Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning(NPTE.L.)

Course Title

Basics of Finite Element Analysis

Lecture – 06
Strengths of FE Method, Continuity conditions at Interfaces

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Hello, welcome to basics of finite element analysis book course, today is the last day of this week and what we will do in today's rather brief lecture will be just to have a quick summary of whatever we have learned till so far in last five lectures so that is what we are going to do.

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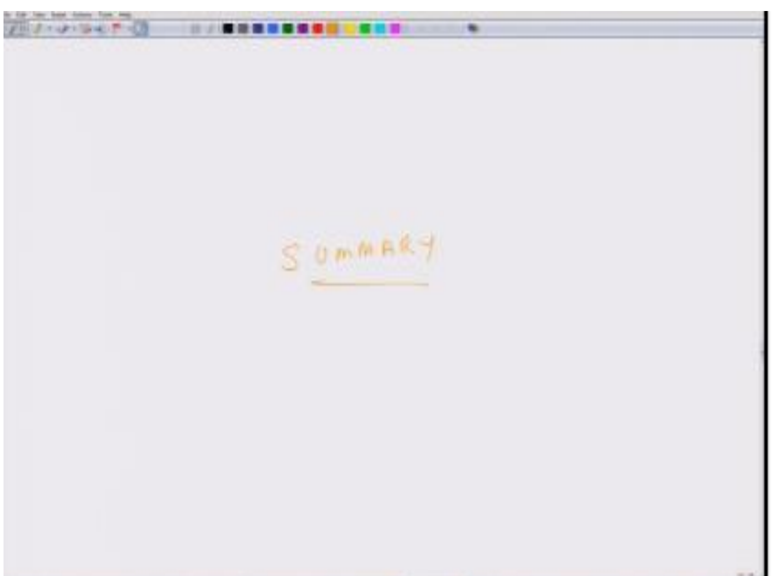

So our theme will be basically summary okay, so the first thing we discussed and we explained is that most of the physical phenomena which we observe for instance heat transfer, problem related to fluid dynamics, or problems related to solid mechanics or diffusion, they can be explained or the, this phenomena can be captured by a differential equation and one thing we discussed in rather detail is that in general we do not have easy solutions for these partial differential equations even though these equations may be linear in nature, when it becomes nonlinear than getting solutions for these things is even more complex.

And the reason for that is that the conditions for an exact solution are pretty stringent, the first condition is that whatever solution we think is going to be the actual solution it has to meet all the boundary conditions so that is the first thing, and then the second thing is that it has to be it has to satisfy the governing partial differential equation at all the points in the a, in the domain. So basically what that means is that the error that is the difference between the left side and the right side if we plug in the assume solution in the differential equation that error has to be 0 for all the points in the domain.

So that is what the realization is and because of this people have said okay if we do not get an exact solution let us try to get solutions which are not necessarily exact but they are fairly close to the exact solution, and in that context techniques like finite different method and finite element method and several other approximate solution methodologies have been developed, and as we explained in several other previous lectures the FEA method has some key advantages.

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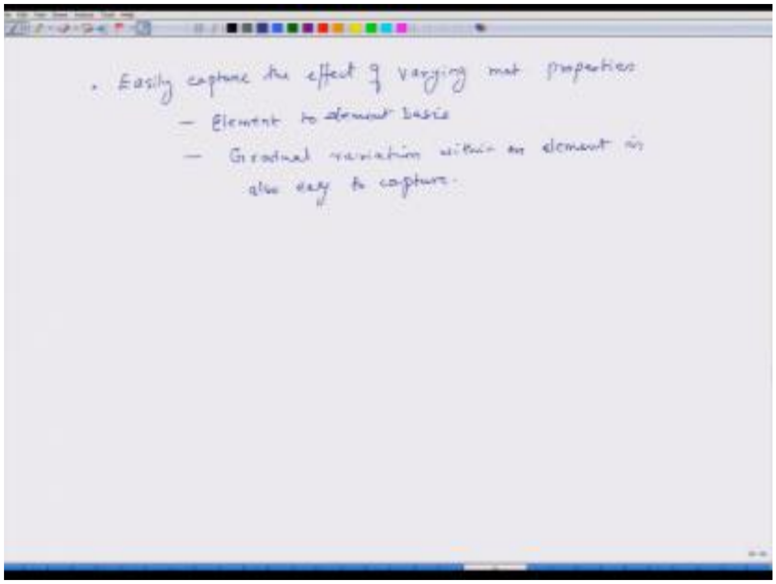

So the first advantage is that it can easily capture the effect of varying material properties either on an element by element basis, so you can have one property in element number one and then if you move on to the second element if the problem requires then you can go to the, you can use another property, so this is one. The second thing is that if you have a situation where material properties are gradually changing in space then that kind of phenomena or that variation, gradual variation can also be captured so gradual variation within an element is also easy to capture. All that we are doing is that we integrate the weighted residue or the error on an element by element basis.

And as we are calculating the error in that we can plug in the variation varying material property and then we do and because this variation is polynomial. polynomial in nature so if we can relatively easily integrate to find the weighted error within the element. The second advantage is

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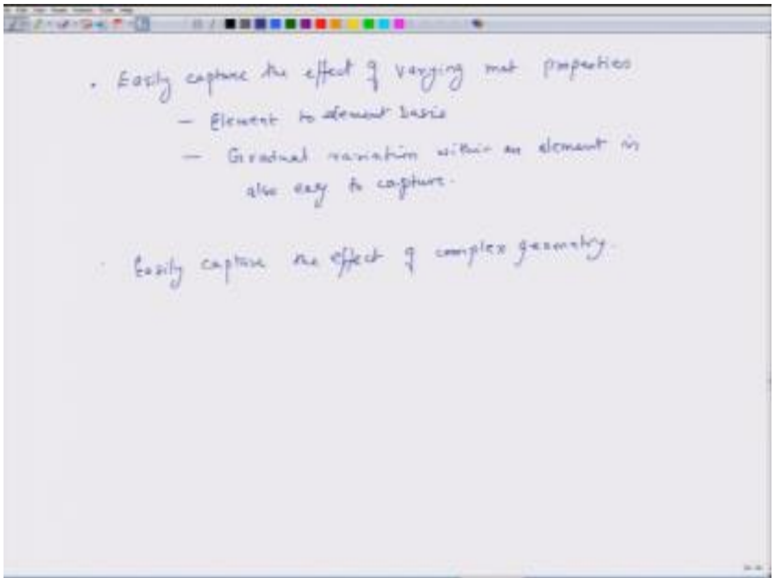

That when you can easily capture the effect of complex geometry okay, so for instance I mean this is not a simple geometry it has some surface here, it has a curvature, there is a hole inside where we have gradually changing radii, so if I have to see how stress is, suppose I am putting a force on this particular element along these two points and if I want to figure out how stress is distributed in this complex surface, if I try to develop an analytical solution it will be pretty difficult probably not possible, forget finding a solution if I want to develop an equation which represents the geometry of this complex surface, that itself may not be easy but in FEA what we do is we break this into small units of either if it is a 2d surface and in, in, in terms of either triangles are quadrilaterals.

Or if it is a 3d volume then I can develop it, break it up into small, small bricks or wedges or tetrahedrons and because of that it does not matter what the geometry is I can easily model the geometry and I can also easily how stresses are changing, and how displacements are varying in this complex surface relatively easily because we rely on the power of discretization.

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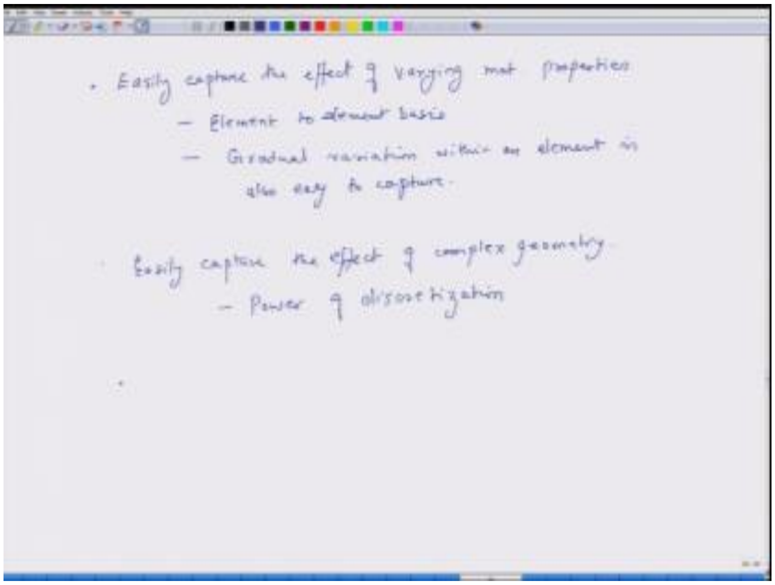

The third thing I wanted to mention is that we had discussed that the scheme in the overall philosophy is that we developed an equilibrium equation, using that we find and then we discretize the geometry on an element by element basis, then for each element we assume some shape functions or polynomial functions which help us explain how the independent variable, the variable in which we are interested to find values of how that dependent variable is varying over the domain of the element, so we assume some functions and then using those functions we calculate the error on an element by element basis, we multiply that error by the weighting function.

And once that weighting function has been multiplied to the error we integrate this weighted error over the domain of the element on the net so, so we get a large number of equations we get a good number of equations for each element and then there are several elements, and so we get a very large number of equations and then we somehow assemble all these elements together or we somehow combine all these equations which have been generated from a single element, and we get an assembly level set of equations, and the philosophy in this assembly process.

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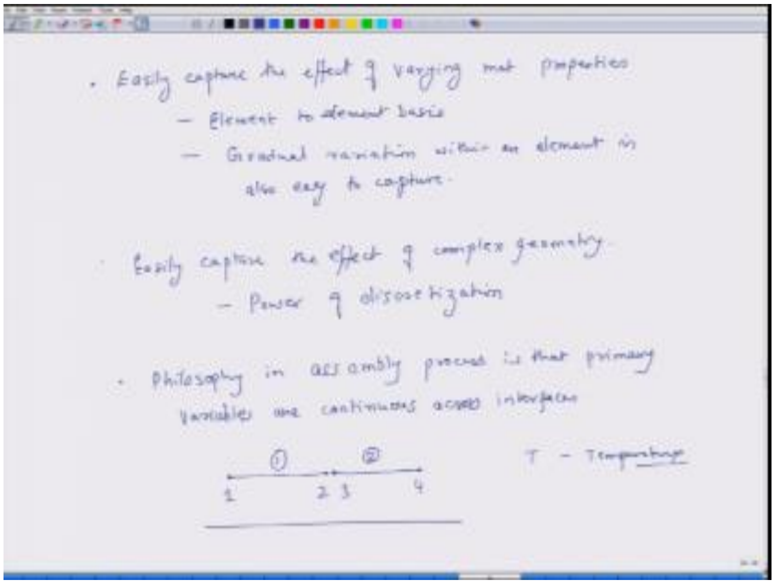

Is, is that primary variables and I will explain this term are continuous across interfaces, so what does that mean? What it means is suppose I have two elements this is element number one and there is another element and I have put a small gap here but in reality this gap will not exist just to designate this element separately I have put a small gap here. So let say us this is node 1, this is node2, this is node 3, and this is node4. So nodes 1 and node 2 are for element number one and nodes3 and nodes4 are for element number 2.

And suppose I am trying to and let us say that this is a problem of heat conduction and we are interested in finding how temperature is varying across the elements across the geometry so this is temperature, and suppose to find temperatures we have to solve the heat conduction equation so in the heat conduction equation temperature is the fundamental variable which depends on X okay, which depends on X and time so that is my primary variable.

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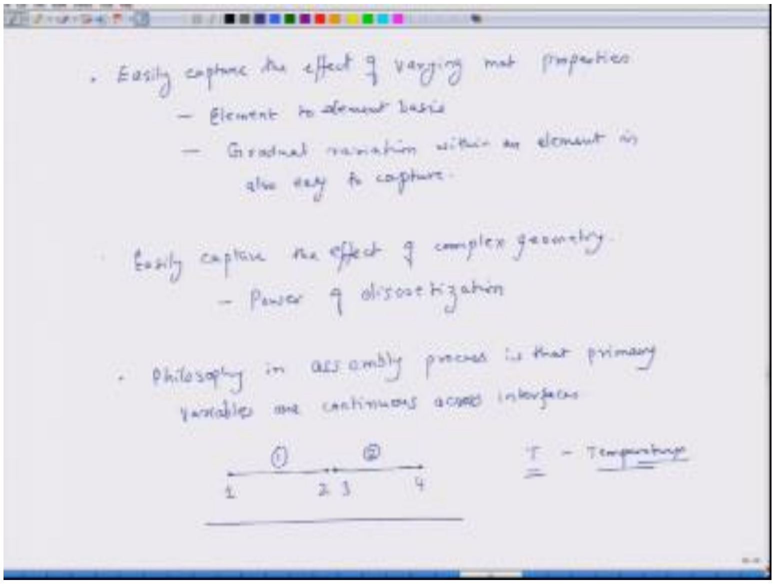

This is the primary variable. Gradients of temperature or heat flux they will be secondary variables, the fundamental variable is temperature. Similarly if I am trying to have a, if I have a problem for of being bending the deflection of the beam at any given point is the fundamental primary variable, slopes and moments and strains and stresses, they are all secondary unknown variables.

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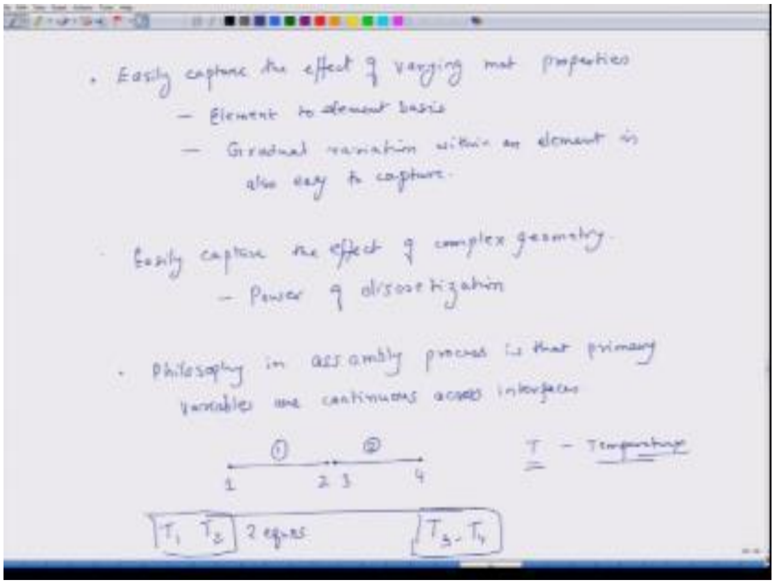

So I will generate, so each element in this case coming back to this example each element in this case has two nodes right, if and this is this is assuming that I take a linear element. If I had a quadratic element then each element would have 3 nodes, 2 of these nodes will be at the boundaries and one node will be in the middle of the element, but coming back so let us assume that we will take each of we will develop each element as a linear element so I have two elements for each excuse me, I have two nodes for each of these elements and.

I develop 2 equations so for element 1 I developed 2 equations and these will be in these equations will be for temperature t1 and t2 right and the, how will we develop those equations we have somewhat we have not explained in great detail but we have somewhat addressed that methodology in previous lectures and similarly for the second element I will have two more equations and these two equations the unknowns will be t3 and t4 okay.

So I will, I will have two equations for element 1 and 2 equations for element 2. Now in the assembly process what we do is that when we join these two equations, and exact mathematics will be discussed later we ensured that because nodes 2 & 3 are same.

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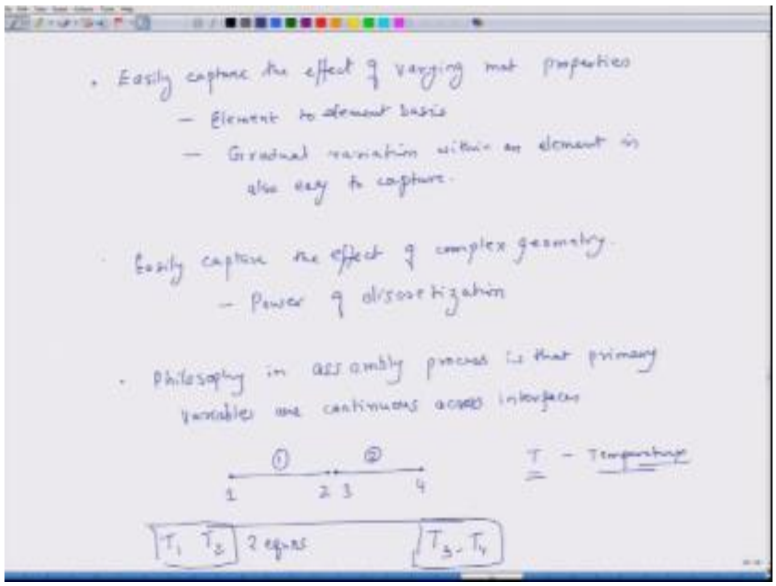

In reality here just for the purposes of explaining I have said that there is a small gap but in reality because nodes 2 and 3 are at the same physically at the same location then there is no reason to think that the temperature of t2 will be different than temperature of t3, so we will ensure while we are doing assembly that the temp, value of temperature t2 the same as the temperature at t3. So these 4 equations will merge and then we will lose one equation in that process.

So when we assemble these 2 equations together we will get 3 equations okay, in through the assembly process and those 3 equations will be for variables t1, t2 and t4 because t3 will have same temperature as t2.

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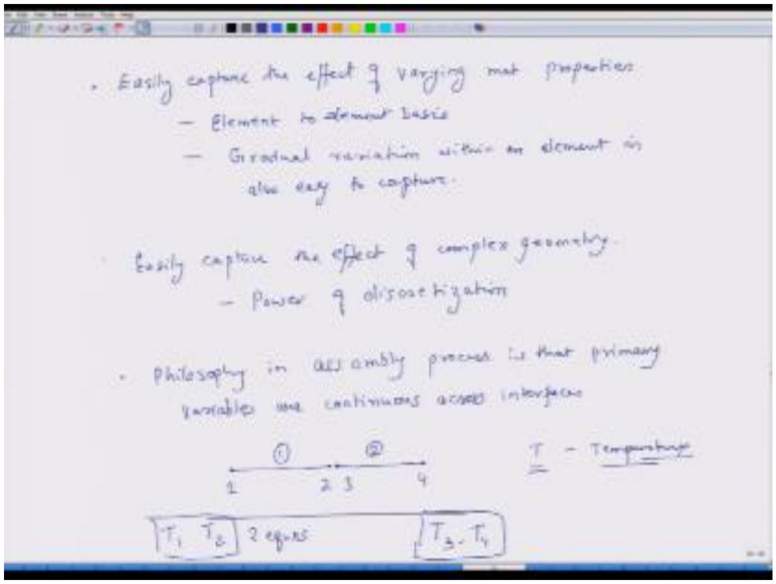

So this is the basic philosophy through which we combine or we combine different element equations at the assembly level, now there could be a situation where

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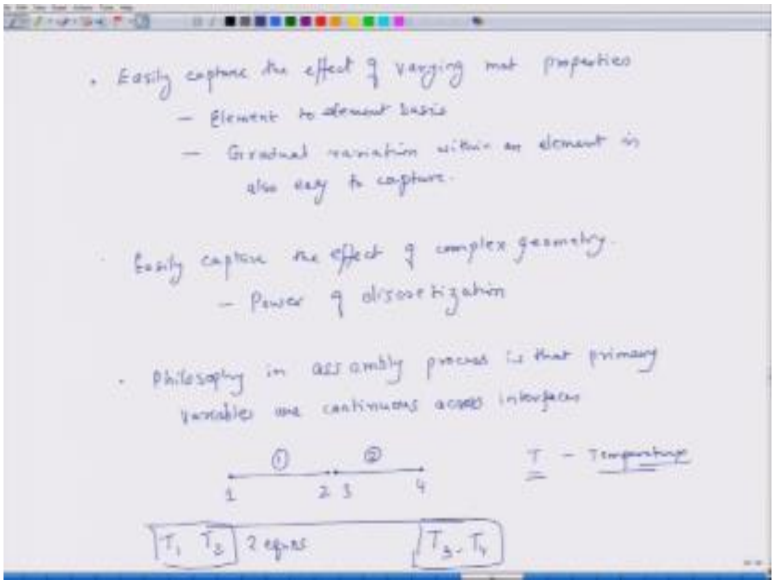

A value of variability through may not be same as the value at t2 what could be the case, for instance suppose I have.

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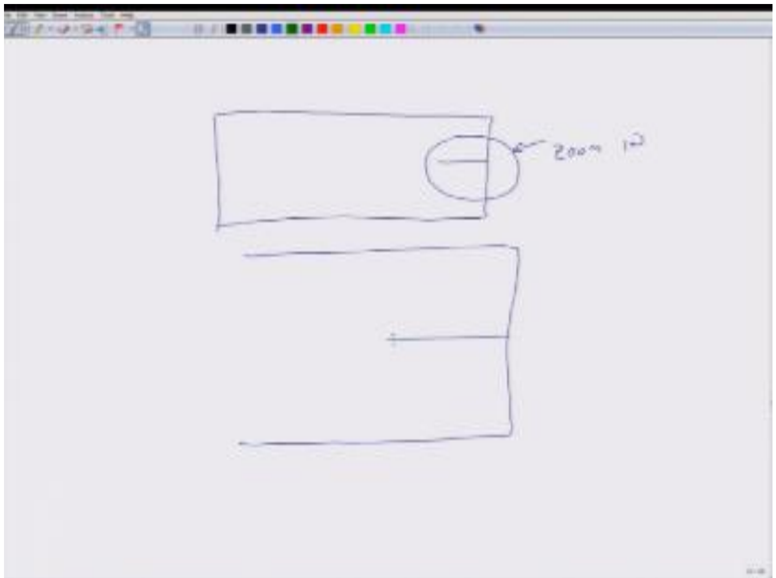

Let us say I have a piece of metal and there is a crack here okay, so this line represents a crack and when I am doing meshing let us say if I zoom in on this area then this is the crack, this is my metal right, now in reality there is I can have one point on this side of the crack and another point on this side of the crack, and in reality both the locations of both these points would be at the same physical location because crack is just a sharp boundary of zero thickness, it could be like that right, in such a case there is no reason to think, so suppose this is point p1.

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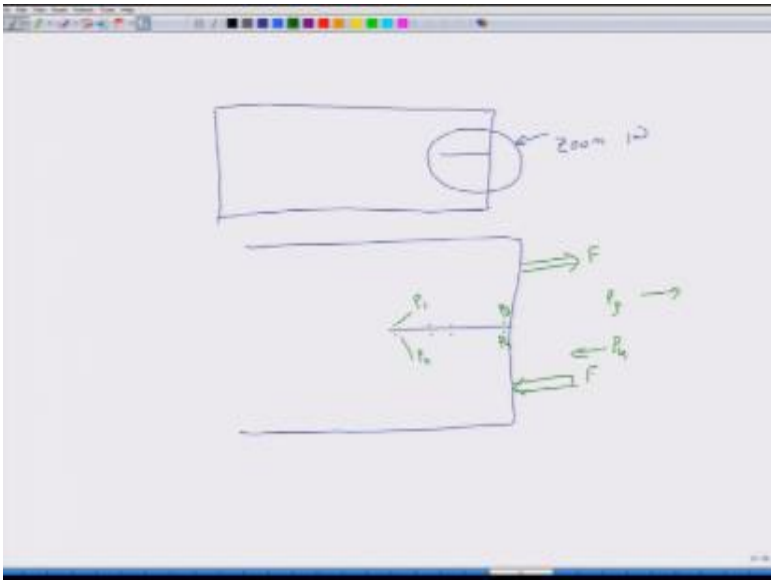

And this is point p2, similarly there could be lots of other points and in all these cases the adjacent points here I have separated them by a finite distance but in reality they may be sitting at the same location right. So let us say this is p3, this is p4, and in such a case when suppose I am applying a force here at this location and here I am pulling it and in this part I am pushing it, so what will happen in reality is that p3 will move in this direction and point p4.

Will move in this direction based on the physical understanding of the situation right, so you have to look at the physics of the situation and then in such a case when I am trying to do the assembly we will not this impose this condition like what we did.

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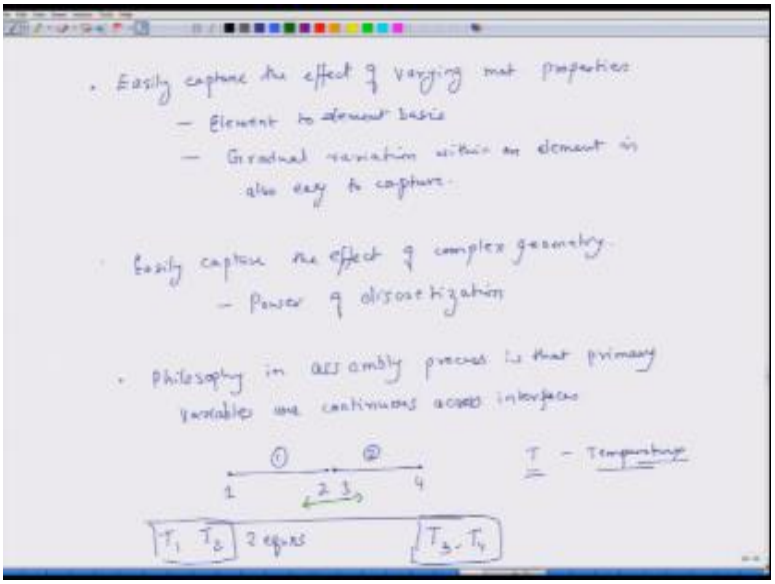

In this case where we had assumed that t2 is same as t3, this is known as imposing the condition of continuity of the primary variable, that the primary variable is continuous across the interface, it is continuous across the interface.

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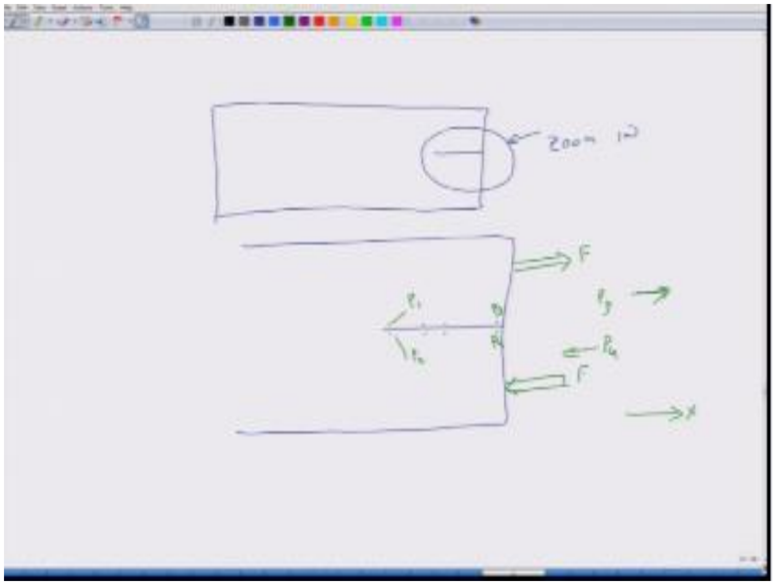

In this case p3 and p4 they are even though they are physically located at the physically same point point p3 I am, is most likely going to move in a positive x direction if this is my X direction and point p4is going to move in negative x direction and their velocities may be different right, the velocities maybe so they will, their velocities will be different, so in such a case we do not impose the condition for continuity of a primary variable even though the points here are both the points are at the same physical location okay.

So whenever when we impose the condition of community we have to look at the physics of the situation, most of the cases we have us the case where continuity of primary variable is actually a reality but in situations like.

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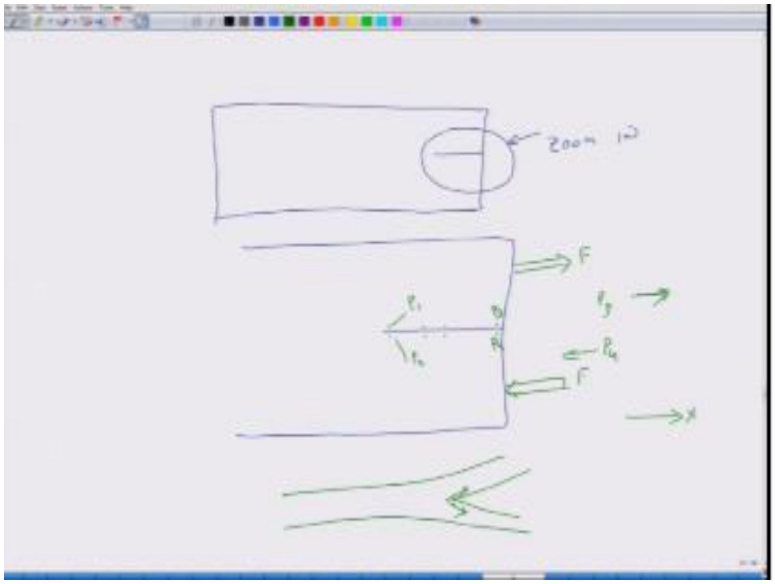

Fracture mechanics or for instance you have a flow and at some point one part of the flow is getting diverted in one direction, another part of the flow is getting diverted in the other direction so it is something like this, so particle which is here in some you know so the, so 1point excuse me, so what so, so, so they will be a discontinuous jump in velocity across this point of bifurcation okay, so these types, these types of situations we do not impose the condition for continuity.

We impose some other conditions but to FEA it does not matter whether situation is continuous or not continuous, if it is continuous.

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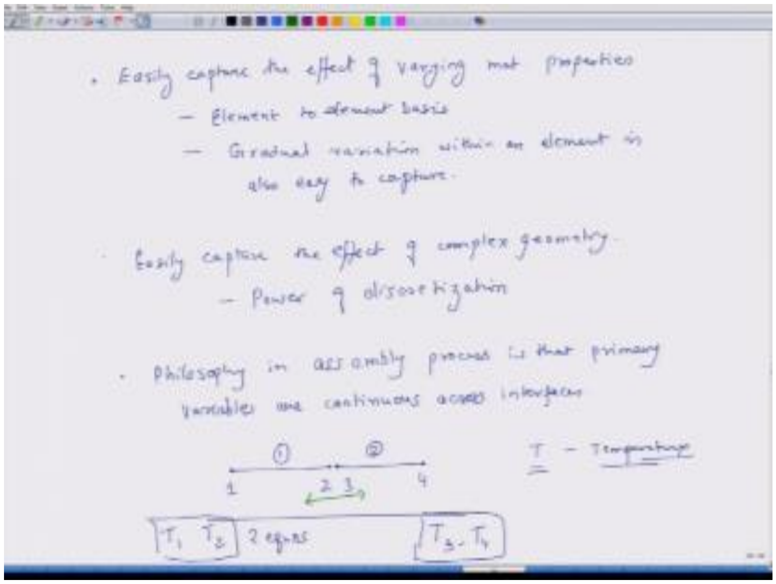

Then we impose this type of condition where across the interface primary variables are the same.

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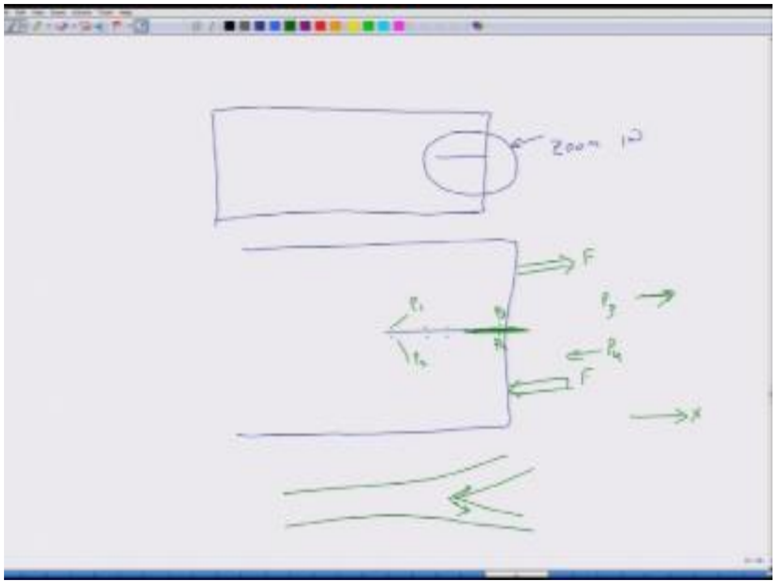

If it is not continuous then we do not do that. In that case we do something different for instance here if there is no friction on the interface boundary then we say that there is no shear force there is no shear force at the location p, p3 so that maybe the other condition so that is the extra condition we impose in case of fracture for this particular kind of problem.

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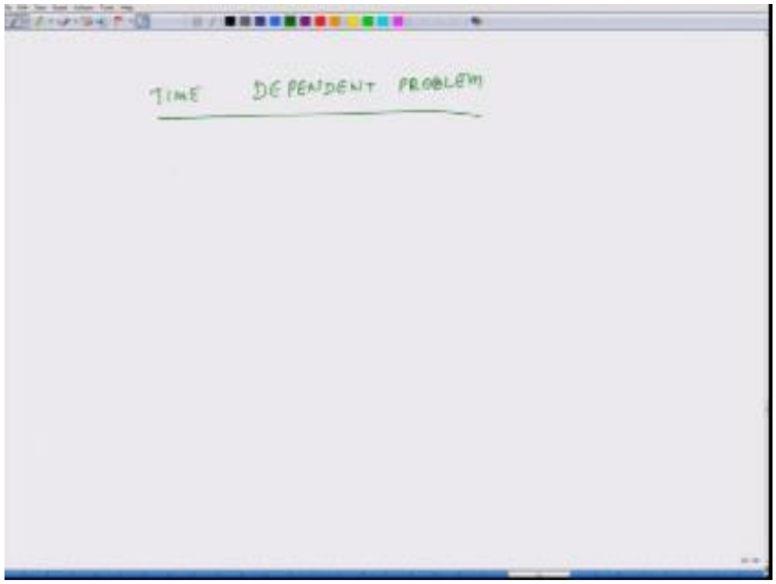

And the last thing I wanted to talk about is time dependent problem, independent problem, so in this case what typically we do is that when we try to find the derivatives of a prime of a variable of an unknown variable, primary variable in space suppose there is a time delay over del X then because we have assumed the function for u as a, as a function for X we can easily find its derivative but then we are trying to and that is how we discuss discretize the space, we discussed discretize the space in elements so we break the geometry into small, small chunks.

And we discretize space, but in time dependent problem we not only have to find solutions in space, for instance with respect to x or y or x y & z directions but we also have to find the solution at time t is equal to 0, t is equal to 1 second, t is equal to 2 seconds and so on and so forth. So in that case in a lot of situations the way we do it is that we take partial derivatives of time in terms of you know so let us say a partial derivative of u with respect to 2 is, with respect to T would be you at time T plus 1 minus u at time T divided by delta T. So we use in a lot of cases not always necessarily in a lot of cases finite-difference approach to discretize time.

And we use finite element approach to discretize space, and we combine these two and solve time related problems. So these are some of the important things I wanted to mention in today's lecture and I hope what you have learned over this week is the basic philosophy, so of how finite element method works and as you have gotten exposed to several of these concepts some of the mathematical details we will work out in the coming week and the week after that, but the overall thrust of this week's lectures has been to help you understand the overall basic philosophy of how the finite element method works and can solve a very large type of problems in, in an automated on in an automated way and with a lot of reliability, so thank you very much and I look forward to seeing you in the next week, bye.

Acknowledgement
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