Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 46
Temporal approximation for parabolic problems: Part-11
by
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Hello again, welcome to basics of finite element analysis, today is the fourth day of this week, yesterday what we had discussed was temporal approximation approach which will help us convert an ordinary differential equation into a set of algebraic equations and in that context we had discussed alpha family of approximations which help us convert derivative in time into differences of time, so today what will we do is we will use that approach to convert the ordinary differential equations which we had developed earlier which are these

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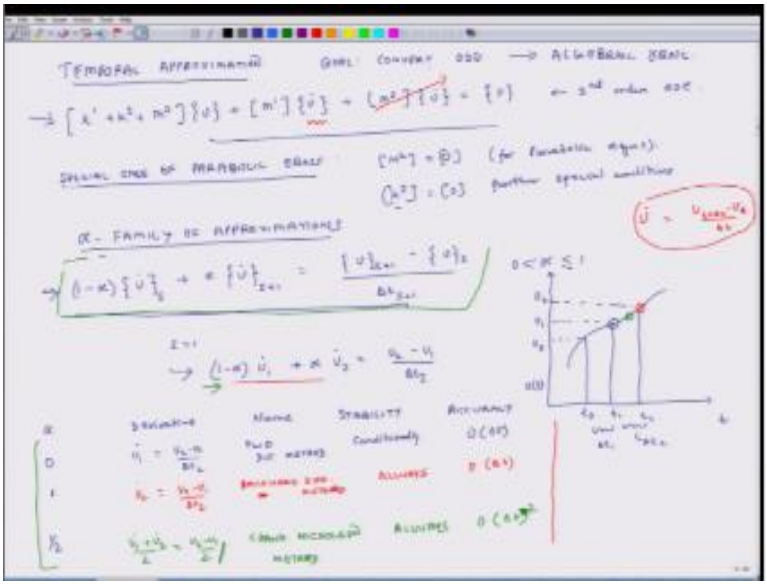

Especially in context of parabolic equations so that is.

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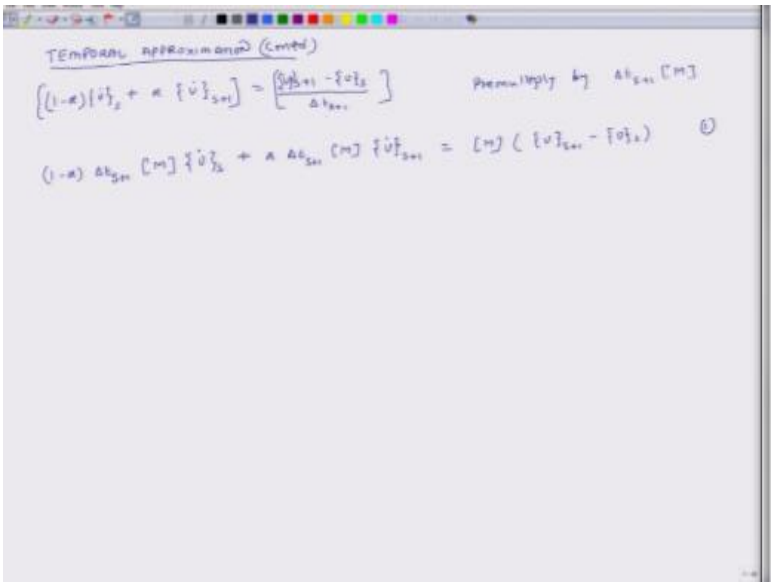

Basically temporal approximation so we are continuing this discussion in today's lecture. α family of approximations tell us that one minus α times velocity at s time step minus α times v time derivative at step number s+1 equals U_{s+1} minus u divided by time step s number s+1. Now what we do is we multiply both sides re-multiply both sides by Δts+1 times n matrix, so what we get is 1minusα Δ_{ts+1} times mass matrix M times U at time step number S minus this is actually this should be plus, so I get a plus sign here plus α times $\Delta_{ts+1'th}$ time step multiplied by mass matrix M times derivative of time at time step S+1, so this is what we have done on the right side and we do the same thing on the left side and this is what we have done on the left side with the same thing on the right side so Δ_{ts+1} cancels away and all what we have is mass metrics $\{U\}_{s+1}$ minus{U} at time step number S so this is equation one, now we go back

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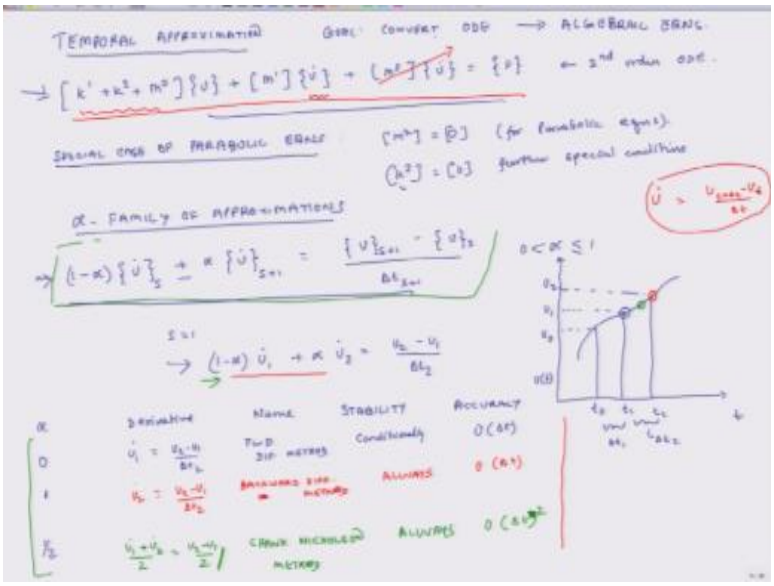

And look at this equation okay and we have already said that for R case M^2 is 0 and also we have said that K^2 is 0 so what we can consider it as that this entire term is one single stiffness matrix we call it K so K is equal toK^{1+} M okay, and m^1 we call it as m matrix so basically it is k times u+ m times u dot equals f that we got, so we will use that terminology

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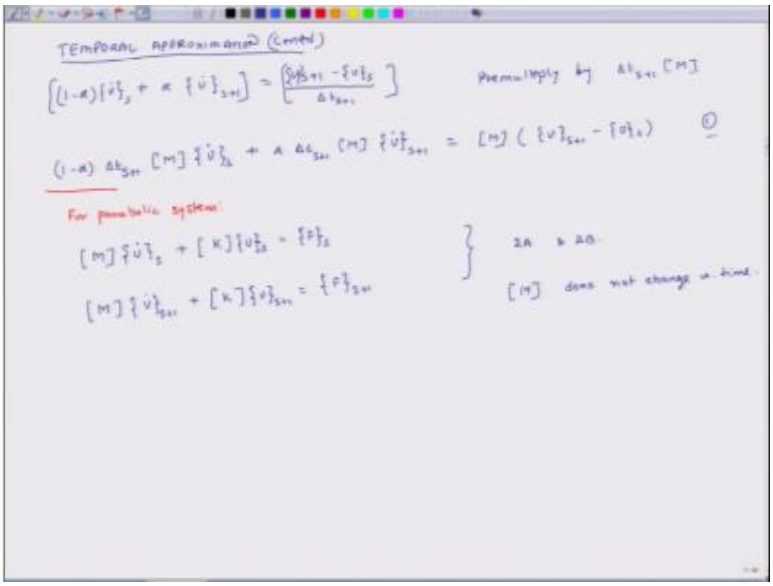

So for parabolic system the equations which we had developed earlier they can be reduced to [M] time {U} dot

And these equations are valid at all time steps so first I will write the equation at time step number s so this is equal to [M]{U}dot +{K} times {U} at s'th time step is equal to F at s'th time step and then the equation is also true to the next time step so I will write that also [M] times {U} dot a time step number s plus one plus [K] times {U} at time step number s plus one equals {F} a time step number s plus one, please remember that these equations are at.

Element level we are not done the assembly of this equations so we have to develop these equations for each element at each time step okay, so we call this equations 2A and 2B so that is equation one this is 2A and 2B. The other thing is we note that this [M] so here we are assuming that [M] does not change with time, does not change with time, that is what we are assuming because typically in lot of system [M] represents the mass so the mass of the system does not change each element level also it does not change with time, the stiffness may change but mass does not typically change

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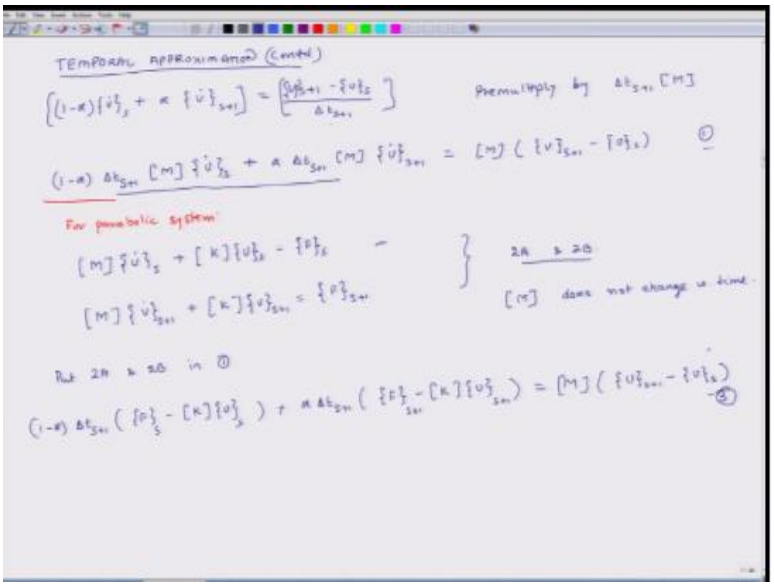

So what we do is that we put equations 2Aand 2B in the LHS which is this side of equation one okay, so in equation one we have [M] times U dot at s and [M] times U dot at s plus one, so we replace [M] U dot at time step s by f_s minus k_{us} from here and we replace [M] times U dot at s plus one by f minus k u at time step s plus one okay. So what we get is so we put 2A and 2B in one and what we get is (1-α) Δt_{s+1} first I will have {F} minus {K} {U} and all these are at s +1+ α Δt_{s+1} ({F} minus [K] { U}) oh it should not be s+1 it should be at time step s here I have time step s +1 and the right side becomes [M] times u S +1 – u s, so that is my equation 3 okay.

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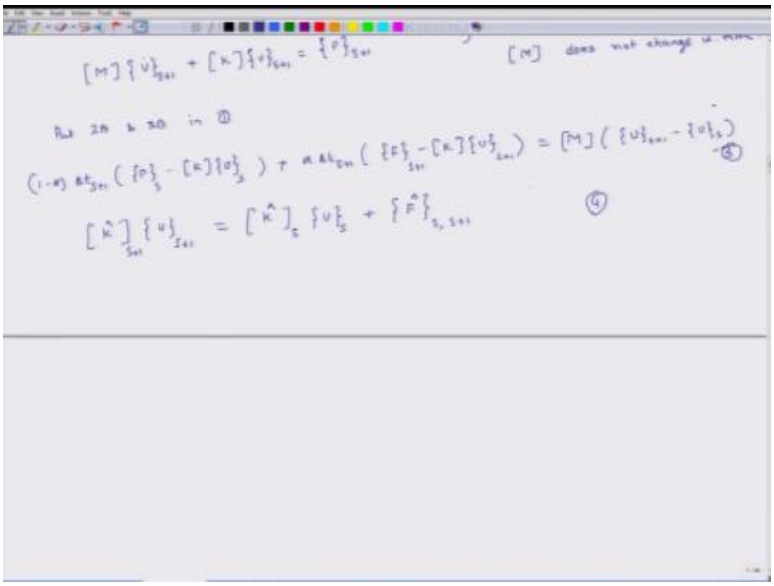

So what is our goal, our goal is to calculate u's at different time steps right, so if I know u at time step s then I should be able to calculate u at time step S+1 okay. So that is my goal I have to calculate times u which is the nodded displacement at time step S + 1 in terms of u add which is the noded displacement at the last time step, so if I in that if I know the initial solution which is a time step t = 0 then if I have this method.

Then I can figure out at time step 1 then in terms of time step 1 I can calculate solution at time step 2 and so on and so forth. So what we will do is that we will collect all the variables at time step 1 on 1 side and all variables at time step S on other side and then we will calculate the solution at time step s + 1 so I can express this entire equation in a compact form and I call it k^, k^ and this k^ is evaluated at time step S+ 1 so here it should be noted that now I M changing k also with respect to time.

And this times primary variable vector at times step S+1 = k^ times evaluated at time step S times u evaluated at times step S+ a force vector and this is called F^, F^ it is different than F and this is evaluated both at time step S and S+1 it dependents on both, so this is equation 4 and this equation 4 is gotten directly from 3 and because.

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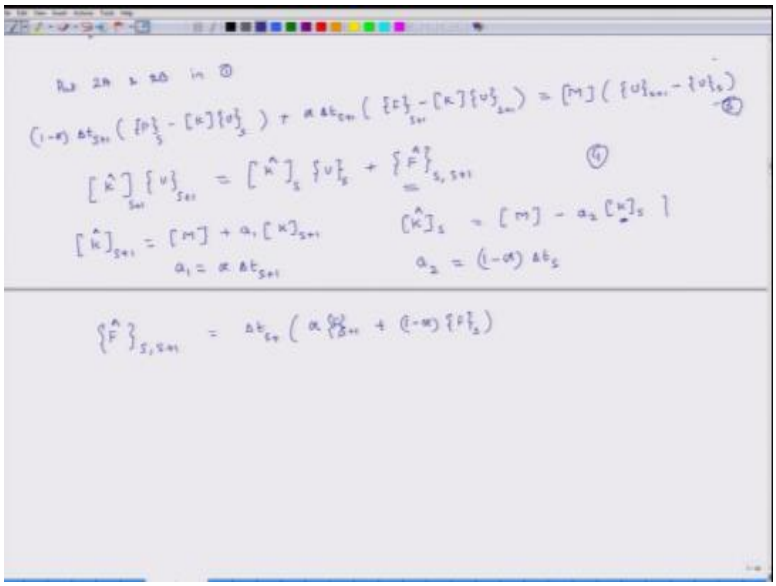

And what is k^, k^ at 9 step S+1 = the mass matrix + a constant a_1 times k of S+1 okay and k^ evaluated time step s it is different it is M – a_2 evaluated at time step s. Now if k is not changing with time then it does not matter but here we have said that K this K matrix not k^ it can change with time if k is changing with time then we have to calculate k^ at s and k^ at S+1 and we have to plug it in and this equation 4 comes directly from 3 if we reorganize it and rearrange these things.

And here a1 =αΔ t that is $_{s+1}{}^{th}$ time step a_2=1- α times Δt that is the s^{th} time step and finally because here we also have this F^ vector so F^ S, S+1=Δ time step S+1 times α F_{S+1} +1- α F_s
Okay so all what we have done how we have gotten all these equation 4 we have just reorganized equation 3 and given some different labels to we have constructed a k matrix which is basically a combination of k^ at s sorry this k, k^ at S+1 is a combination of M and k of S+1 and same thing for k^ at S so this is there.

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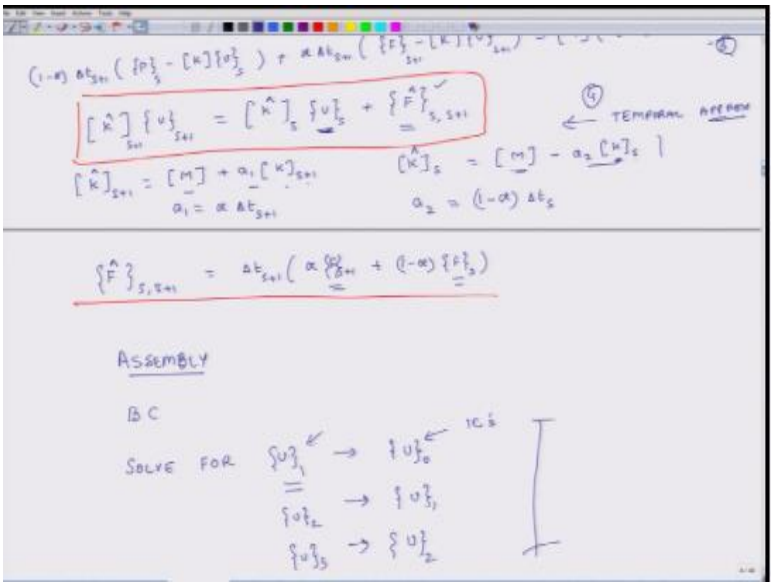

So once we have developed this equation F^ is Δt corresponding to S+1 time step, we know all the time steps how we are breaking up so we know that, we know how is F changing with time so I know F at S+1 and F^ at S so I can calculate f^ at S , S+1, so the so this term can be calculated at all times u at time step S should be known and k^ at s k^ at s is M – a_{2k} , k at S so again k^ at S is also known, k^ at S+1 is again M+a_1 times k S+ 1this is also known so the only thing which is not is u at S +1th time step.

So if we know u at S time step then I can use this equation to calculate u at the next time step S+1th time step okay, so this is the temporal approximation, to the temporal approximation of the of the overall partial differential equation and we are still at the element level but what we have been able to do through this somewhat long process is that we have converted the partial differential equation first into an OD and now into several algebraic equations.

So now the next 2 steps are fairly straight forward what we have to do is we have to do assembly, we have to do assembly, second then we have to apply the boundary conditions and then I start solving for u(S+1) I have 2 then I start solve for u at first time step, for that I have to

know u at 0 otherwise if I do not know this side then I cannot solve, so this information comes through initial conditions okay in the ones I have calculated u1 then I calculate u2 which is at the next time step in terms of u1, then I calculate u3 in terms of u2, so in this way I keep on marching ahead.

In time and at each point of time I calculate the solution in terms of the solution which existed in the last time step so when I am starting the process I have to know what is the solution at the beginning otherwise I cannot solve the for use at the end of first time step so that is why initial conditions become important and what we see is that because our original partial differential equation.

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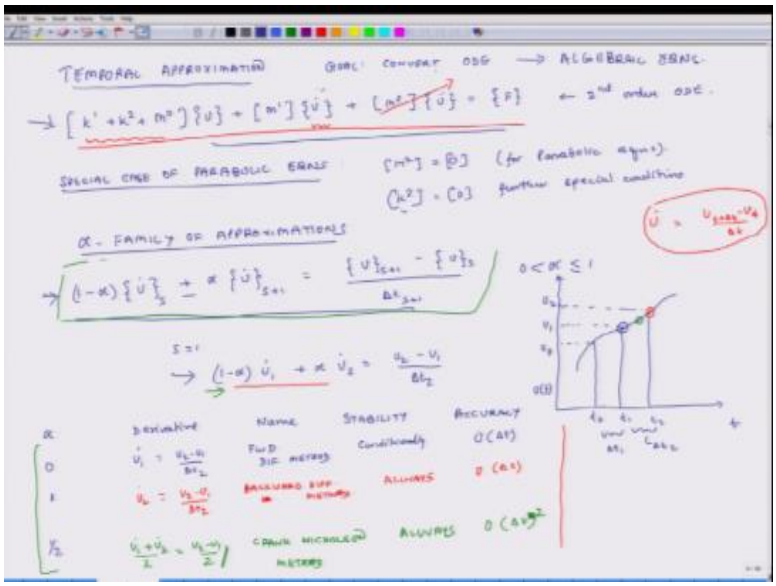

Had only derivative, first derivative in time so we need only.

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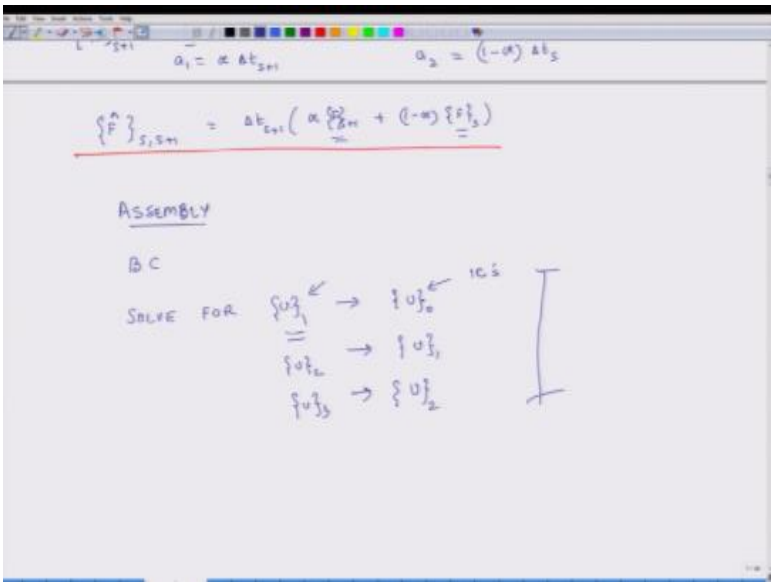

U0 which is one set of initial conditions we do not have, we do not necessarily need to know the velocity and others so we need only one set of initial conditions because the v the original differential equation at only first order derivative in time, so that is the overall process for solving hyperbolic equations and I hope that you would be able to use this and you can read a little bit more about this from the text book which I have discussed, I mentioned earlier or through other texts.

So this completes our discussion on parabolic equations I am sorry not hyperbolic, parabolic equations and tomorrow we will cover and develop a similar method for hyperbolic equations, so that concludes our discussion for today and look forward to seeing you all tomorrow, thanks.

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