Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course title
Basics of Finite Element Analysis

Lecture -05
Types of Errors in FEA, Overall FEA Process & Convergence

by
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Hello, welcome to basics of FEA, in the last lecture which was yesterday, we had discussed the overall process of doing finite element analysis, and what we had explained was that we start with the governing equation and the first step what we do is we break the domain into small sub domains each domain being known as an element, and then for each of these sub domains we develop element level equations which are developed through use of differential governing equations.

And then we do the assembly process, and then finally we impose the boundary conditions to get the final set of equations which are having a certain number of unknowns, and the number of equations is also same as that number. And then when we solve these linear algebraic equations we are able to get the solution for the overall problem.

So now what I wanted to explain is continuing that discussion further is that what kind of errors could happen in this entire process, what are the sources of errors?

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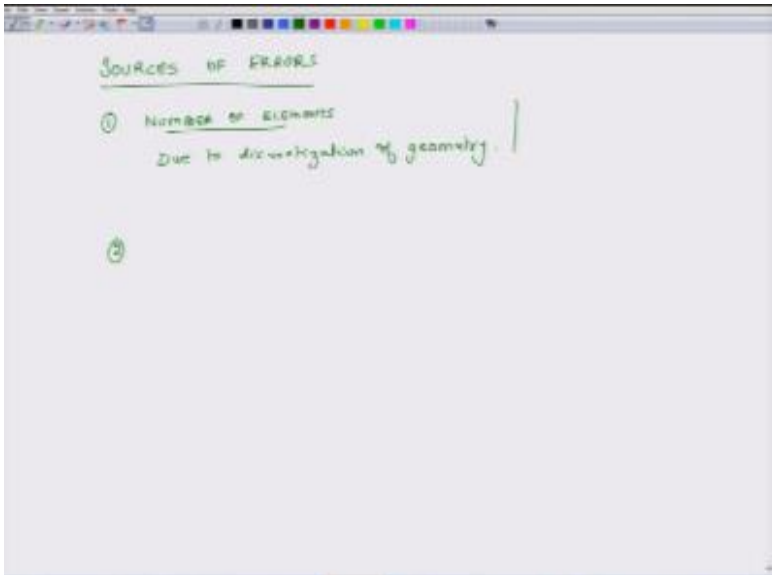

Okay, so if you go back to the first example when we started discussing the philosophy of finite element method and we have -- what we had done was that we try to calculate the area under a particular complicated curve, we saw that the first source of error is driven by number of elements okay. If this number is more the error is smaller, as this number becomes larger and larger the error goes down okay.

So in other words we can say that errors are introduced due to discretization of geometry, discretization of geometry. If the discretization is extremely fine, then error will be extremely small, if the discretization is large, then the error will be large or appreciable okay. So this is the first source of error, discretization error. The second source of error is driven by we have seen by the type of interpolation functions we choose okay.

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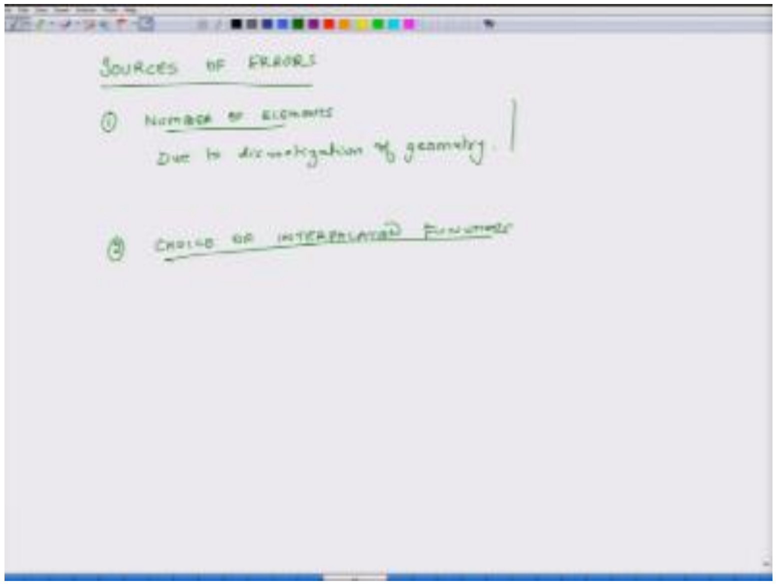

So choice of interpolation functions, in general we saw in context of the example which we discussed earlier, that if we have a higher order interpolation function then the error tends to be less, if the order of interpolation function is low then error tends to be more okay.

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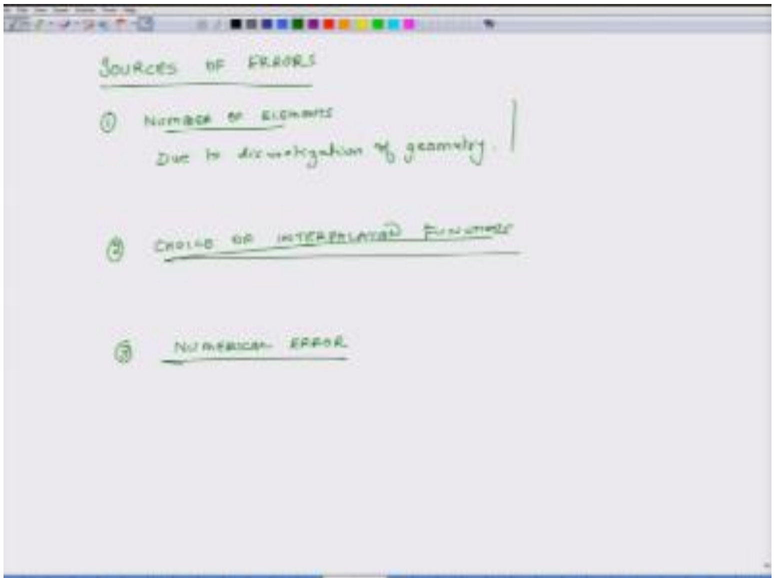

So this is due to choice of interpolation functions. And these are the two sources of errors which we had discussed earlier, but there is a third choice source also, third source of error is numerical error okay. Numerical error, so what does it mean, so in this whole process we are doing a very large number of calculations, we are multiplying and dividing and taking squares and taking cubes, a very large number of times.

And then when we do inversion of matrices, we do millions and millions of operations okay. So if in each operation, suppose if in each operation if the number of digits to which we try to compute the accuracy of the operation is not sufficiently large then in each operation these errors tend to add up and slowly become significant in the sense that they become visible and appreciable.

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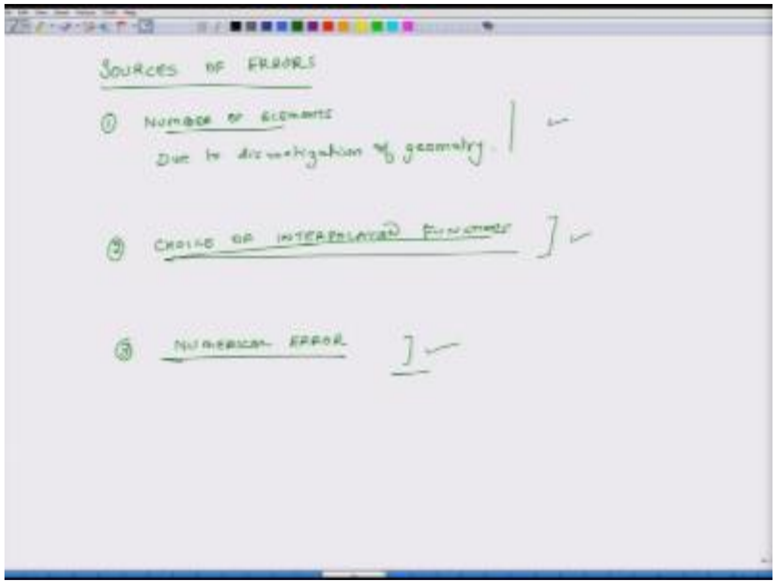

So as we are trying to ensure accuracy of a finite element procedure, we have to make sure that we have a reasonably large number of elements, our choice of interpolation functions is appropriate, and also numerical errors are managed by making computations to large number of places of decimal, that is that is one straight way of ensuring that errors remain moderate.

So that is one thing, in context of this I wanted to say of couple of other things also. So what we have seen is that you have.

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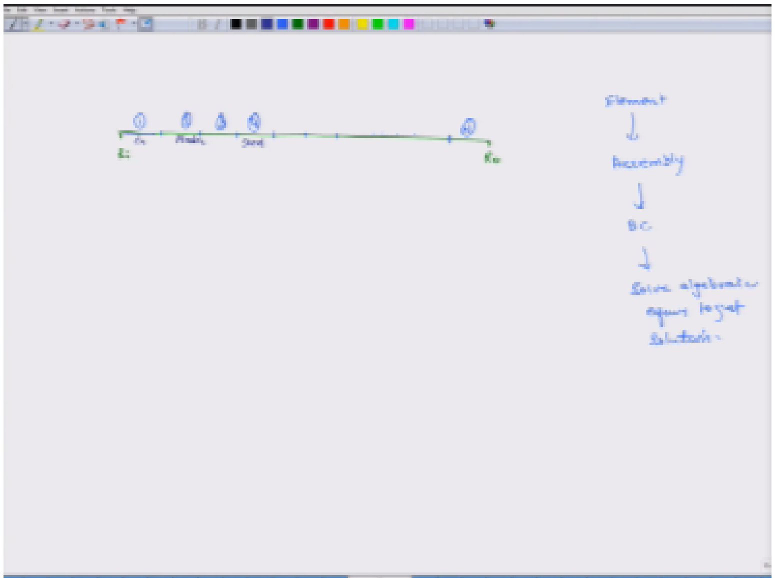

Let us say in this case, this was your domain r_i to r_o right. And there are lots of elements, so this was element 1, this was element 2, this was element 3, this was element 4 to the n^{th} element. And what we had explained was that first we develop element level equations, then we do assembly, and then we do boundary conditions, algebraic equations to get solution.

In this process is a very powerful feature of finite element method, and what is that feature, that it could be possible that element 1 is made up of copper, and element 2 is made up of – let us say some plastic, and element 4 is made up of steel, okay. This could be a real situation based on how we discretize it, it could be steel. If we had to use a traditional way of getting you know analytical exact forms.

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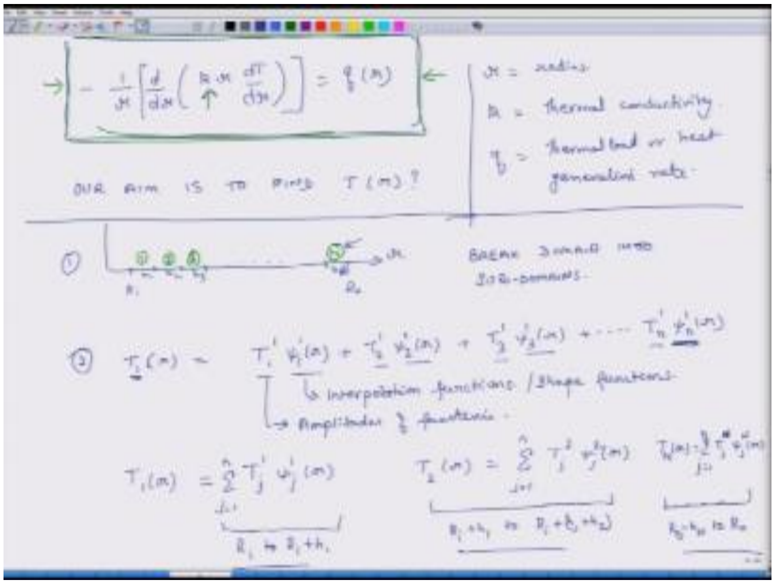

So inherent in this method is the fact that it can handle variation of material properties very easily, if we use traditional way to solve this problem then we will have to integrate the differential equation and while we are integrating the differential equation we cannot assume that k is a constant, because k is varying. In this case we have defined it in such a way that k is jumping, maybe initially it is 1, then after some distance it becomes 2, then after some distance at all of a sudden becomes 3, so it is jumpy.

And these types of jumps cannot be easily handled by these differential equations in a continuous, because everything is continuous there. In other situations the material properties could be changing for instance, you can have hardened steel where hardness on the outside surface is very high and hardness on the interior of the steel is very low, and it is slowly changing.

And again there also the material property is changing with position. So that makes these -- those things equation nonlinear, and it becomes extremely hard to develop exact solutions for those kinds of problems.

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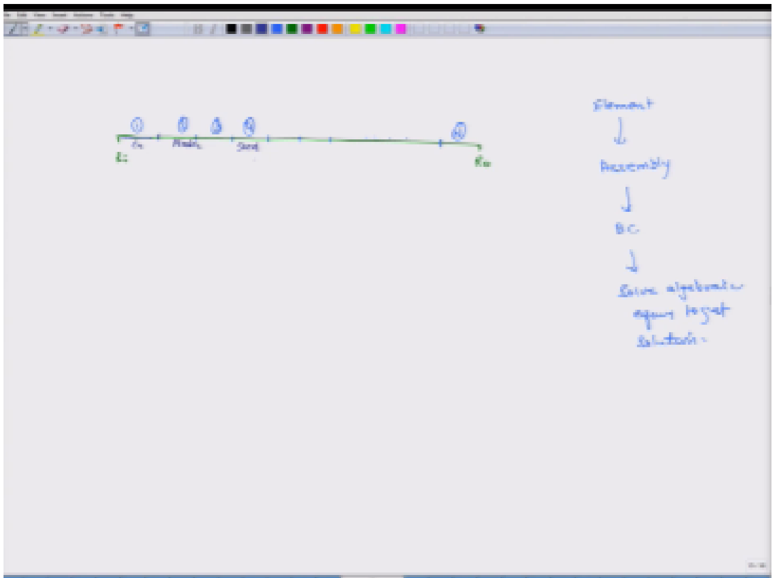

But then you have element level formulation you can avoid these types of problems, because if you have very jumpy material properties for instance, you have core of copper, outside you have plastic, then outside you have steel, and so on and so forth, what you can do is, you can make the first element or some number of elements such that the boundary of the last element terminates with copper.

So when you are developing element level equations for this you will put in the value of k as that of copper.

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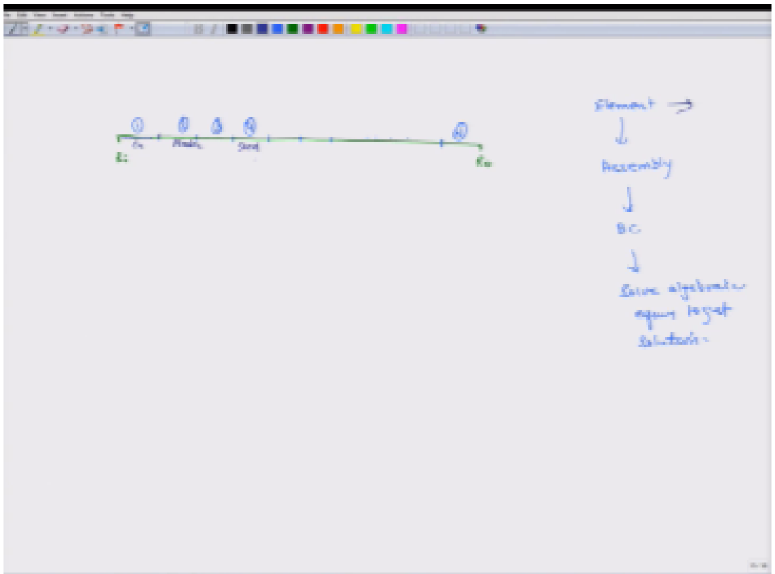

So first five elements you are doing element level equations such that the value of k is that of copper. Then the next three elements if they are of plastic you use plastic material, then maybe the next four elements are made of steel, then you use the steels property okay. So you can specify this element by element material properties and you can have very complex material parameters which cannot be handled by traditional, you know solution methods.

But finite element method is able to handle significantly large changes in material properties; because all you are doing is you are computing element level equations. That is nowhere, and then when you are doing assembly it does not matter all you are doing is adding some constants here and there and coefficients of different things.

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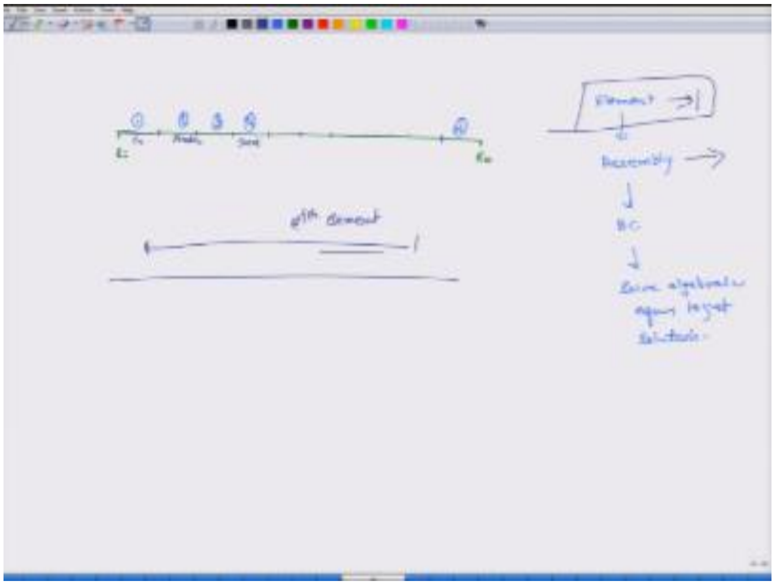

So at assembly level you do not worry about that, at element level you compute element by element, you know the terms of that matrix, and there you handle the variation of material properties relatively easily. Also if within a material, if within an element suppose this is one element, e^{th} element; let us say the material property is slowly changing, slowly changing.

Then what you do is, you can -- like you have a shape function for displacement you can also say that k is a function of x. And you can assume some function polynomial function for that also, and integrating polynomial functions is easy. So again you take that and integrate it, and again you will easily get the element for -- so material property does not even have to be constant over an element it can be varying over an element itself.

And you can integrate it easily, because all you are doing is integrating polynomials at least in context of finite elements. So this makes it very powerful, this makes it x2, this feature of FEA makes it a very powerful tool that it can handle very complex assemblies, complex material properties, graded material properties, and so on and so forth.

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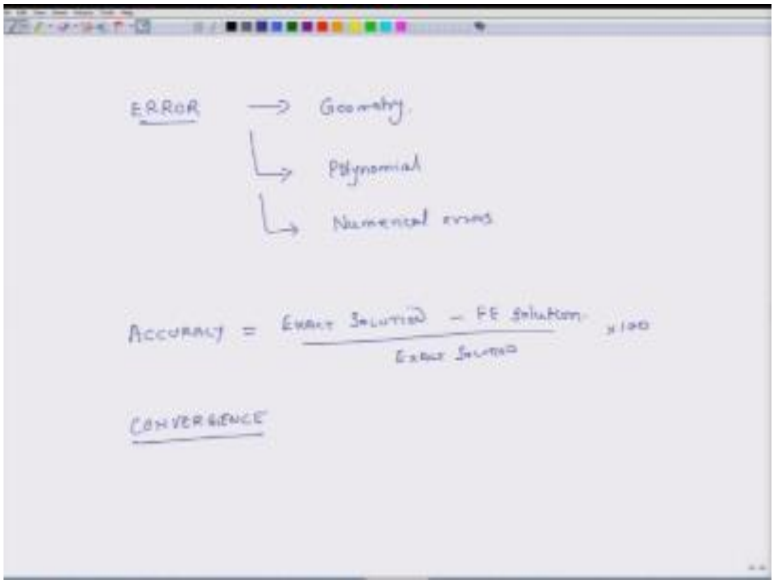

There are couple of other points I wanted to make, so I had mentioned that the error it is driven by three parameters how accurately we capture geometry right. And that is driven by the size of the element; the second is how accurately we capture the variation of a function on a thing, so that is polynomial order, and the third one is numerical errors. Now in context of finite element method we define accuracy is the difference between exact solution minus FE solution.

Where if we have to see, so this is the absolute value of accuracy if we have to calculate it in percentage terms then we divide it by exact solution times hundred okay, there is anything that is good to know. And finally there is a term in finite element method known as convergence. And it is very often used and the question often asked is that is your solution converged.

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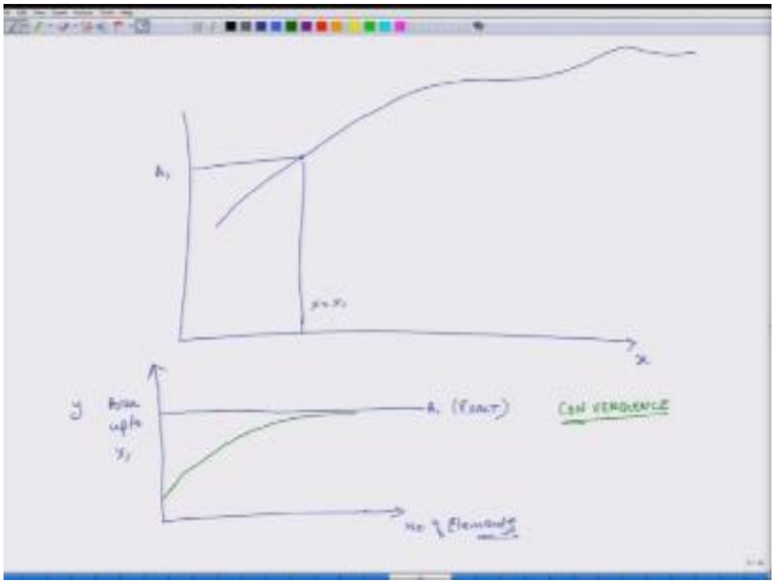

So what is convergence, so again we go back to the example of area under the curve, and we see, suppose the area under the curve as x is increasing, maybe the area under the, area under the curve will also increase, it will do something like this right? And suppose I want to calculate area under the curve up to this point, so this is x = x1. So if I plot it on this thing, so y is area up to x1 and on the x-axis I am plotting let us say number of elements.

So exact solution will be, let us say this exact solution is even. So exact solution is even this is exact, that does not change whether my element size is 1 or 100 it is whatever it is right. What happens to the finite element solution, when my number of elements is very small error is large and slowly I start approaching this even line okay. So as my number of elements increases I start converging to the exact solution. This phenomena is known as convergence.

For linear problems I never cross this line okay. So I -- my solution of primary variable temperature or displacement, it will always be below the exact solution okay. The primary variable temperature and why is it below, we will see that later in several classes, but I will never -- if I am trying to compute displacement in a beam and I am putting a force here, if I have one element only the beam will appear very stiff and it will deflect maybe only a very little amount, while the real beam may deflect a lot.

If I put two elements it becomes a little more flexible and it deflects more, but it does not double. If I put one million elements it becomes extremely close to the exact solution, but it does not become more than that, it never crosses that line okay, this will never crosses that line. So in a set of -- in a specific set of in a class of problems which are linear and conservative systems, and why this is conserve – what is the meaning of conservative and linear we will just discuss that later.

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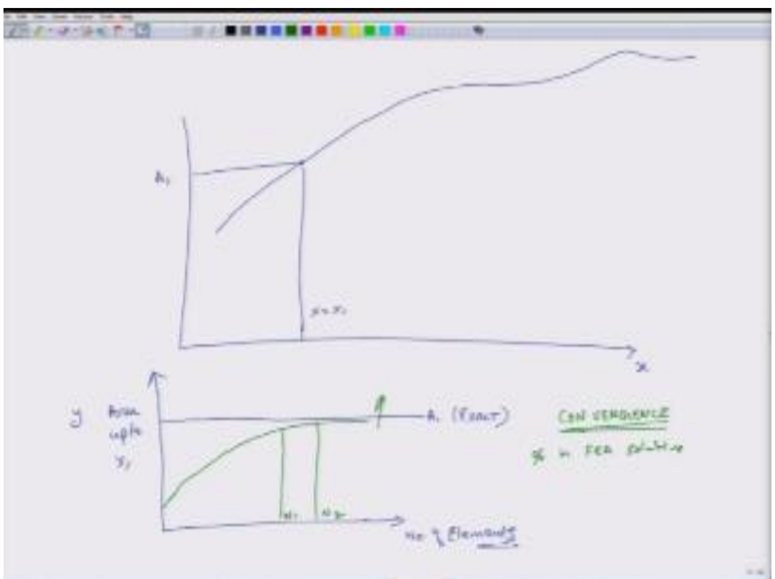

We do not cross this line, in some special problem we may, but we do not cross this line. So this is called convergence, so if I have to see whether my solution is accurate I have to see okay, here number of elements was n1, here number of elements was n2. What was the percentage change in FEA solution? If it changed by maybe twenty percent have I converged, I have not converged.

If it changed by maybe one percent and if I feel comfortable by that one percent, I will say I am fine with that solution. If it changes 0.01 percent I am probably perfectly with that kind of solution okay. So this is what is known as convergence. So and convergence is always related to how many number of elements we have used, as number of elements increase we approach the actual solution. So this closes the lecture for today and we will continue this discussion tomorrow, thank you very much.

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