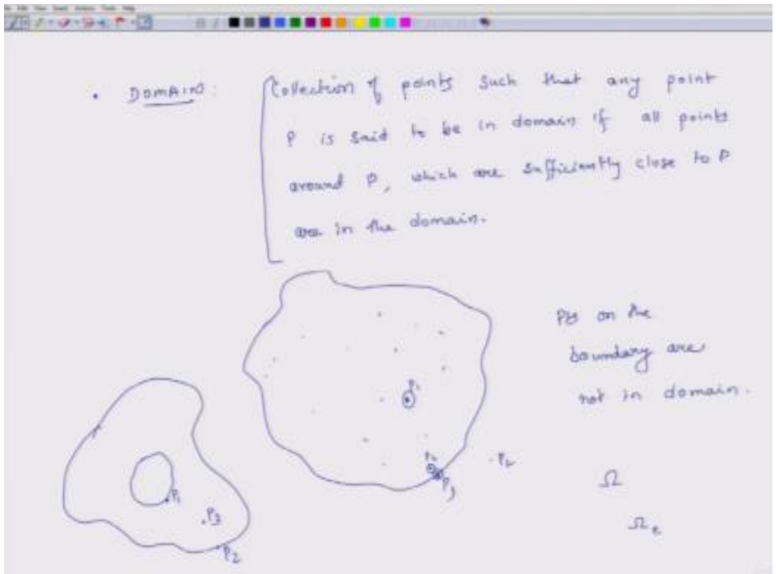

Of points, that is the first thing but then it is not just any arbitary collection such that any point P is said to be in domain if all points around P which are sufficiently close to P are in the domain okay. This is what it says, so what does that mean suppose this is, so there are lots of points infinite points and what it says is that a point, so here we are describing defining domain in a roundabout way so we say that is a collection of points.

Now there could be a point here also, there could be a point here also, there could be a point here also and there could be a point here also okay so let us number these P_1, P_2, P_3 and P_4, so what does it say that a point is in the domain if all the points which are sufficiently close which are sufficiently close to the domain are in the domain, so when we look at point P_1 all the points which are very close to P_1 are in the domain

So point P_1 is said to be in a domain so similarly we find out what all points lie in the domain and that collection of points is called the domain. Now you go to P_4 which is very close to the boundary of the domain but still as long as it is not in the boundary all the points around it which are sufficiently close they are still in the domain, so point P_4 is also in the domain.

Now you look at point at P_3 here there are some points, there are some points around P_3 which are in the domain and some points which are not in the domain, so we do not say that point P_3 is in the domain, and so for similar reason point P_2 is also not in the domain okay, so what it directly implies is that points on the boundary are not in domain okay, another is so this is another it could be domain and it could have an internal boundary.

So it could have an internal boundary and an external boundary, so then points which are here let us say this point P_1 this is not in the domain, point P_2 this is not in the domain, but point P_3 it is in the domain. In a lot of our further discussions we will designate domain by this symbol Ω and this is also a fairly standard symbol used for to denote domain in a lot of technical literature.

If I say that my domain is an element lot of times we will say that Ω subscript e which means this is the domain of an element

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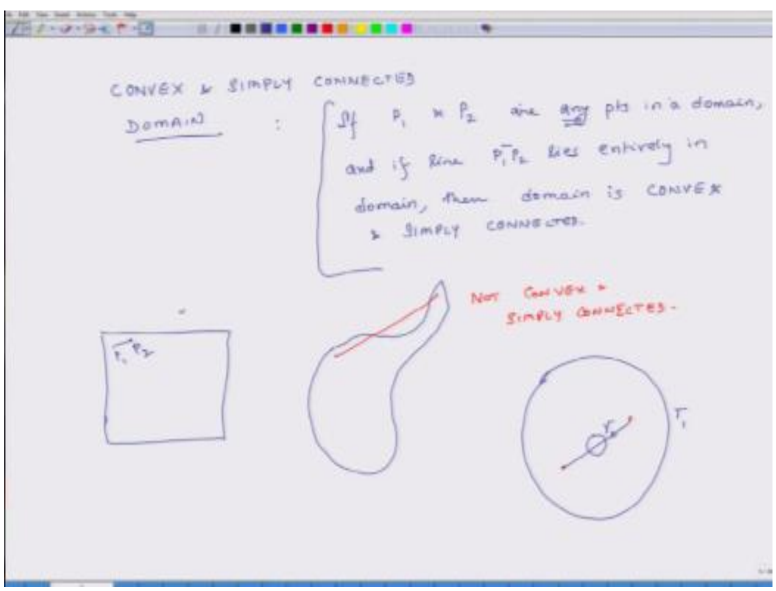

Third concept, convex and simply connected domain, so what does so a domain could be convex and simply connected or it may not be convex and simply connected domain so what is a domain which is convex as well as simply connected it is, this is how we figure out if P_1 and P_2 if P_1 and P_2 are any points in a domain and if line P_1, P_2 lies entirely in domain, then domain is convex and simply connected okay.

What does that mean, consider this domain, this is a rectangular domain, now you take any two points P_1 this, this statement has to be true for all sets of all the points in the domain so you take any two points and if you join them through a line that line will always remain in the domain okay, so this is an example of simply connected as well as convex domain.

Now you take another domain okay if I take this point and I take this point and I draw this line, this line is not is totally in the domain, so this is not convex and simply connected. Okay another example a circle is a simply connected domain but if I make a small circle within a circle okay then it has two boundaries, boundary one and boundary two and then I can have two points this is point one, this is point two in the domain and the line which connects them does not light entirely in the domain.

So again this is an example of not convex and not simply connected system, okay the reason why I use the term convex is that convex you will see you know what is convex lens, so it tries to focus all the rays to point it tries and then so that is

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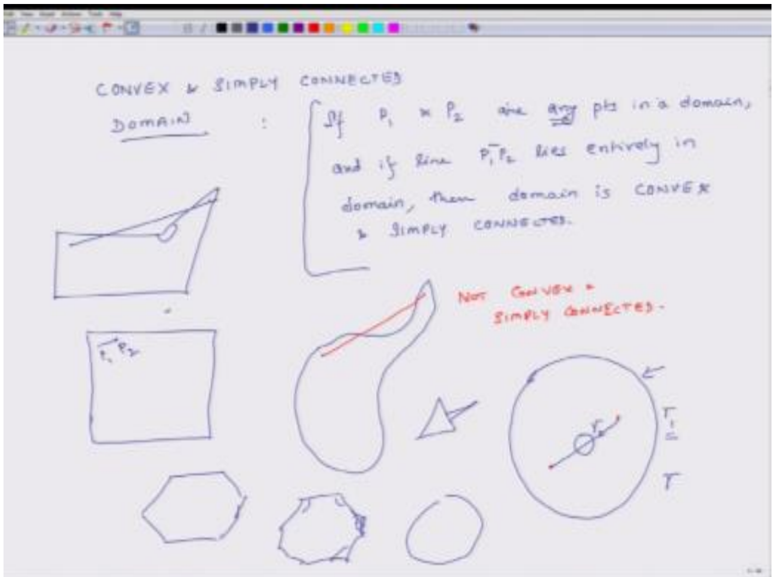

A very good that is a feature of lot of for instance you have a regular hexagon, this is simply connected domain, you have a octagon this will also be simply connected, you can have a circle you can have a triangle the moment you have some non convex, see here all the angles are less than 180 degrees but if you have a, like this, this angle is not less than 180 degrees and because of this you cannot say that this will be simply connected because of one okay

So this is what a simply connected convex domain means and I should have mentioned that we always define symbol symbolize boundary as this γ okay, so in this case for this example there are two boundaries γ_1 and γ_2 so whenever we try to integrate a function on the boundary of a domain, on the boundary of a domain, it is not that we just only worry about the outside boundary we have to integrate it on all the boundaries whether they are internal boundaries or external boundaries it is important to understand.

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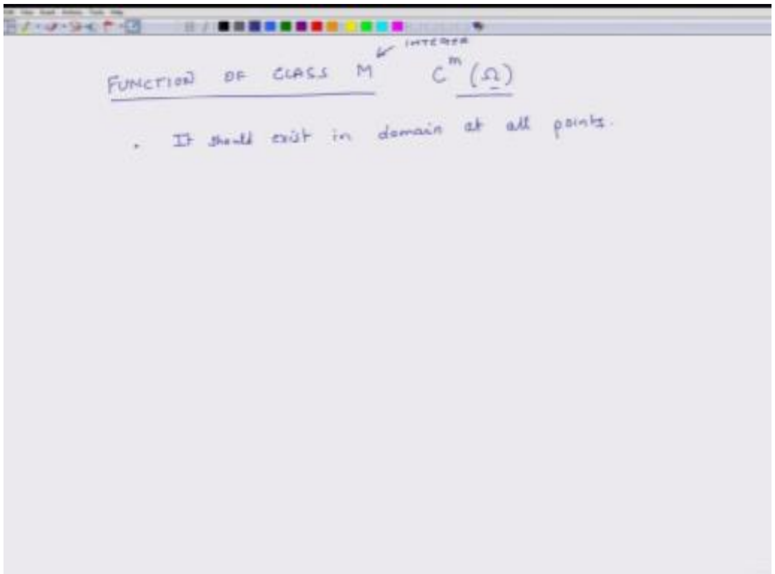

The next term is function of class M there could be a function it can have a class 1, it can have class 0, can have class 2, M is a number integer and it is designated as C^m C is class m is the integer, so a function is of class m in a particular region not region domain okay what does this mean, a function is of class m in a domain it has to satisfy two requirements, it should exist in domain at all points.

So when it exists in domain at all points do we, does it require that it has to be on the boundaries no because the points for the boundary are not necessarily not in the domain okay.

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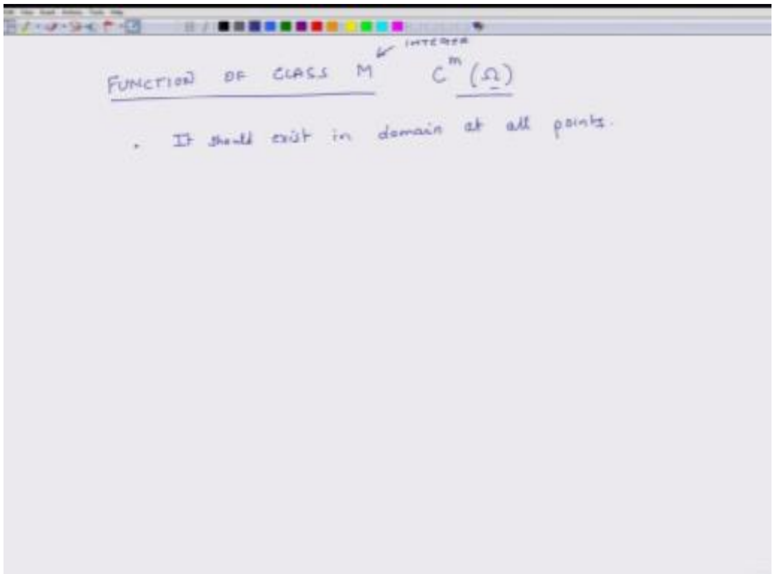

And the second thing is

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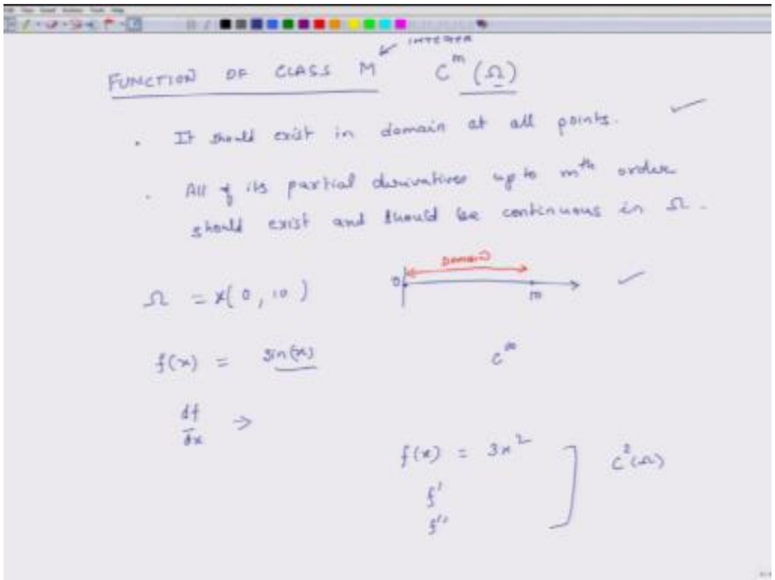

All of its partial derivatives up to m^{th} order should exist and should be continuous in domain this thing okay, so for instance so this domain could be one-dimensional, two-dimensional, three-dimensional does not matter right for instance let us say my domain is a set of points from 0 to 10 x okay.

So what does this mean my domain is on the x-axis this is 0 this is 10, oh I am sorry is here, so, so this is my domain and then I have a function let us say and let us say it is equal to sine (x), so then I see that sine(x) is a well-defined it exists at all the points between 0 and 10, so this condition is true and then I also see that df over ds, dx exists for all the points in this domain 0 to 10.

And all of its derivatives actually exist I can keep on differentiating it, so this is a class infinity right it is infinitely differentiable but if I have an another function f(x) and let us say it is equal to $3x^2$ right then I have the first derivative, first derivative exists, second derivative exists, the third derivative and onwards does not exist, so this is a C^2 function in this domain okay understood.

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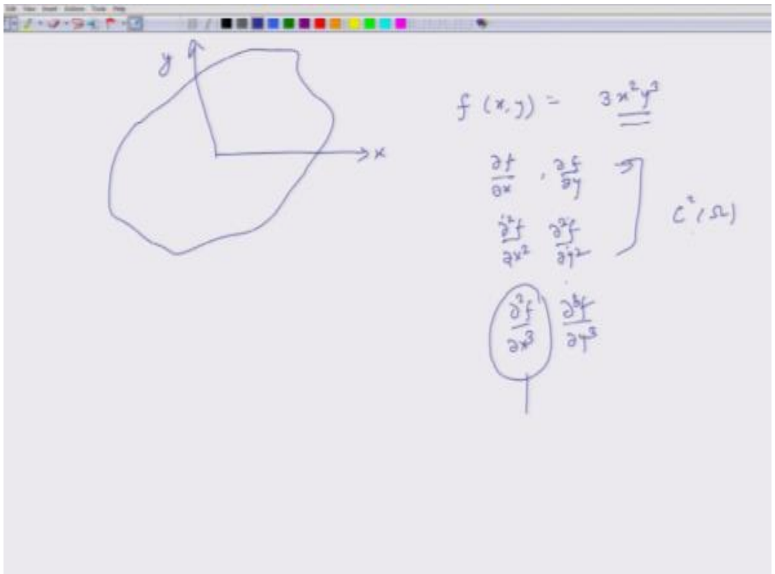

Now if you have a 2D domain let us say you have some complicated shape, you have a 2D domain and there is a function and let us say there is a function $f(x, y) = 3x^2 y^3$ then the way we figure out the classes that all so does a ∂f over ∂x and ∂f over ∂y exist so the answer is yes so it is certainly m is 1 or higher and you go on figuring out the partial derivatives right.

So then you take the second order derivatives and if they exist then it will be C^2 if higher ones do not exist so ∂^2 f over ∂x^2 and ∂^2f over ∂y^2 they also exist for this function and but this guy, this guy does not exist and higher terms right so this is C^2 okay

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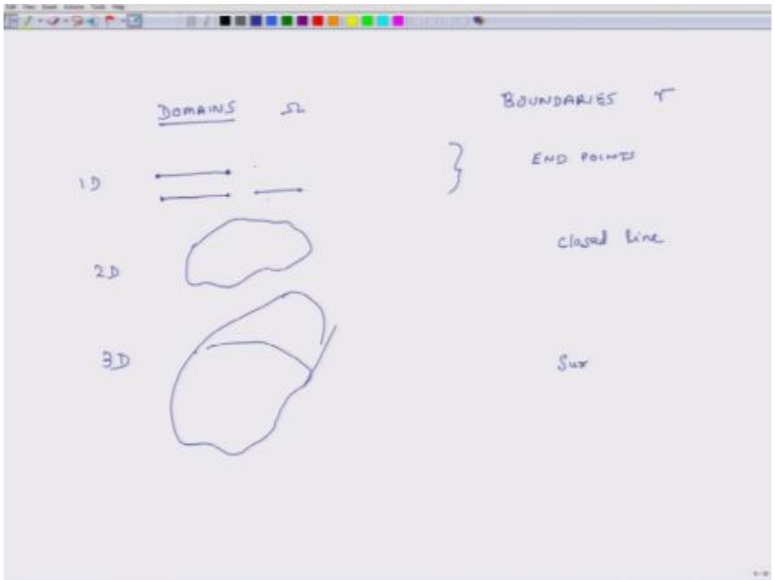

So now domains could be one dimensional in nature, two dimensional in nature, and three dimensional in nature, actually they could be of higher dimensions also but in context of this course I think we will be here and then their boundaries, so a domain could be like this, this is one example of a domain these are the end points, another example of a domain now this one this domain is not simply connected.

Right so this is an example 1 this is example two because there is a point here and there is a point in this one and if I connect them then all the intermediate points between these two points do not lie on the line, so this is a 1D domain simply connected this is a 1D not simply connected right in this case the boundaries are just end points. A 2D domain is a basically some surface it could be a flat surface or it could be a curved surface also, could be a curved surface also.

So here it is that is a line closed line alright and here this could be a 3D object and here it is a surface.

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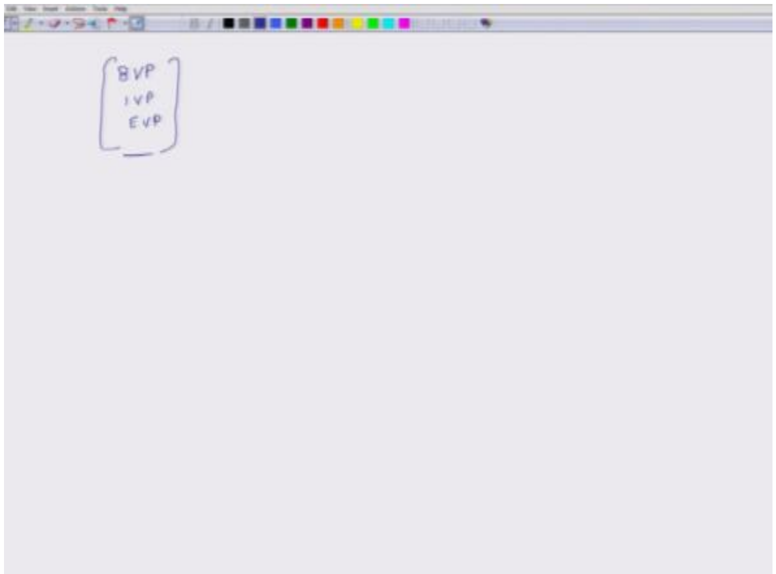

Finally we will talk about what our boundary value problems, initial value problems and Eigen value problems we will explain what are these, so these are the three types of problems we will solve in our problem in our course okay so I will explain this by an example.

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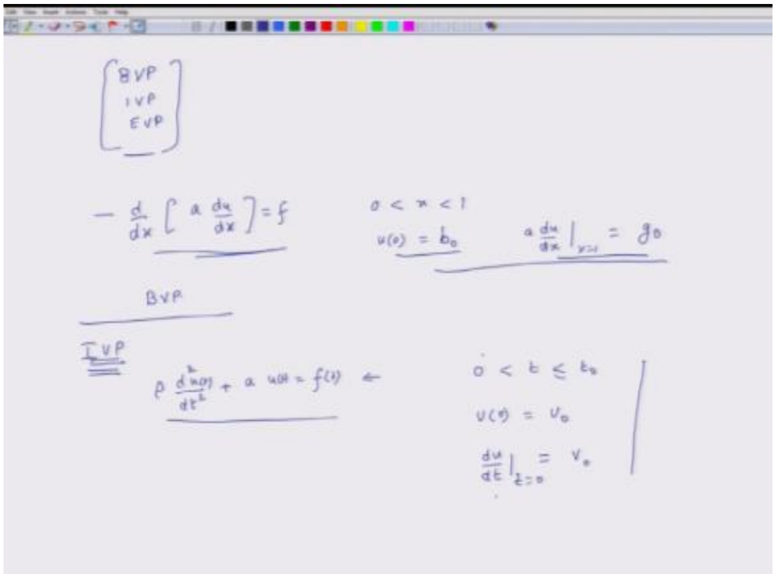

So let us say my differential equation is this –d over dx a d over dx =f, so here u is the dependent variable and it depends only on x which is the independent variable and this is valid between, let us say this differential equation governing equation is valid in this region and the value of u(0) is equal to some constant b_0 and the value of a times du over dx at x is equal to 1 equals g_0 okay.

So what you see is that this is the governing differential equation and I have prescribed the primary value a variables and some functions of its derivatives at the boundary, what are the boundaries 0 and 1 so I have the prescribed either I prescribe the primary variable or the dependent variable at the boundaries so this is a boundary value problem, okay this is the example of a boundary value problem.

The second category of problems are again I will explain through example so this is my second categories initial value problem, in the first case we are prescribing primary variables or their dependence or that is their derivatives and stuff like that at the boundaries. In the initial value problem consider this equation, so here u is a function of time, okay this is so this would also be a function of time and u is a function of time okay this is like an your equation mx double dot plus kx is equal to C it is something like that.

And this equation works for different values of time and the value of u at time t is equal to 0 is u_0 and the value of its derivative v_0 okay so this is the initial value problem, so in the first case I was prescribing the values at the boundaries physical bodies they are physical boundaries, here I am prescribing the value of primary variable and its functions not at only one boundary which is t = 0

There is no t is equal to something else other boundary right, so I am prescribing u_0 and its derivative at the beginning of time which is t = 0 so that is why it is called initial value problem because I am with am prescribing conditions which exist initially not something sometime later okay, so that is the initial value problem and then we can have problems which are both.

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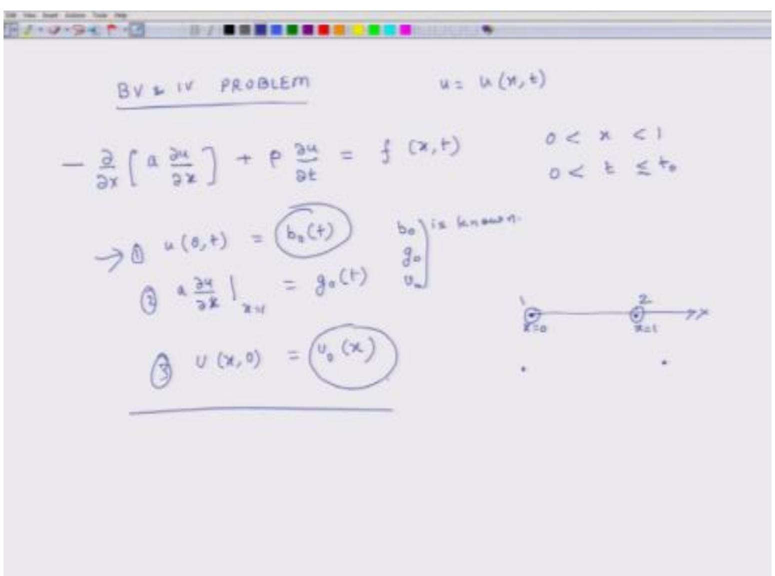

Boundary value as well as initial value, so as you are developing solutions FEA solutions we have to look what is the type of problem okay so here I will again write down an equation, so here u not only depends on x and it not only depends on time alone but it depends on both so u is a function of x as well as time, so here we have two independent variables and one dependent variable and this equation I say that it is valid between x is equal to 0 and 1 and also between time more than 0 and sometime T_0

And here I am saying that okay value of u at all times at x is equal to 0 is b_0 of t some function of time and b_0 is known, then value of a times u over ∂x at t is equal to 1 excuse me at x equals 1 is g_0 of t and finally u(x, 0) is $u_0(x)$ so all this $b_{0,}$ $g_{0,}$ and u_0 they are all known functions b_0 and g_0 are known functions of time and u_0 is a known function of x okay.

So what are we specifying here we are specifying the conditions at the boundary, this is this is my x-axis so this is x is equal to 0 this is x is equal to 1, so I am specifying all throughout the time the value of u at x is equal to 0 this is condition 1, this is conditioned 2 okay, so I am specifying the value of u for all values of time at x is equal to 0 and x is equal to one, so I am specifying stuff on both the boundaries.

So in that because of that it is a boundary condition and I am also specifying at all the points, at all the points at time t is equal to 0, so this is so this is the condition 3 and that is why it is a boundary and an initial value problems, so I am stating what the system looks like initially and how it behaves at the ends at all times okay. Another thing they are sometimes people say that this is a homogeneous boundary condition and some and this is a non-homogeneous boundary condition.

So if b_0 was 0 then we would have said that this particular condition is homogeneous boundary condition, if this u_0 of x was 0 then it would be a homogeneous boundary condition, if it is non zero then it is a non-homogeneous boundary condition, so this is the last another thing and last thing.

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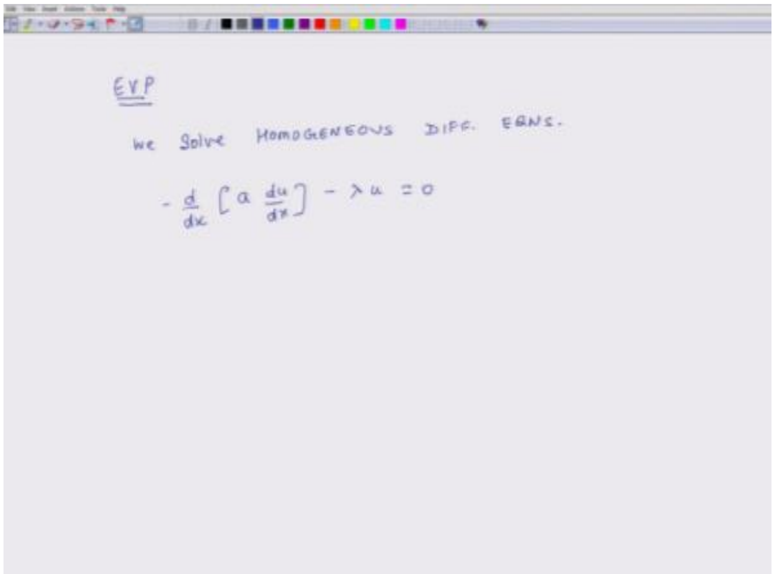

We will talk about Eigen value problem EVP, so we have talked about BVP, IVP BV+IVP and this is the last one is Eigen value problem, so in Eigen value problems we solve homogeneous differential equations. See earlier we talked about homogeneous boundary conditions here we solve the homogeneous differential equations so what is a homogeneous differential equation, for instance d over dx a du over dx - λ u is equal to 0.

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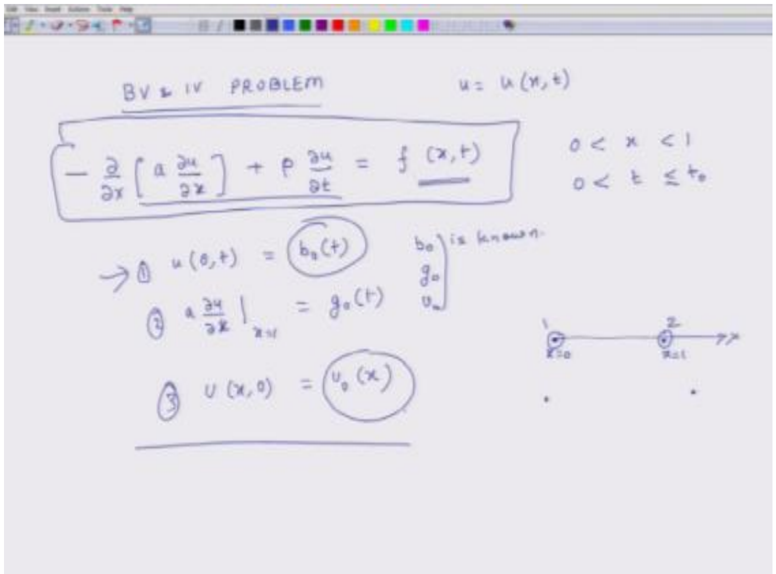

So first let us look at this equation, this is a non-homogeneous equation y because the right side of the equation okay so our first thing is that on the left side we have put all things related to u which is the dependent variable on the right side there is no u term, so the non u term is FXT it is a forcing function okay it is a forcing function and here forcing function is nonzero.

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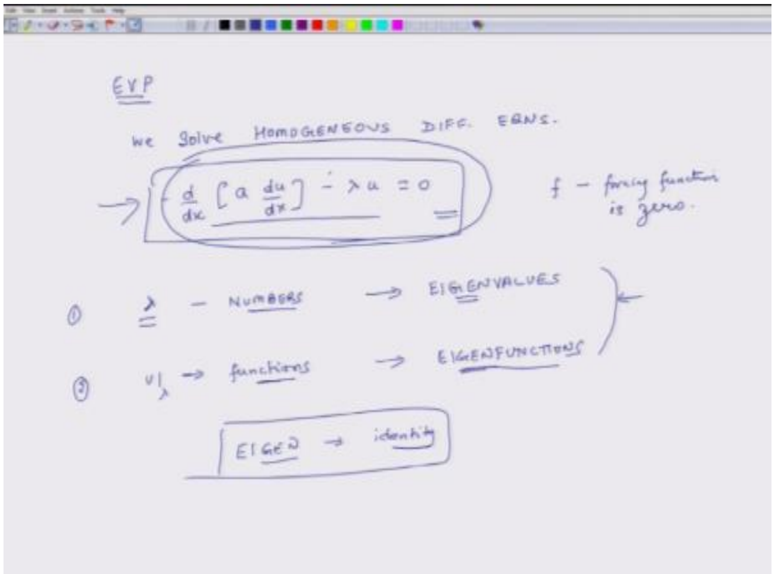

Now we look at this equation, here all terms on the left are again in u or their derivatives but on the right side the forcing function is 0 so this is a homogeneous differential equation because F our forcing function is zero, it is a homogeneous differential equation. And in this case we solve this equation and we solve it to find the values of λ okay, first we solve the values for λ so this is 1 and then 1 so we will get several values of λ okay we may we get several values of λ and then for each value of λ we find value of not u not value of u we find okay.

So this is these are numbers and for each value of λ we find u | λ and these are functions okay. These numbers are called Eigen values and Eigen values is one single word it is not two words it is one single word and these are the known as Eigen functions, Eigen function is also one single word, so here I have just written one particular type of homogeneous differential equation but any homogeneous differential equation does not have to stick to this form is of, is where we find some numbers like this λ and these numbers are called Eigen values and associated with each of these numbers is a u function and these functions are known as Eigen functions.

Okay and that last point I wanted to mention is that in German this term called Eigen it relates to something called something relates to identity it may not be the exact translation it relates to identity, so here there is no external force okay, so the system the behavior of the system is purely dependent on the internal characteristics of the system, it is purely dependent on the internal characteristics of the system it is so these λ are basically the identity if there is nothing external so howsoever the system is going to behave will be purely driven from the internals of the system.

So that is why this idea of identity of the system is related to this thing called Eigen okay so that is there, so this closes our today's lecture and we will continue this discussion tomorrow, thank you very much, bye.

Acknowledgement
Ministry of Human Resource & Development

Prof. Satyaki Roy
Co-ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
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Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta

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Shivendra Kumar Tiwari

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