Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 48
Explicit and implicit method, diagonalization of mass matrix, closer

by
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Hello, welcome to basics of FEA, today is the last day of this course and in today's lecture we will cover two themes, explicit schemes and implicit schemes and then after discussing these two particular topics we will close our discussion for this course.

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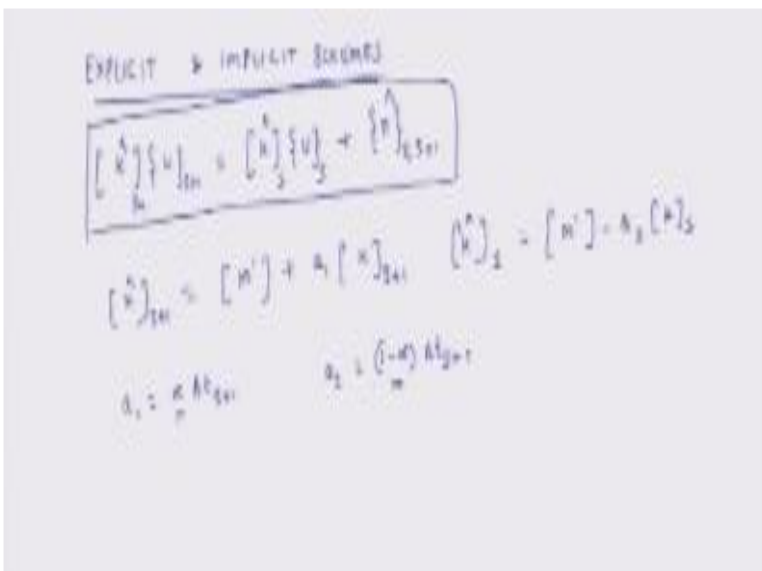

So what we are going to discuss is, and implicit schemes. And we will explain it by an example so that things will become clearer. So we will consider the example of the parabolic system of equations. So our parabolic equation we had defined as it was expressed like this. So this equals okay. So this is the equation we had developed about three, four lectures back in time and here we had explained that K^ evaluated as (s+1) time step equals m1 + a1 [k] at s+1.

And k ^ at sth time step is equal to m1-a2 [k] at s and similarly we had, so this is our basic differential equation not differential the algebraic set of equations F was also defined earlier. Now A1 was defined as alpha times Δt(s+1) and a2 was defined as 1-α times Δt time at s+1. Now this alpha relates to the alpha family of approximations which we had discussed earlier.

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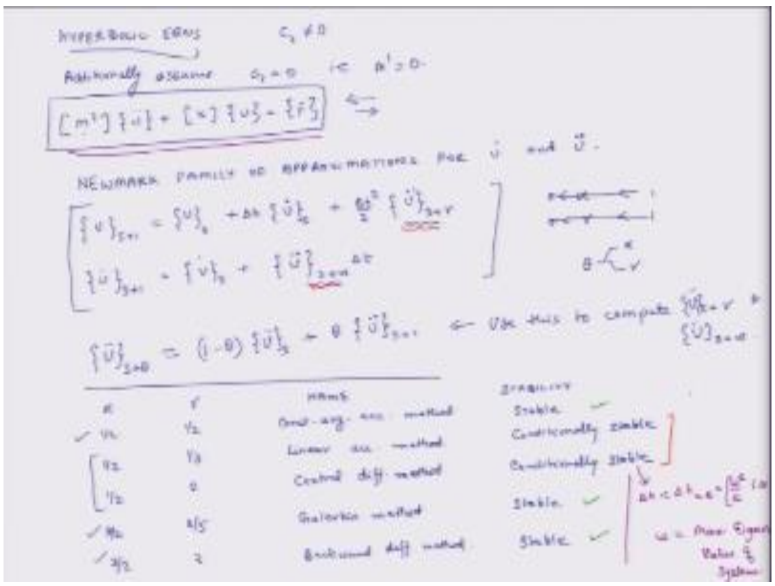

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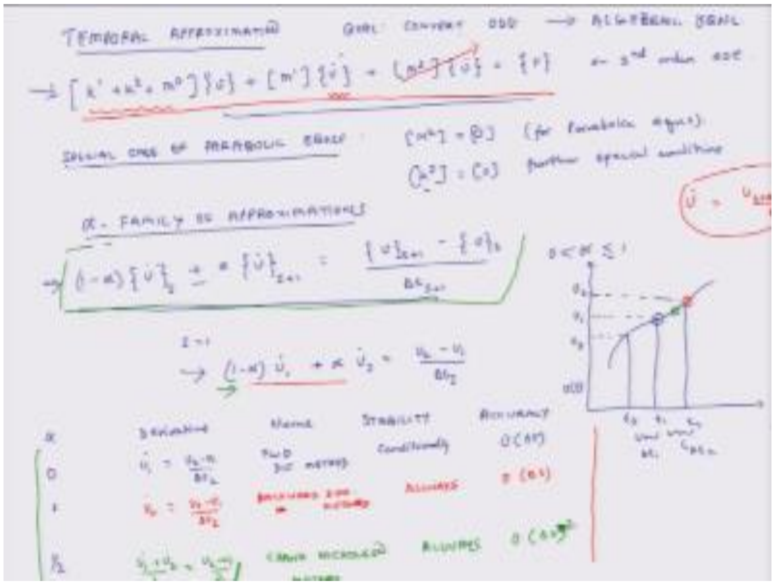

And there were several choices of alpha’s we could make which you see here. So alpha could be zero for forward difference method it could be one for backward difference method, it could be half for Crank Nicolson method, we had also said that it could be two thirds for Galerkin method. So we could choose one of these values of alpha and solve the equation which we have discussed earlier.

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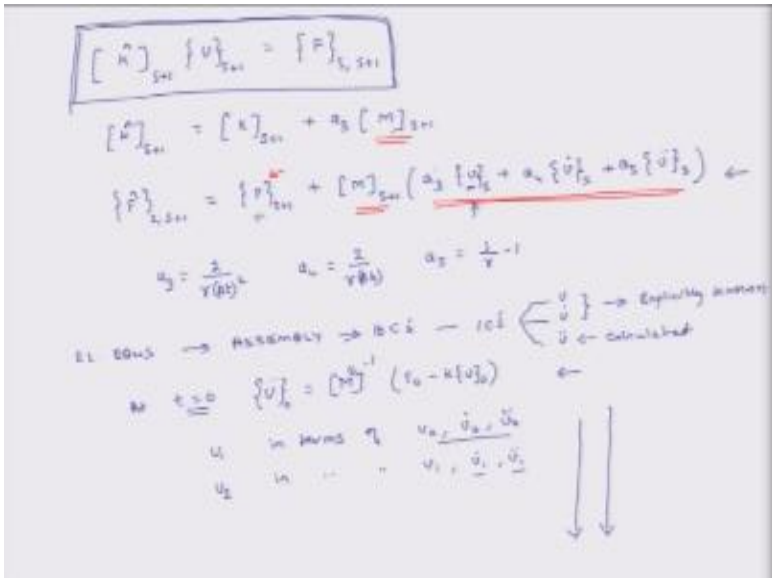

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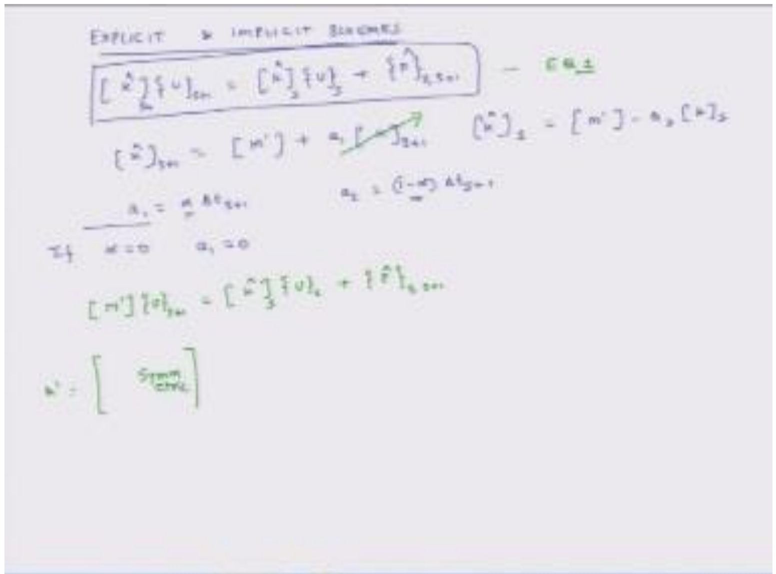

Now consider what happens if α = 0. So if α = 0 then a1 = 0 which means that in the k^ this term goes away because a1 is zero. So then this equation, so this is equation one it can be written as m1 times u(s+1) equals k^, k^ times u at s time step plus f^ s plus 1th time step.

That here the left side of the equation the k^ matrix becomes essentially m1 matrix and it does not change with time, it does not change with time, okay. Now in general, in general my mass matrix not in general my mass matrix is always a symmetric. This mass matrix, why is it symmetric? Because when I see its definition of m1.

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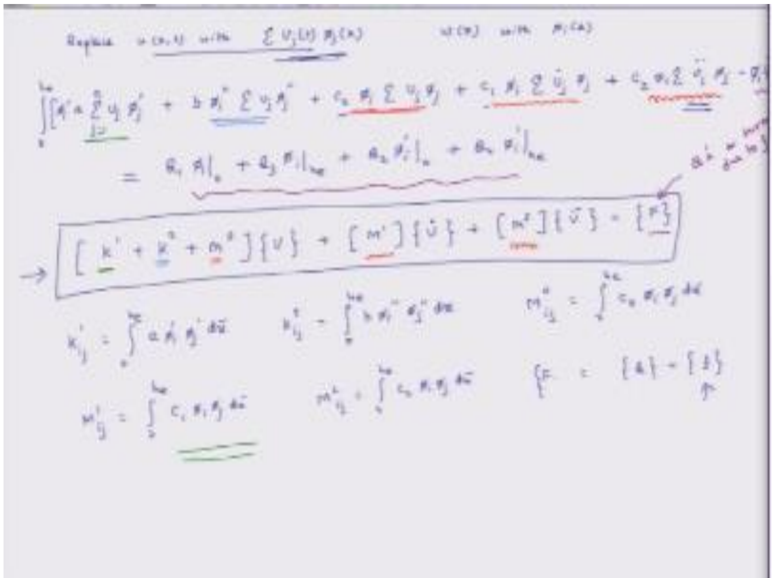

Then m1 corresponds to this function. So it is Ø1 Øi times Øj. So m1 is always symmetric matrix.

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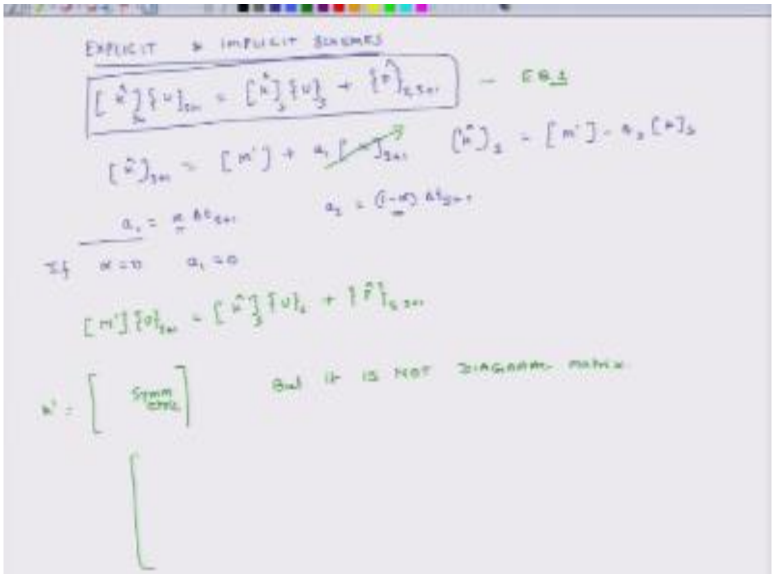

But it is not so this m1 is but it is not, it is not diagonal matrix. It is symmetric but not diagonal, if we can somehow make this matrix diagonal, then what will happen? What we will be able to do is, that we will have non zero terms here, suppose the matrix is 3/3 or 4/4 suppose it is four-by-four then what will happen? So I am not saying the m1 is diagonal, suppose we are somehow able to modify m1 into a diagonal mass matrix.

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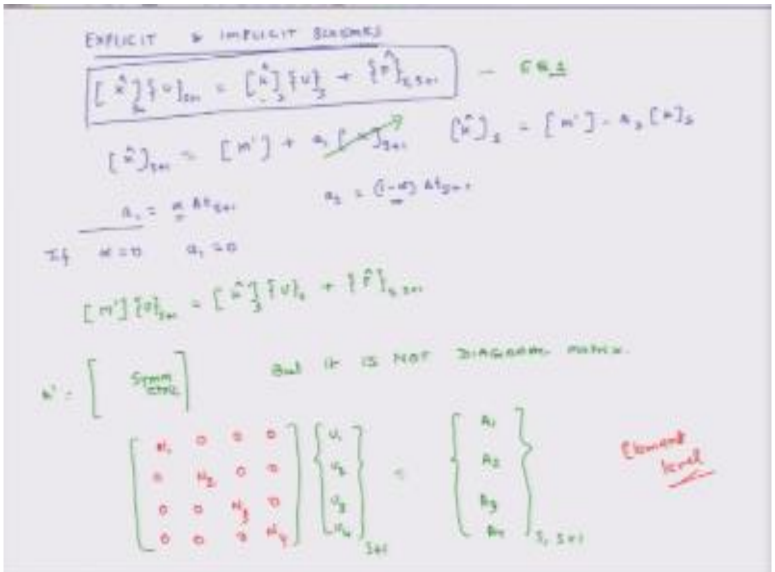

And let us say the diagonal mass matrix is a four-by-four term. So this term is not zero so all these are zeroes. But the diagonal term, all the terms in this are zero except for the diagonal terms. So the diagonal term is a non-zero number, this is a non-zero number, this is a non zero number, this is a non-zero number, okay. So basically I am rewriting equation one with a diagonal mass matrix.

Which is not same as m1 and I have not explained how I am going to make it diagonal but somehow by magic I am able to make it diagonal then this is multiplied by u1, u2, u3, u4, let us say these are four nodes of an element, these are evaluated at time step (s +1) and this equals this k ^ times US times F, so essentially this will be some vector, right? Which is a combination of s and s+1 time step, right? Associated with s and s+1 time step.

So this is some number let us call it a1, a2, a3, a4 and these numbers are n1 non zero numbers. So n1, n2, n3, n4, okay, so this is at an element level. Now when I assemble all these element level matrices I get the assembly level then I do boundary condition I apply initial condition do all that good stuff but essentially what I will be left with is.

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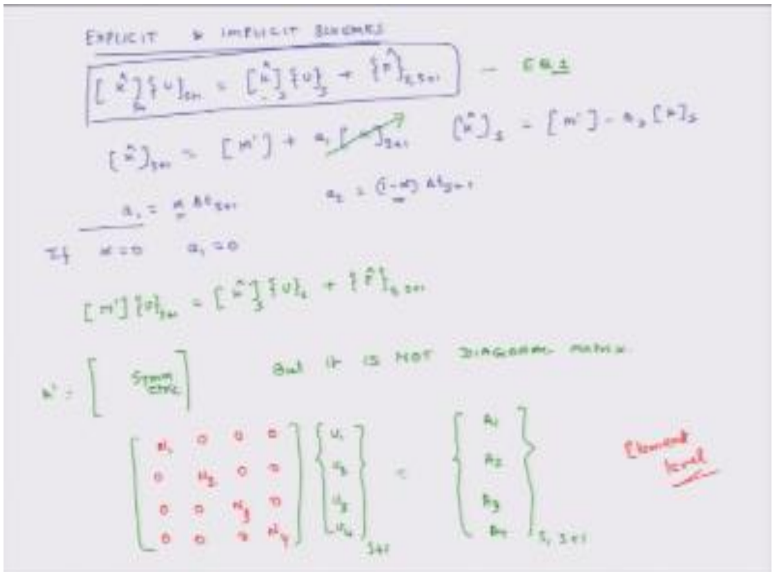

A set of equations of n where this matrix, the mass matrix on the, the assembled mass matrix on the left side will be diagonal. So I do not have to invert this assembled mass matrix to solve for u1, u2, u3, u4 because what is the first equation? N1 times u1 equals a1. So I can directly calculate u and s, a1/n1 understood? So if so what does that mean? That if I choose alpha to be zero and what that means is that my k^ matrix becomes same as m1.

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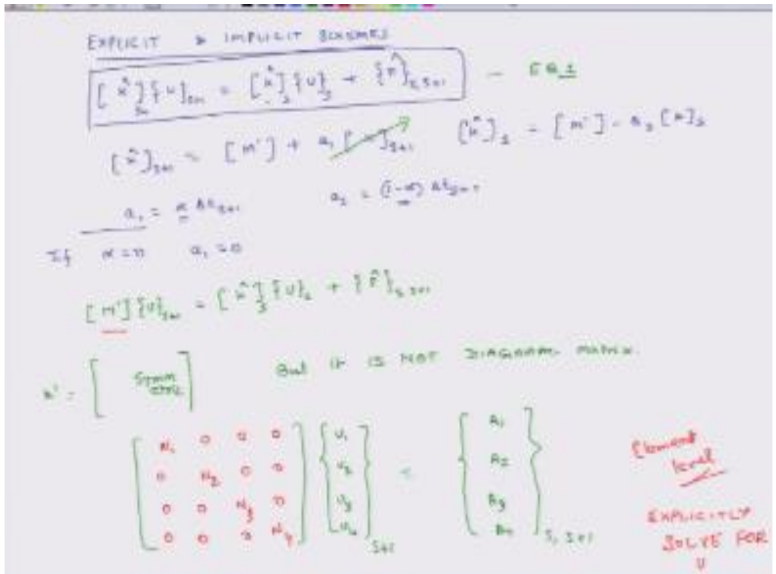

And then if I do some special trick to make my mass matrix as diagonal then I can explicitly solve for u's, I can explicitly solve for u and that is a much faster way because to invert a matrix is much more numerically computationally intensive but I can explicitly solve for u and that is a very fast method of solving the problem, okay. So this method where we choose alpha to be zero and somehow diagonalise the matrix is known as explicit method of solving the problem, a time dependent problem so this description is valid for time-dependent problems.

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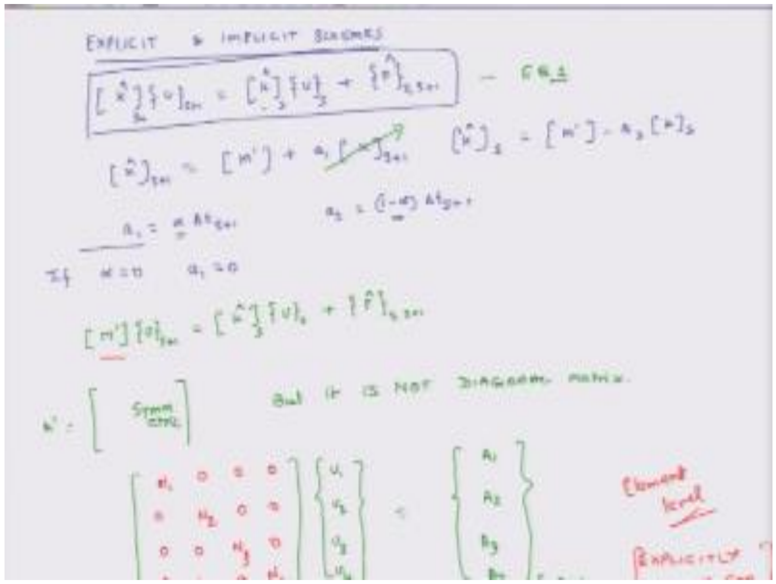

By default we do not do that, so that default method is known as the implicit method it is an implicit scheme for solving the problem, okay. But in this case this is an explicit method where I am not inverting the mass matrix or the stiffness matrix and I directly solve explicitly solve for the unknowns which I use. So this is known as explicit method. So that is what I mean by explicit scheme and implicit scheme.

In implicit scheme we have to invert we have to invert because the matrix on the left side is not diagonal, in explicit schemes we do not need to do that and we are able to get to that situation by making two things happen, first we choose alpha to be zero and second one is we diagonalize the mass matrix.

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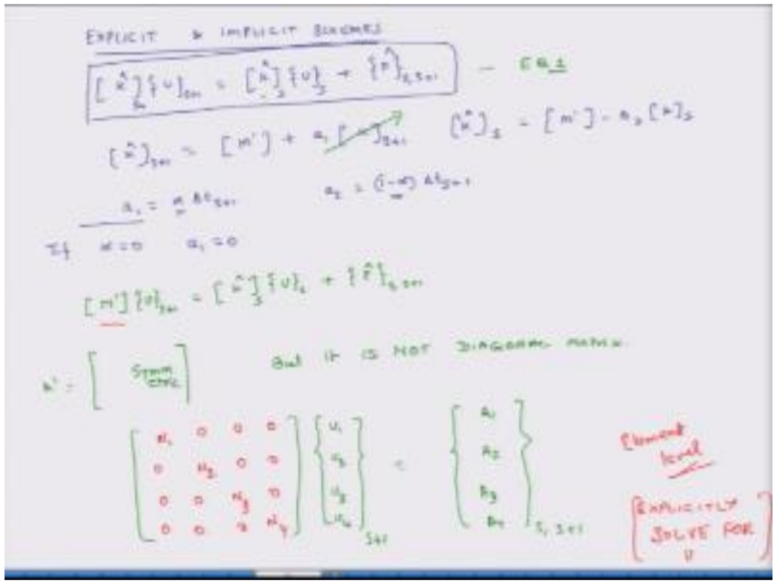

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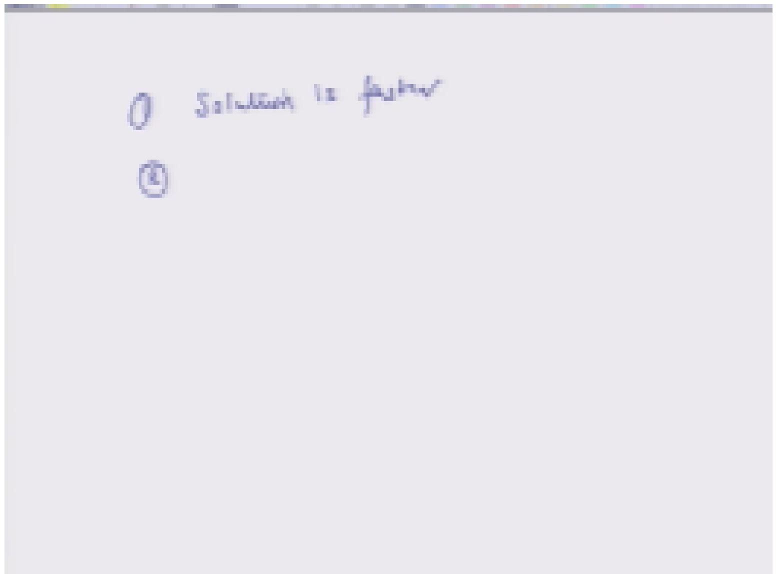

Now the advantage of explicit scheme is one is that solution is faster, okay. Solution is faster that is one advantage, the second thing is that when you diagonalise the mass matrix it just happens that the value of t critical.

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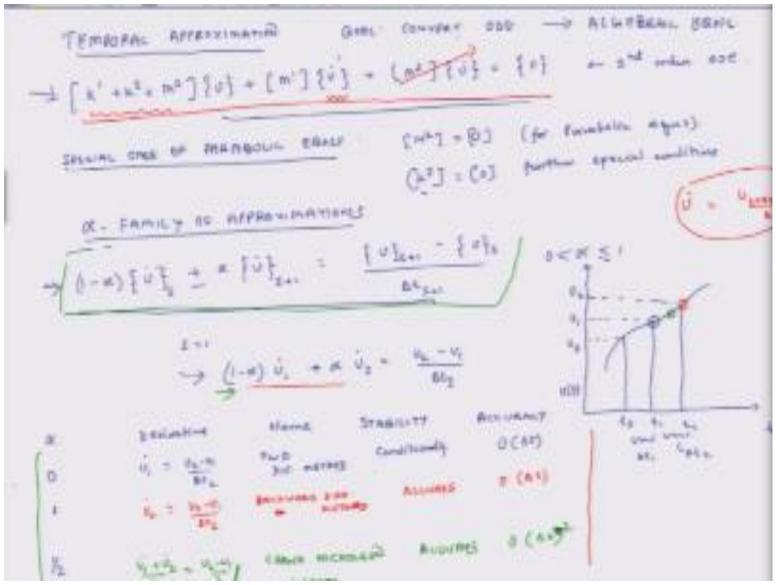

Now if you remember the forward scheme is conditionally stable, okay. Forward scheme is conditionally stable, now that conditional stability means that my take time increment has to be smaller than the critical value.

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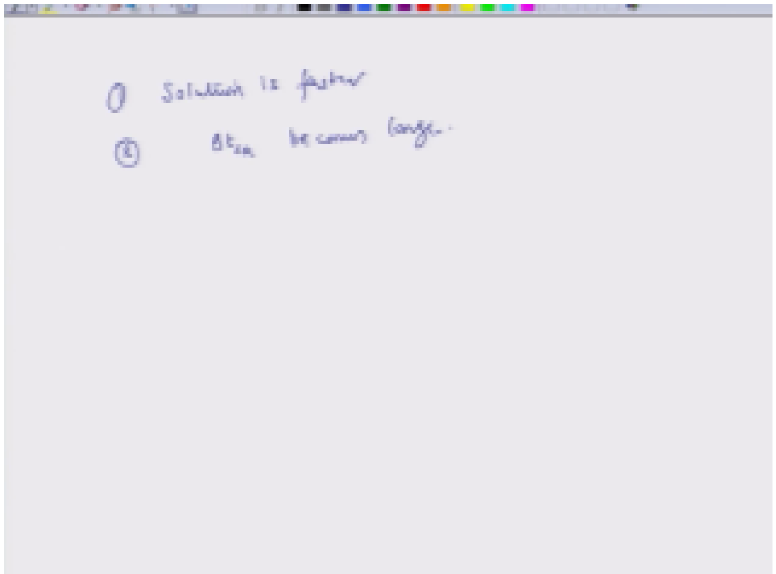

It just happens that when you diagonalise the mass matrix the value of t critical becomes large. It becomes large. So I can jump from 1 time to second time to third time in larger steps. So I get two benefits one is solution is faster because my solution process is explicit.

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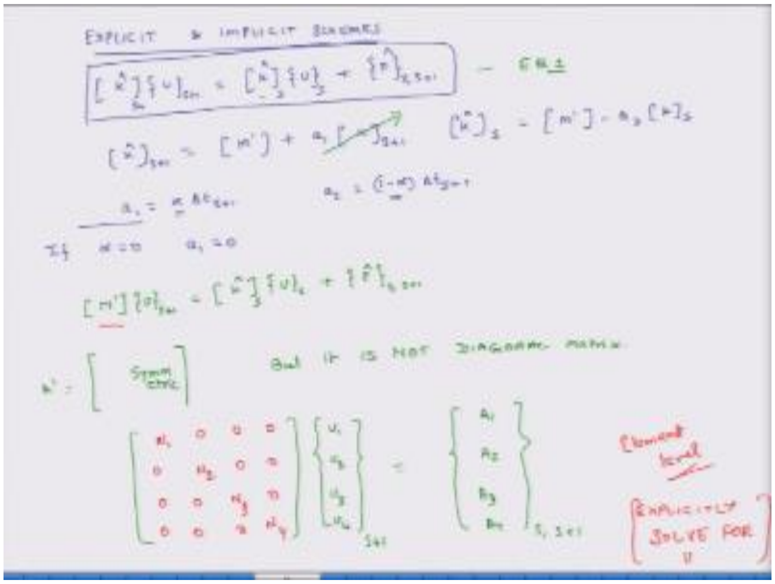

I do not have to invert the matrix.

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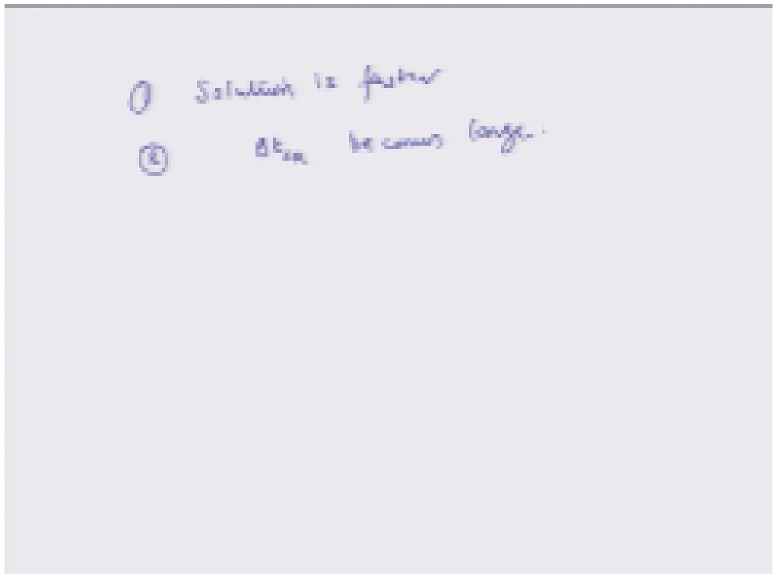

The second one is t critical becomes large so I can take larger time increments and that also gives me another advantage. So because of these two factors explicit schemes are in general faster ways of solving the problem but they may not be applicable in all sorts of situations in special situations we can use the advantage of this thing, that is what I wanted to talk about explicit and implicit schemes so the next thing in this context is we will learn how to diagonalize, we will learn how to diagonalize the mass matrix okay.

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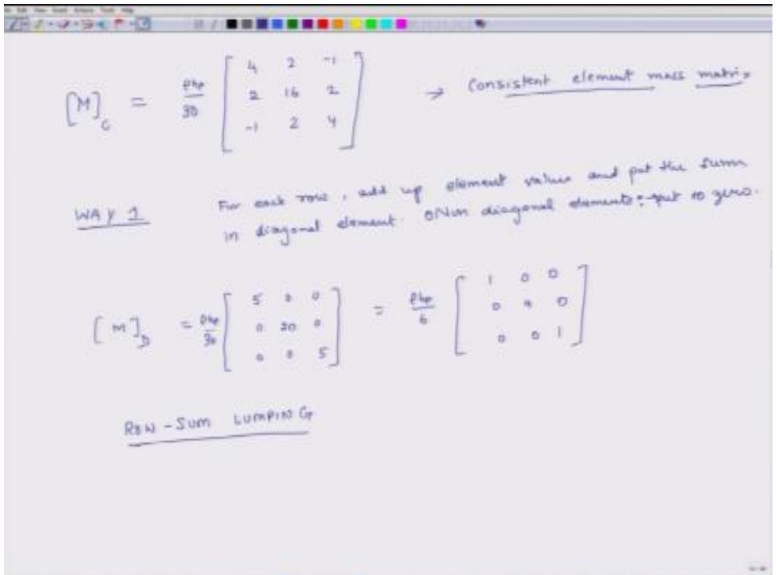

We will so suppose my original mass metrics at let us ay original mass metrics is a 3/3 matrix suppose it is a 3 noded element so let us say and the values are row h_e over 30 and the internal values are 4 2- 1 2 16 2 -1 2 4 okay. Now this is the original mass matrix which I have calculated from the relations which we discussed earlier using the weak formulation. So this is known as consistent mass matrix, actually consistent element metrics because the way we calculated these numbers is consistent with the original theory which we have developed.

But now we will do some approximations and we will try to diagonalize it, there are two ways to diagonalize way 1 okay, what you do is you add up so for each row add up element values and put the sum in diagonal element, other non diagonal elements, values are put to 0 so for non diagonal elements you put the value to 0 so what do we do, so this is consistent so I am putting this subscript C so, so what we do is for each row we add up the element values and we put the sum of those values in the diagonal element and all other non diagonal elements are set to 0.

So this is my consistent mass matrix M_C and my diagonal matrix and I will call it M_D and some people call it M_L as because we are lumping all the elements in one position so M_L or M_D and this is equal to row h_e over 30 and what is the sum of all the elements in the first row it is 4+2 + -

1 so it is 5, so that is the diagonal element value and other 2 values are 0, the second row if I add up all the elements it is 2 + 16 + 2 so that comes to 20 so that goes in the second diagonal element and other values are 0.

In the third row the sum is -1+2+4 so it is 5 and the non diagonal elements are 0 okay, so this I can simplify it as row h^e over 6 1000, 4100 excuse me this should be 0 1 this way of diagonalizing the mass matrix is known as row sum lumping, is known as row sum lumping because we are summing up all the row elements and are lumping it into one particular position of the diagonal, one diagonal element okay so we can lump it in this way.

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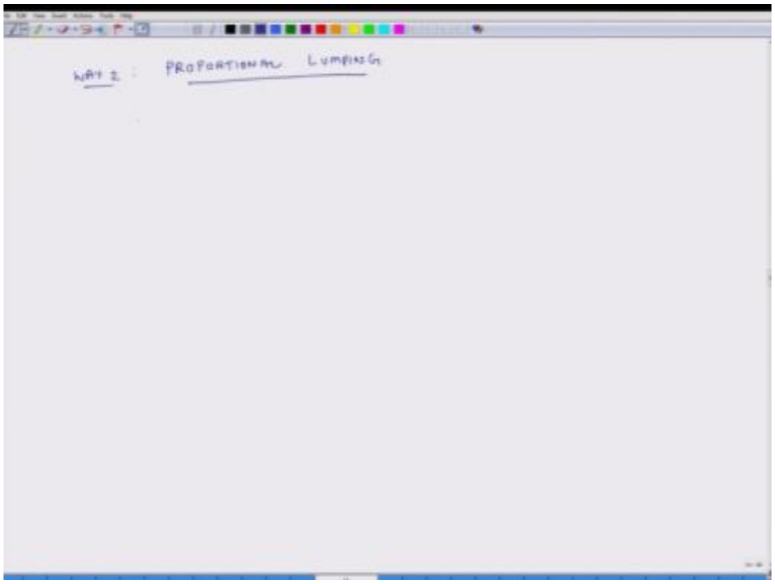

Now the other way to do it is, so second way and this is known as proportional lumping okay so in proportional lumping

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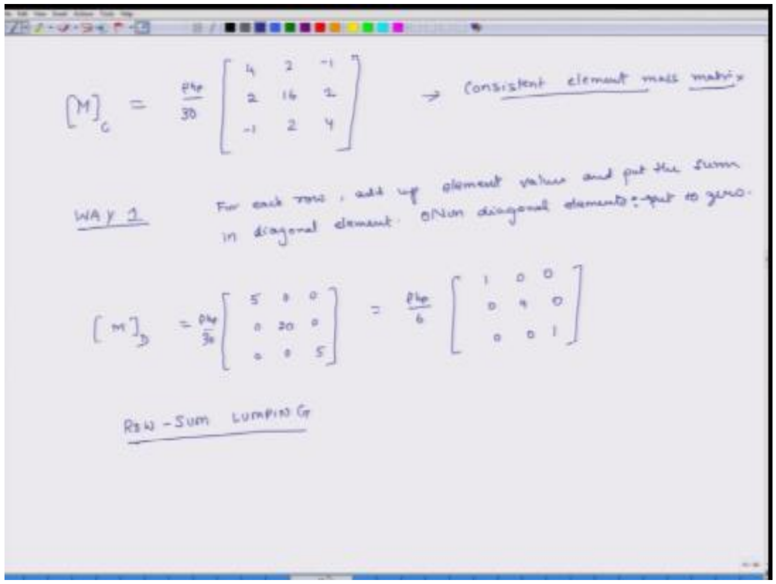

Our original mass matrix.

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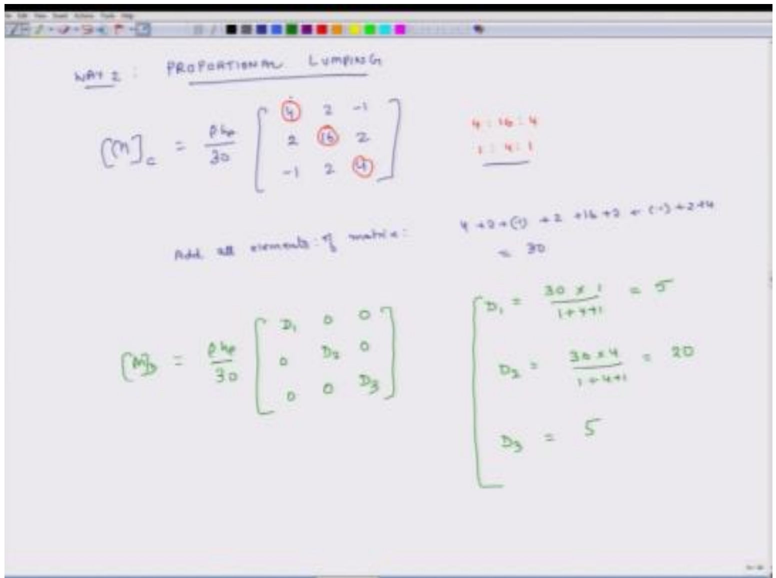

The non diagonalized version which is M_C was row h^e over 30,4 2-1216 2-1 2 and 4 okay so what we see is we look at the diagonal elements and we see that they are in the ratio is 4:16 is to 4 or you can call it 1:4:1. So what I do is first add all elements of the matrix, so what do I get 4+2+ - 1 +2+16+2+- 1+2+4 so that equals 30, this equals 30 and now I split it amongst the diagonal elements in this proportion so let us say this I call this so M diagonal is equal to row h^e over 30 and I have to figure out what are the values of D_1, D_2 and D_3.

So $D_{1\,=}$ 30 x 1/1+4+1 D3= 3= x4/1+4+1 so this equals what is it 5 and this equals 20 and D_3= 5, so this is known as proportional lumping approach it just turns out that.

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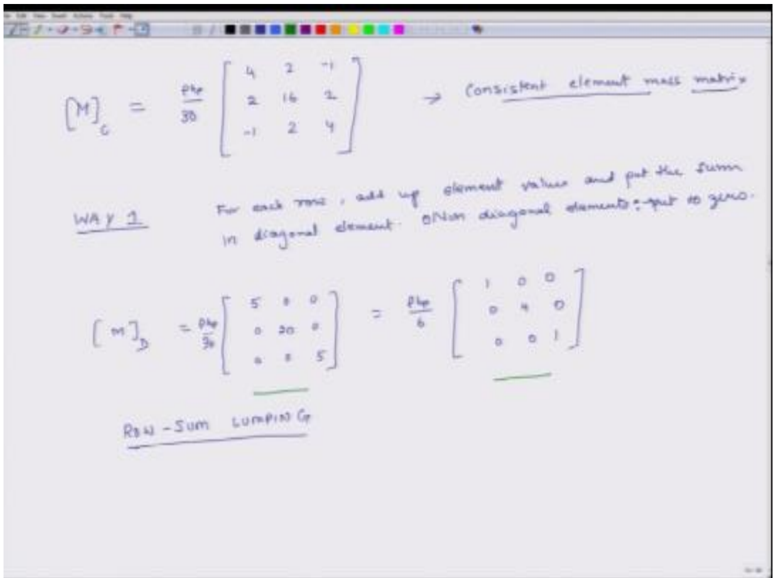

In this case the proportional lumping and the row sum lumping

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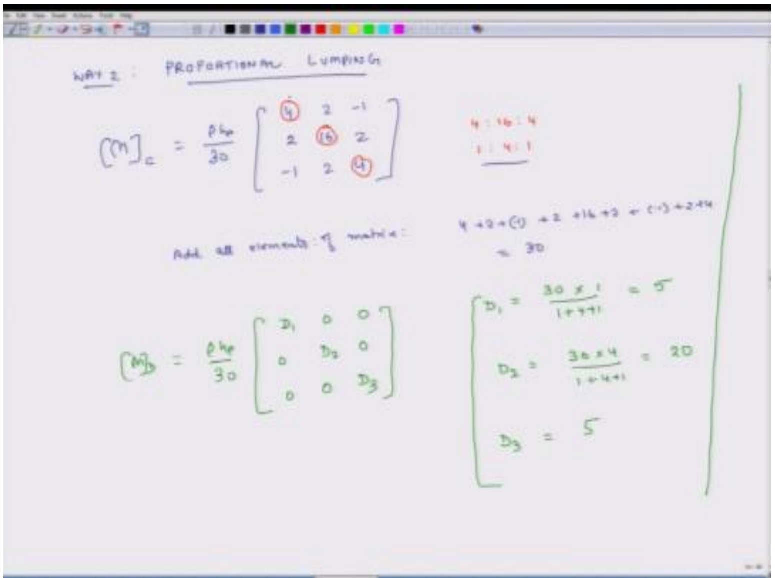

Gives us the same values but this may not necessarily be always true, it may not be always true but in these two met ways we can do the diagonalization, it is important to note that when we do proportional lumping.

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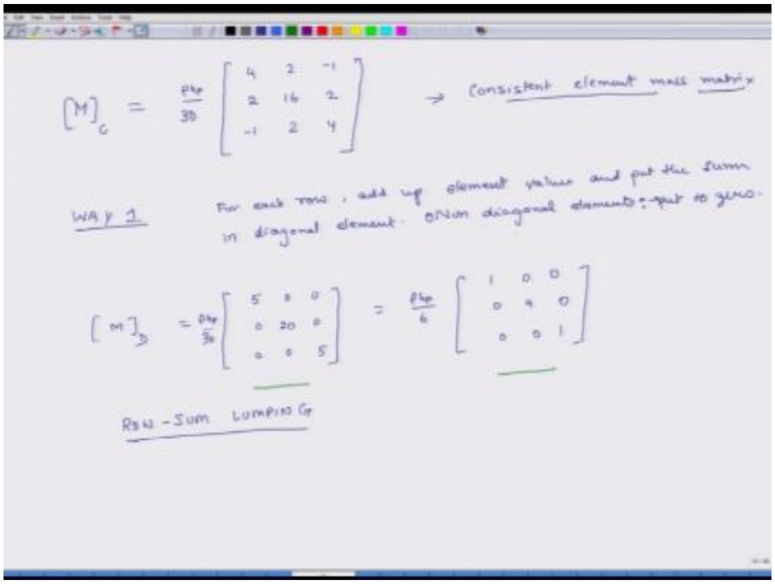

Or row sum lumping we have to make sure that.

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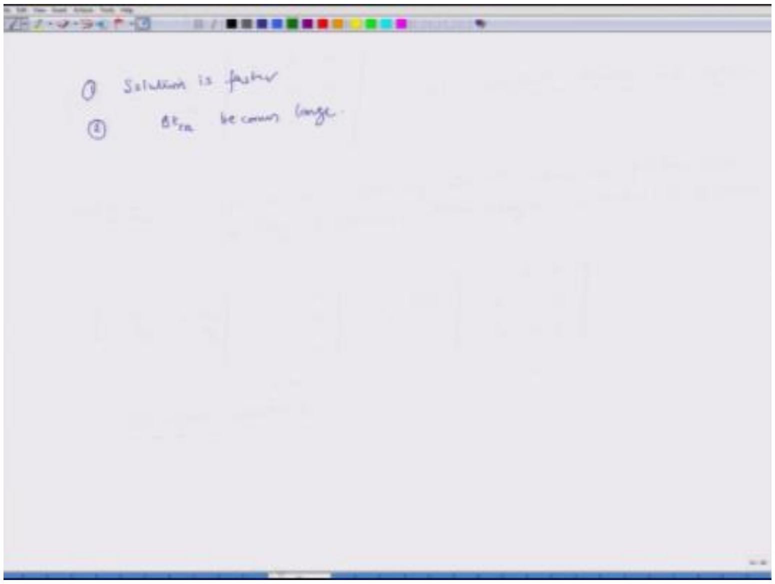

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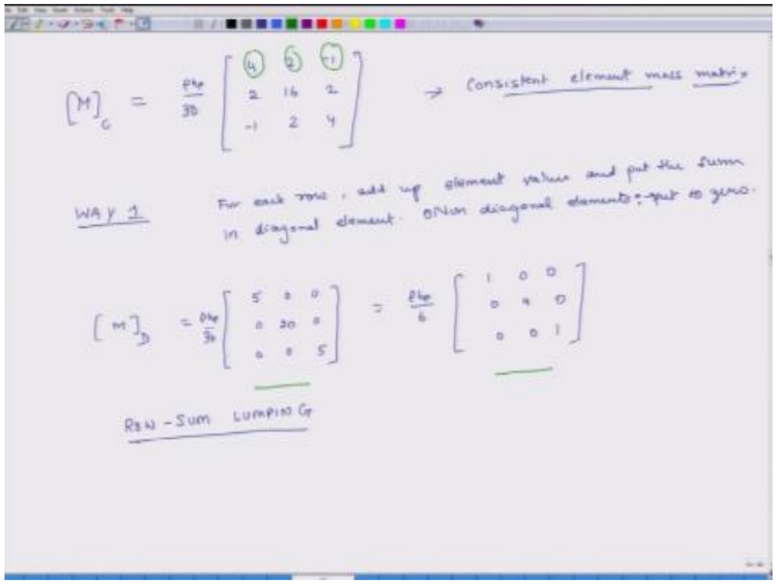

See this mass matrix is multiplied by a vector of unknowns now it just happens that if we are solving a 1D problem and I have only one single primary variable let us say U then does not matter, but in beams there are two primary variables W and W prime okay then we cannot add these terms in a straightforward way because the first term will correspond to the displacement, the second term will respond to the slope, third term, so then we have to when especially.

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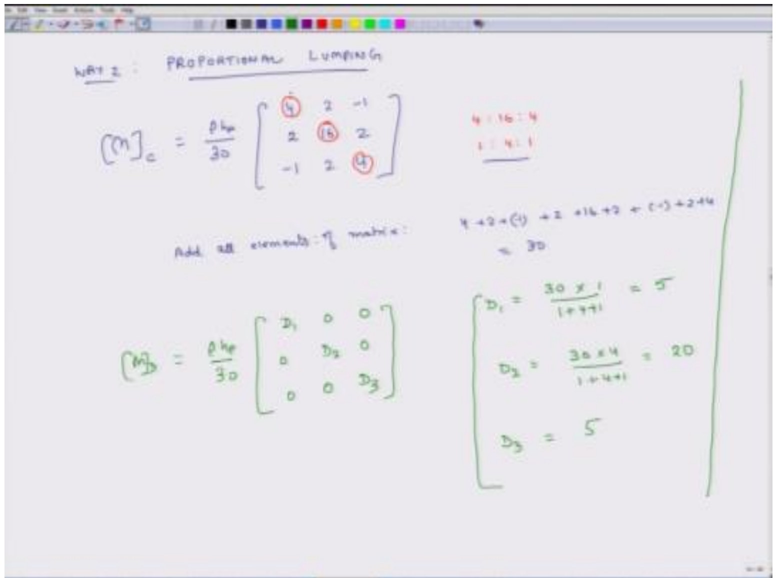

When we are doing proportional lumping we have to separately find out the mass and the moment of inertia terms separately and then do proportional lumping accordingly understood, the row is also we have to make sure of that thing. So I think this concludes our discussion for this course, this has been an eight week long course and we started this course by making introductory remarks, we explained the philosophy of finite element method, we explained what are nodes, shape functions, our approximation functions, elements, we figured out how to calculate these approximation functions for different situations.

We also learned stuff about residual approaches, variational methods, weighted residual methods and then how from weighted residual method we come down to weak formulation strategies to solve differential equations and, and then we started applying some of these methods in context of beams, bars, heat conduction problems, and once we were became comfortable with static problems then we started working on Eigen value problems and time-dependent problems, so this concludes our discussion of this course.

And the course comes to an end. I am sure that all of you who have been patiently listening to these eight week long lectures you have benefited from this course and so your exams will be due on 20th and 27th of March so please prepare for them, all these exams both of these exams, so you will be expected to take only one exam but there will be two dates on which this exam is going to happen, so these exams will be multiple choice questions but to solve these multiple choice questions you will be expected to do detailed calculations, so it will not be just so you have to review your FEA course in detail, practice it, and I am sure that you will do fine and if you have any questions please send us emails or communicate to us and we will be happy to address any of your questions. Once again thank you very much patiently being a part of this course and have a great day and best wishes for future, thank you.

Acknowledgement
Ministry of Human Resource & Development

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Co-ordinator, NPTEL IIT Kanpur

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Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta
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